2024 Fall Math 140 Week-In-Review

Week 13: Final Exam Review

Disclaimer: **This is by no means a comprehensive exam review. These problems do not cover all topics or all the ways in which the topics covered could be asked.**

1. For the given matrices, perform the indicated operations, if possible.

$$A = \begin{bmatrix} 1 & 2 \\ a & b \end{bmatrix} \qquad B = \begin{bmatrix} -1 & x & 5 \\ y & 3 & -2 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 1 & z \end{bmatrix}$$

(a) $3A - 2B^T$ subtracting 2 matrices, must be exact same size

$3A \Rightarrow 3(2\times2) \to 2\times2$ & $2B^T \Rightarrow 2(2\times3)^T \Rightarrow 2(3\times2) \Rightarrow 3\times2$

$(2\times2) - (3\times2)$

$\boxed{\text{can't happen}}$

(b) $4\boxed{CA}$ ← main thing is $C \cdot A \longrightarrow (3\times2)\cdot(2\times2) \longrightarrow 3\times2$

must match

$$4D = \begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 1 & z \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ a & b \end{bmatrix}$$

☆ rows from 1st matrix & columns from 2nd matrix ☆

$$4D = \begin{bmatrix} d_{1,1} & d_{1,2} \\ d_{2,1} & d_{2,2} \\ d_{3,1} & d_{3,2} \end{bmatrix}$$

$d_{1,1} = (1)(1) + (-1)(a) = 1 - a$ $\quad d_{1,2} = (1)(2) + (-1)(b) = 2 - b$

$d_{2,1} = (-2)(1) + (2)(a) = -2 + 2a$ $\quad d_{2,2} = (-2)(2) + (2)(b) = -4 + 2b$

$d_{3,1} = (1)(1) + (z)(a) = 1 + az$ $\quad d_{3,2} = (1)(2) + (z)(b) = 2 + bz$

$$D = \begin{bmatrix} 1-a & 2-b \\ -2+2a & -4+2b \\ 1+az & 2+bz \end{bmatrix} \rightarrow 4D = \begin{bmatrix} 4(1-a) & 4(2-b) \\ 4(-2+2a) & 4(-4+2b) \\ 4(1+az) & 4(2+bz) \end{bmatrix} = \boxed{\begin{bmatrix} 4-4a & 8-4b \\ -8+8a & -16-8b \\ 4+4az & 8+4bz \end{bmatrix}}$$

one matrix = one matrix

2. Use the given $\boxed{\text{matrix equation}}$ to solve for each variable.

$$\underbrace{5\begin{bmatrix}(3-x) & 2\\ 9y & -4\end{bmatrix}+6\begin{bmatrix}-1 & 4\\ 0 & 3z\end{bmatrix}}_{\text{simplify}}=\begin{bmatrix}11 & 34\\ -8 & -7\end{bmatrix}$$

$$\begin{bmatrix}(15-5x) & 10\\ 45y & -20\end{bmatrix}+\begin{bmatrix}-6 & 24\\ 0 & 18z\end{bmatrix}=\begin{bmatrix}11 & 34\\ -8 & -7\end{bmatrix}$$

$$\begin{bmatrix}15-5x+(-6) & 10+24\\ 45y+0 & -20+18z\end{bmatrix}=\begin{bmatrix}11 & 34\\ -8 & -7\end{bmatrix}$$

$9-5x=11$

$-5x=2$

$\boxed{x=-2/5}$

$34=34$ ✓

$45y=-8$

$\boxed{y=\frac{-8}{45}}$

$-20+18z=-7$

$18z=13$

$\boxed{z=13/18}$

3. The points $(-4,-4)$ and $(2,7)$ form a line. Determine the equation of the line in point-slope form and standard form. Leave all numbers in exact form.

① $m=\dfrac{y_2-y_1}{x_2-x_1}=\dfrac{7-(-4)}{2-(-4)}=\dfrac{11}{6}$

② Point-Slope: $y-y_1=m(x-x_1)$ where (x_1,y_1) is any point on line

Slope-Intercept: $y=mx+b$ where $(0,b)$ is the y-int

Standard: $Ax+By=C$ where $A, B, \& C$ are whole #s

Point-Slope: $y-(-4)=\frac{11}{6}(x-(-4))$ OR $\left(y-7=\frac{11}{6}(x-2)\right)\cdot 6$

$6y-42=11(x-2)$

$6y-42=11x-22$

Standard: $-11x+6y=20$

$V(t)=mt+(\text{purchase price})$ $t=\text{time (years)}$

$(y=mx+b)$ $V=$"value" or worth ($)

$V(t) = mt + (\text{purchase price})$ t = time (years)

$(y = mx + b)$ V = "value" or worth (\$)

4. A particular item depreciates at a rate of \$118.27 per year. Two years after purchase, the item's value is \$1,488.46.

aka "this is the slope"

$(x, y) \to (2, 1488.46)$

$m = -118.27$

so $V(t) = -118.27t + b$

(a) Determine the purchase price of the item.

$1488.46 = -118.27(2) + b$

$1488.46 + 118.27(2) = b$

$b = 1725$

$\boxed{\text{the purchase price is } \$1{,}725}$

(b) If the scrap value of the item is \$665, how long will it take for the item to reach its scrap value? Round your answer to the nearest tenth of a year.

one decimal place

lowest \$ amount the item can be worth

Set $V(t)$ = scrap value & solve for t

$$V(t) = -118.27t + 1725 = 665$$
$$-1725 \qquad -1725$$
$$-118.27t = -1060$$

$$t = \underbrace{\frac{-1060}{-118.27}}_{\text{(exact)}} \approx 8.9625\ldots \to \boxed{t \approx 9.0 \text{ years}}$$

5. A business produces and sells Product Zeta. When the company produces 15 units of the product, the total cost is \$7,143.75 and when the company produces 25 units, the total cost is \$7,406.25. When the company sells 42 units of the product, the total revenue is \$3,507. Determine linear functions representing the total production cost, $C(x)$, and the total revenue, $R(x)$, for producing and selling x units of Product Zeta.

cost: ☆ $(15, 7143.75)$, $(25, 7406.25)$ — we make a cost line out of this

Cost = (cost per item)$\cdot x$ + (fixed cost)

$$m = \frac{7406.25 - 7143.75}{25 - 15} = 26.25$$

$C(x) = 26.25x + b$

$7143.75 = 26.25(15) + b$

$b = 6{,}750$

$\boxed{C(x) = 26.25x + 6750}$

revenue: $(42, 3507)$

Revenue = (selling price per item) $\cdot x$

$3507 = (\text{price})(42)$

$$\text{selling price} = \frac{3507}{42} = 83.5$$

$\boxed{R(x) = 83.5x}$

6. The cost and revenue functions for x units of particular product are given by $C(x) = 122x + 11,117.25$ and $R(x) = 213.5x$, respectively. Determine the profit function, $P(x)$, for the product and discuss any break-even points.

★ Profit = (Revenue) − (Cost) ★

★ don't forget to distribute this negative thru the cost function

$P(x) = R(x) - C(x) = (213.5x) - (122x + 11,117.25)$

$$\boxed{P(x) = 91.5x - 11,117.25}$$

Breakeven:

1. Revenue = Cost
2. Profit = 0

Break-even: $P(x) = 91.5x - 11,117.25 = 0$

$91.5x = 11,117.25$

$x = \frac{11,117.25}{91.5} = 121.5$ # of items

★ unless otherwise specified, we assume x-values must be whole #s ★

There is no "true" break-even point

7. For Item Z, the market price-demand function is given by $D(x) = p(x) = -0.01x + 12.15$. For the same item, producers are willing to supply 80 items at a price of \$4.35 each and are willing to supply 126 items at a price of \$5.50 each. Determine any equilibrium points for Item Z.

$D(x) = -0.01x + 12.15$

Supply: $(80, 4.35)$ & $(126, 5.5)$

$m = \frac{5.5 - 4.35}{126 - 80} = 0.025$

$S(x) = 0.025x + b$

$4.35 = 0.025(80) + b$

$b = 2.35$

$S(x) = 0.025x + 2.35$

supply = demand

$S(x) = D(x)$

$0.025x + 2.35 = -0.01x + 12.15$

$0.035x = 9.8$

$x = \frac{9.8}{0.035} = 280$ items

we plug-in to either $D(x)$ or $S(x)$ to get the y-value

$D(280) = -0.01(280) + 12.15$

$= \$9.35$

$$\boxed{(280, 9.35)}$$

8. Determine the solution(s), if any, for the given system of equations. If the case of a parametric solution, use t as the parameter.

(i) substitution
(ii) addition/elimination
(iii) rref in calculator

$$8x + 6y = -12$$
$$3y = -4x + \frac{3}{2}$$

Substitution: solve for one variable in one equation, then plug that in to the other equation

$$8x+6y=-12$$
$$6y=-8x-12$$
$$y=-\frac{8}{6}x-\frac{12}{6}$$
$$y=-\frac{4}{3}x-2$$

$$3\left(-\frac{4}{3}x-2\right)=-4x+\frac{3}{2}$$
$$\cancel{-4x}-6=\cancel{-4x}+\frac{3}{2}$$
$$+4x \qquad +4x$$
$$-6=\frac{3}{2} \quad \text{(always false)}$$

$\boxed{\text{no solution}}$

Elimination: multiply one or both equations by a constant so that the x or y variables then cancel (equations must be in same order)

$$8x+6y=-12$$
$$-2\cdot\left(4x+3y=\frac{3}{2}\right)$$

$$\cancel{8x}+\cancel{6y}=-12$$
$$+\ \cancel{-8x}-\cancel{6y}=-3$$
$$0=-15 \quad \text{(always false)}$$

$\boxed{\text{no solution}}$

RREF: all variables on left & constants on right

$$8x+6y=-12$$
$$4x+3y=3/2$$

augmented matrix

$$\left[\begin{array}{cc|c} 8 & 6 & -12 \\ 4 & 3 & 3/2 \end{array}\right] \xrightarrow{\text{RREF}} \begin{array}{ccc} x & y & \text{constant} \\ \hline \end{array} \left[\begin{array}{cc|c} 1 & 0.75 & 0 \\ 0 & 0 & 1 \end{array}\right]$$

2nd → X^{-1} (Matrix) → Edit

enter the matrix

Quit to HomeScreen

2nd → X^{-1} (matrix)

→ Math

→ RREF

→ RREF(A)

gross decimals?

Math → Enter → Enter

$x+0.75y=0$ & $0x+0y=1$

always false

$\boxed{\text{no solution}}$

9. Determine the solutions(s), if any, for the given system of equations. If the case of a parametric solution, use t as the parameter.

★ 3 equation system w/ 3 variables, RREF is the only viable option

$$\left.\begin{aligned} 2x + 3z &= 30 - 5y \\ 0.6z - 6 &= -0.4x - y \\ 6x + z + 11 &= 7y \end{aligned}\right\} \to \begin{aligned} 2x + 5y + 3z &= 30 \\ 0.4x + y + 0.6z &= 6 \\ 6x - 7y + z &= -11 \end{aligned}$$

$$\begin{array}{cccc} x & y & z & \text{constant} \\ \end{array}$$

$$\left[\begin{array}{ccc|c} 2 & 5 & 3 & 30 \\ 0.4 & 1 & 0.6 & 6 \\ 6 & -7 & 1 & -11 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{ccc|c} 1 & 0 & 13/22 & 155/44 \\ 0 & 1 & 4/11 & 101/22 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x + \frac{13}{22}z = \frac{155}{44}$ $\qquad$ $y + \frac{4}{11}z = \frac{101}{22}$ $\qquad$ $0 = 0$ (always true)

let t be the variable in both equations

good indicator we probably have ∞ solution

$t = z$ $\qquad$ $x = \frac{155}{44} - \frac{13}{22}t$ $\qquad$ $y = \frac{101}{22} - \frac{4}{11}t$

$$\boxed{(x, y, z) \to \left(\frac{155}{44} - \frac{13}{22}t,\ \frac{101}{22} - \frac{4}{11}t,\ t\right)}$$

10. Graph the given inequality.

$$3x + 5y > -10$$

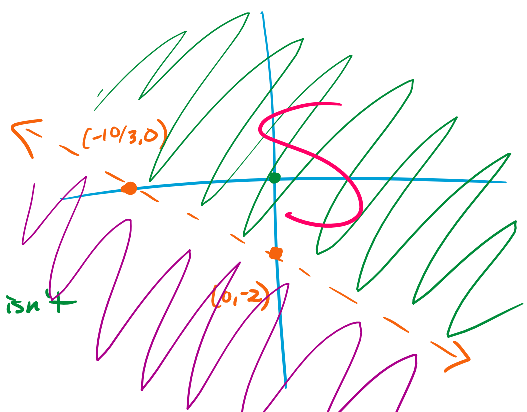
easiest way is to find x & y intercepts

$\underline{x=0}$: $5y = -10$ $\qquad$ $(0, -2)$

$y = -2$

$\underline{y=0}$: $3x = -10$ $\qquad$ $(-10/3, 0)$

$x = -10/3$



pick a test point (choose (0,0) if it isn't on the line)

$3(0) + 5(0) > -10$

$0 > -10$ (true)

= true shading

= false shading

11. For the given system of inequalities, determine the solutions set and its corner points, the point(s) where $F = 3x + 6y$ is minimized, and the point(s) where $G = 10x + 8y$ is maximized.

$-80 \geq -5x - 4y \rightarrow$ Line 1: $-80 = -5x - 4y$

$x \geq 28 - 2y \rightarrow$ Line 2: $x = 28 - 2y$

$x \geq 0,\ y \geq 0 \rightarrow$ Line 3: $x = 0$ → Line 4: $y = 0$

L1: $x=0$: $-80 = -4y$, $20 = y$, $(0,20)$

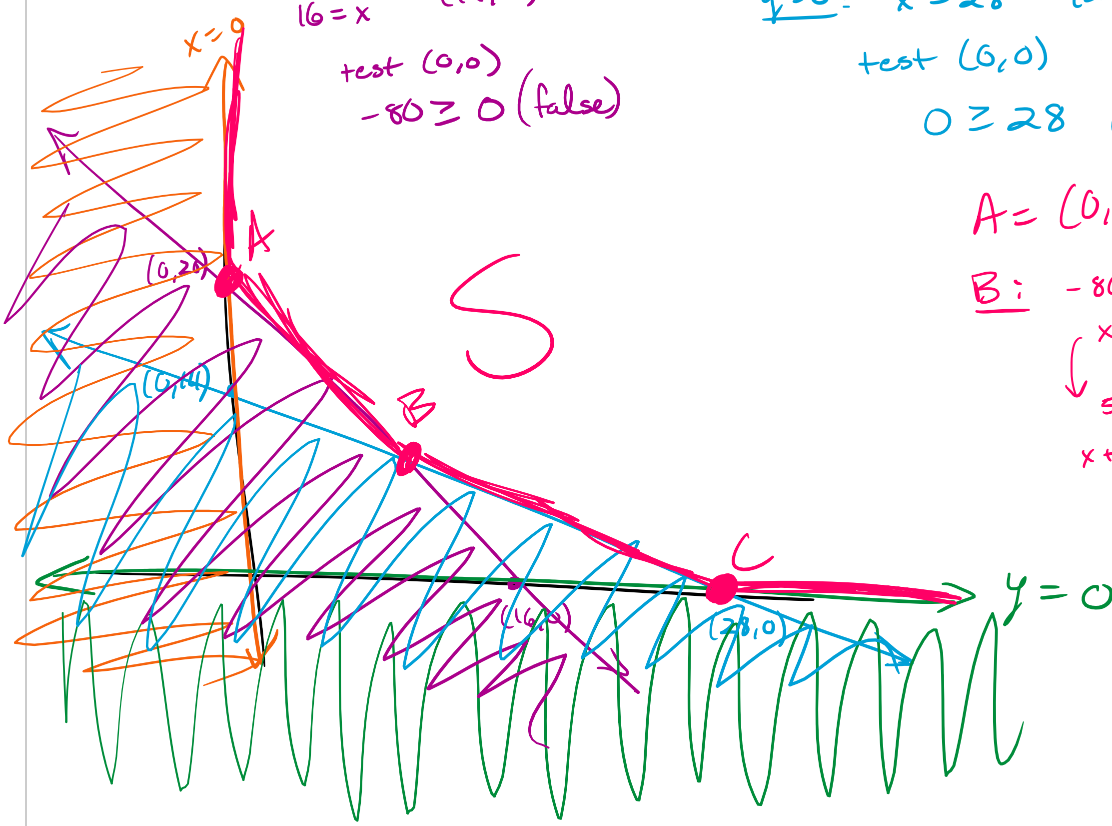
$y=0$: $-80 = -5x$, $16 = x$, $(16,0)$

test $(0,0)$: $-80 \geq 0$ (false)

L2: $x=0$: $0 = 28-2y$, $2y = 28$, $y = 14$, $(0,14)$

$y=0$: $x = 28$, $(28,0)$

test $(0,0)$: $0 \geq 28$ (false)



$A = (0,20)$ $C = (28,0)$

B: $-80 = -5x - 4y$
$x = 28 - 2y$

$5x + 4y = 80$
$x + 2y = 28$ $\rightarrow \left[\begin{array}{cc|c} 5 & 4 & 80 \\ 1 & 2 & 28 \end{array}\right]$

rref

$\left[\begin{array}{cc|c} 1 & 0 & 8 \\ 0 & 1 & 10 \end{array}\right]$

$B = (8,10)$

Points	(min) $F=3x+6y$	(max) $G=10x+8y$
$(0,20)$	$F=120$	
$(8,10)$	$F=84$	
$(28,0)$	$F=84$	

there is no max b/c S is unbounded in the first quadrant

both give a minimum

we need the line segment that connects them: $x = 28-2y$, $x - 28 = -2y$ → $\boxed{-\frac{1}{2}x + 14 = y \text{ (line) from } 8 \leq x \leq 28}$

12. For the following standard maximization problem, rewrite all inequalities as equations using slack variables and set up the initial simplex tableau.

(all variables on left) $\leq$ (0 or positive #)

Maximize: $H = 0.13x + 0.08y + 0.11z$

Subject To:

$3x + 9y - 2z \leq 455$

$-1\cdot(4x - 4y - 4z \geq -380)$ $\quad -4x + 4y + 4z \leq 380$

$2x - y + 3z \leq 420$

$x \geq 0,\ y \geq 0,\ z \geq 0$

$3x + 9y - 2z + s_1 = 455$

$-4x + 4y + 4z + s_2 = 380$

$2x - y + 3z + s_3 = 420$

$-0.13x - 0.08y - 0.11z + H = 0$

$\Rightarrow$

$$
\begin{array}{cccccccc}
x & y & z & s_1 & s_2 & s_3 & H & \text{constant} \\
\end{array}
$$
$$
\left[\begin{array}{ccccccc|c}
3 & 9 & -2 & 1 & 0 & 0 & 0 & 455 \\
-4 & 4 & 4 & 0 & 1 & 0 & 0 & 380 \\
2 & -1 & 3 & 0 & 0 & 1 & 0 & 420 \\
\hline
-0.13 & -0.08 & -0.11 & 0 & 0 & 0 & 1 & 0
\end{array}\right]
$$

13. For the given simplex tableau, determine any basic or non-basic variables, the corner point, the solution, and if it is the optimal solution. If it is not the optimal solution, determine the next pivot element.

basic: x, s_2, s_3, V

non-basic: y, z, s_1

(automatically assigned a value of zero)

$$
\begin{array}{cccccccc}
x & y & z & s_1 & s_2 & s_3 & V & \\
\end{array}
$$
$$
\left[\begin{array}{ccccccc|c}
1 & 3 & -\frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & \frac{455}{3} \\
0 & 16 & \frac{4}{3} & \frac{4}{3} & 1 & 0 & 0 & \frac{2960}{3} \\
0 & -7 & \frac{13}{3} & -\frac{2}{3} & 0 & 1 & 0 & \frac{350}{3} \\
\hline
0 & \frac{31}{100} & -\frac{59}{300} & \frac{13}{300} & 0 & 0 & 1 & \frac{1183}{60}
\end{array}\right]
$$

~~$455/3 \div -\frac{2}{3}$~~ no negative allowed

$2960/3 \div \frac{4}{3} \rightarrow 740$

$350/3 \div \frac{13}{3} \rightarrow 26.92\ldots$

corner point: $(x, y, z) = (455/3, 0, 0)$

solution: $V = 1183/60$

Optimal: Not optimal b/c there is still a negative # in the bottom row

Pivot:
1. biggest negative in bottom row
   - column 3
2. ratios: constant / entry in that column
3. pick smallest ration
   - row 3

$\boxed{\text{pivot on } 13/3 \text{ in column 3, row 3}}$

14. An experiment consists of drawing a marble out of a bag and recording the color (B=blue, G=green, and Y=yellow), then flipping a coin and recording the results (H=heads, T=tails).

★ 2-stage experiment → every outcome has 2 parts

(a) Write the sample space for the experiment. What are the total number of events for this experiment?

one single outcome

$$S = \{ \underbrace{(B,H), (B,T), (G,H), (G,T), (Y,H), (Y,T)}_{n=6 \text{ simple events}} \}$$

$$\text{total \# events} = 2^n = 2^6 = \boxed{64 \text{ total events}}$$

(b) If there 3 blue marbles, 2 green marbles, and 6 yellow marbles, what is the probability that the marble drawn is not yellow?

$$\star \ \text{Probability} = \frac{\text{\# favorable outcomes}}{\text{total \# outcomes}}$$

$$P(Y^c) = \frac{\text{\# blue} + \text{\# green}}{\text{total \# marbles}} = \frac{3+2}{3+2+6} = \boxed{\frac{5}{11}}$$

(c) Using the symbols above, write the symbolic notation of the event: "$\boxed{\text{neither}}$ a blue or yellow marble is drawn, but the coin lands on tails".

$\cup$

B Y

complement

T

and

↓

$\cap$

$$(B \cup Y)^c \to B^c \cap Y^c$$

$$\boxed{\begin{array}{c} (B \cup Y)^c \cap T \\ \text{OR} \\ B^c \cap Y^c \cap T \end{array}}$$

15. Let J and K be two events of a sample space S. If $P(J \cap K^C) = 0.24$ and $P(J^C) = 0.48$, determine the following:

Venn Diagram $\quad P(K) = .44$

$J = \{x, y\} \quad K = \{y, w\}$

$J^c = \{w, z\} \quad K^c = \{x, z\}$

$J \cap K^c = \{x\} \rightarrow x = 0.24$

$J^c = \{w, z\} \rightarrow w + z = 0.48$

$x + y + z + w = 1$

$\star P(J^c) = .48 \rightarrow P(J) = 1 - .48 = .52$

$x + y = .52$

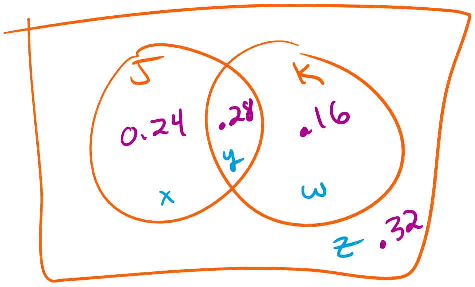
$.24 + y = .52$

$.28 = y$

$P(K) = .44 = y + w$

$.44 = .28 + w$

$.16 = w$



(a) $\underbrace{P(J \cap K)}$

$J \cap K = \{y\} \rightarrow \boxed{P(y) = .28}$

(b) $\boxed{P(K) = .44}$

(c) $P(\underbrace{J^C \cap K^C})$

$J^c \cap K^c = \{z\}$

$\boxed{P(z) = .32}$

16. For the sample space $S = \{x_1, x_2, x_3, x_4, x_5\}$, it is known that $P(x_1) = \frac{3}{17}$, $P(x_2) = \frac{2}{17}$, $P(x_4) = \frac{7}{17}$, and $P(x_5) = \frac{1}{17}$.

(a) Construct a probability distribution for S.

X	x_1	x_2	x_3	x_4	x_5
$P(X)$	$3/17$	$2/17$	$4/17$	$7/17$	$1/17$

all must add to 1

(b) For the events $A = \{x_1, x_4\}$ and $B = \{x_1, x_3, x_5\}$, determine $P(A^C \cap B)$

$A^c = \{x_2, x_3, x_5\} \cap B = \{x_1, x_3, x_5\}$

$= \{x_3, x_5\}$

$P(A^c \cap B) = P(x_3) + P(x_5) = \frac{4}{17} + \frac{1}{17} = \boxed{\frac{5}{17}}$

(c) What is the expected value for the sample space?

$E(X) = (x_1)(P(x_1)) + (x_2)(P(x_2)) + \cdots$

$\boxed{E(X) = (x_1)(3/17) + (x_2)(2/17) + (x_3)(4/17) + (x_4)(7/17) + (x_5)(1/17)}$

17. A game consists of rolling a standard fair 4-sided die and then a standard fair 5-sided die. The game costs \$4 to play. If you roll a sum of 9, you win \$9. If you roll a double, you win \$8. If you roll a sum of 5 you win \$5. With any other roll you don't win.

	1	2	3	4	5
1	1,1	1,2	1,3	1,4	1,5
2	2,1	2,2	2,3	2,4	2,5
3	3,1	3,2	3,3	3,4	3,5
4	4,1	4,2	4,3	4,4	$\boxed{4,5}$

roll 9 → $\frac{1}{20}$

double → $\frac{4}{20}$

roll 5 → $\frac{4}{20}$

any thing else → $\frac{11}{20}$

(a) Construct a probability distribution for the game where X is the net winnings.

↳ (winnings) − (cost to play)

	\$9 − \$4	\$8 − \$4	\$5 − \$4	\$0 − \$4
X	\$5	\$4	\$1	−\$4
P(X)	1/20	4/20	4/20	11/20

(b) What is the $\boxed{\text{expected}}$ net winnings for the player in this game? Is this a fair game?

$$E(X) = (5)(1/20) + (4)(4/20) + (1)(4/20) + (-4)(11/20)$$

$$E(X) = -0.95$$

$\boxed{\text{the expected net winnings are } -\$0.95 \text{ and it is not fair b/c } E(X) \neq 0}$

18. For the given polynomial functions, determine the domain, degree, leading coefficient, constant term, and end-behavior.

only 3 restriction:
① denominator ($\neq 0$)
② even root (inside ≥ 0)
③ log (inside > 0)

(a) $f(x) = -10x^3 + 7x^4 - 2x^9 + e^{13}$ just a #

D: $(-\infty, \infty)$

degree: $n = 9$

leading coefficient: $a_n = -2$

constant: e^{13}

end-behavior: odd w/ negative

as $x \to -\infty$, $y \to +\infty$

as $x \to +\infty$, $y \to -\infty$

(b) $g(x) = 3x(x-9)(x-5)^2(x+2)(x+4)$

$x^1 \cdot x^1 \cdot x^2 \cdot x^1 \cdot x^1$

x^6

D: $(-\infty, \infty)$

degree: $n = 6$

leading coefficient: $a_n = 3$

constant: no constant $\to a_0 = 0$

end-behavior: even w/ positive

as $x \to -\infty$, $y \to +\infty$

as $x \to +\infty$, $y \to +\infty$

19. For the given functions, determine the domain, the vertex, the minimum, the maximum, the y-intercept, and any real roots/zeros.

domain: $\mathbb{R}$ b/c no restrictions

vertex: $x = \frac{-b}{2a}$ then $y = f\left(\frac{-b}{2a}\right)$

minimum, the maximum: open up/down?

real roots/zeros: x-intercepts

(a) $f(x) = 5x^2 + 8x - 3$

D: $(-\infty, \infty)$

$x = \frac{-b}{2a} = \frac{-8}{2(5)} = \frac{-4}{5}$

$y = f(-4/5) = -31/5$

vertex: $(-4/5, -31/5)$

since $a = 5$, open up

Max: none

Min: $y = -31/5$

$x=0$: $y = f(0) = -3$

$(0, -3)$

roots/zeros: $5x^2 + 8x - 3 = 0$

(factor or quadratic formula)

$x = \frac{-8 \pm \sqrt{8^2 - 4(5)(-3)}}{2(5)}$

$x = \frac{-8 \pm \sqrt{124}}{10}$

$\left(\frac{-8 \pm \sqrt{124}}{10}, 0\right)$

(b) $g(x) = (7 - 2x)(x - 4)$

multiply out

$g(x) = 7x - 28 - 2x^2 + 8x$

$g(x) = -2x^2 + 15x - 28$

D: $(-\infty, \infty)$

$x = -\frac{15}{2(-2)} = \frac{15}{4} \to y = g\left(\frac{15}{4}\right) = \frac{1}{8}$

vertex: $\left(\frac{15}{4}, \frac{1}{8}\right)$

since $a = -2$, open down

Max: $y = 1/8$

Min: None

$x=0$: $y = g(0) = -28$

$(0, -28)$

roots/zeros:

$(7-2x)(x-4) = 0$

$7 - 2x = 0$, $x - 4 = 0$

$-2x = -7$ $\quad$ $x = 4$

$x = 7/2$

$(7/2, 0)$ $\quad$ $(4, 0)$

20. For the given functions, determine the domain, any intercepts, any holes, and any vertical asymptotes.

(a) $h(x) = \dfrac{x^2 - 25}{2x^2 + 7x - 15} = \dfrac{\cancel{(x+5)}(x-5)}{(2x-3)\cancel{(x+5)}}$

☆ Find domain BEFORE cancelling ☆

$(2x-3)(x+5) \neq 0$

$2x - 3 \neq 0 \qquad x + 5 \neq 0$

$x \neq 3/2 \qquad x \neq -5$

$\dfrac{x-5}{2x-3}$ restrictions still present are VAs

VA: $x = 3/2$

D: $(-\infty, -5) \cup (-5, 3/2) \cup (3/2, \infty)$

$x=0$: $y = \dfrac{0-5}{2(0)-3} = \dfrac{-5}{-3} = 5/3$

$(0, 5/3)$

$y=0$: $\left(\dfrac{x-5}{2x-3} = 0\right) \cdot (2x-3)$

$x - 5 = 0$

$x = 5$

$(5, 0)$

Hole: $(-5, 10/13)$

$y = \dfrac{-5-5}{2(-5)-3} = \dfrac{-10}{-13} = \dfrac{10}{13}$

(b) $k(x) = \dfrac{2\cancel{x}\cancel{(x-3)}(x+7)^2}{-9\cancel{x}(x-3)^{\cancel{2}}(x+1)} \to \dfrac{2(x+7)^2}{-9(x-3)(x+1)}$

$x = 3, x = -1$ are VAs

$-9x(x-3)^2(x+1) \neq 0$

$-9x \neq 0 \qquad (x-3)^2 \neq 0 \qquad x+1 \neq 0$

$x \neq 0 \qquad x \neq 3 \qquad x \neq -1$

so $x = 0$ is a hole

$y = \dfrac{2(0+7)^2}{-9(0-3)(0+1)} = \dfrac{2(7)^2}{(-9)(-3)(1)}$

$y = \dfrac{98}{27}$

D: $(-\infty, -1) \cup (-1, 0) \cup (0, 3) \cup (3, \infty)$

VA @ $x = 3$ & $x = -1$

Hole @ $(0, 98/27)$

no y-intercept b/c $x \neq 0$

$\dfrac{2(x+7)^2}{-9(x-3)(x+1)} = 0 \Rightarrow 2(x+7)^2 = 0$

$x = -7$

$(-7, 0)$

21. Fully simplify the given expression.

☆ factored form ☆

$$2\left(\frac{x^2-9}{x+1}\right)\left(\frac{x}{x+3}\right) - \frac{x(2x+1)}{x-2} \div \frac{x+1}{x-2}$$

$2 \cdot \dfrac{(x-3)\cancel{(x+3)}}{x+1} \cdot \dfrac{x}{\cancel{x+3}} - \dfrac{x(2x+1)}{\cancel{x-2}} \cdot \dfrac{\cancel{x-2}}{x+1}$

$\dfrac{2\overbrace{(x-3)(x)}^{(x^2-3x)}}{x+1} - \dfrac{x(2x+1)}{x+1}$

$\dfrac{2x^2 - 6x}{x+1} - \dfrac{2x^2 + x}{x+1}$

$\dfrac{(2x^2 - 6x) - (2x^2 + x)}{x+1} = \boxed{\dfrac{-7x}{x+1}}$

22. Fully simplify the given expression. Write your answer using only positive exponents.

$$\left(\frac{28x^{-4}y^5}{(2xy)^3}\right)^{-3/2}$$

★ start by simplifying the inside

$$\frac{28x^{-4}y^5}{(2xy)^3} \rightarrow \frac{28x^{-4}y^5}{2^3x^3y^3} = \frac{28x^{-4}y^5}{8x^3y^3} = \frac{28y^{5-3}}{8x^{3-(-4)}}$$

$$= \frac{28y^2}{8x^7} = \frac{7y^2}{2x^7}$$

$$\left(\frac{7y^2}{2x^7}\right)^{-3/2} \rightarrow \left(\frac{2x^7}{7y^2}\right)^{3/2} \rightarrow \frac{2^{3/2}\cdot(x^7)^{3/2}}{7^{3/2}\cdot(y^2)^{3/2}} \rightarrow \frac{\sqrt{2^3}\cdot x^{(7)(3/2)}}{\sqrt{7^3}\cdot y^{(2)(3/2)}}$$

$$\rightarrow \boxed{\frac{\sqrt{8}\cdot x^{21/2}}{\sqrt{343}\cdot y^3}}$$

23. Rewrite the following functions in radical form and determine their domain.

★ $x^{a/b} \rightarrow \sqrt[b]{x^a}$ ★

(a) $f(x) = 2(19-4x)^{7/6}$

$f(x) = 2\cdot\sqrt[6]{(19-4x)^7}$

even root: $(19-4x)^7 \geq 0$

$\sqrt[7]{(19-4x)^7} \geq \sqrt[7]{0}$

$19-4x \geq 0$

$19 \geq 4x$

$\frac{19}{4} \geq x$

D: $\left(-\infty, \frac{19}{4}\right]$

(b) $g(x) = \dfrac{5x}{(7x+2)^{-4/3}}$

$g(x) = 5x\cdot(7x+2)^{4/3}$

$g(x) = 5x\cdot\sqrt[3]{(7x+2)^4}$ b/c odd root → no restrictions

D: $(-\infty, \infty)$

24. Setup and begin to simplify the difference quotient for the given functions.

(a) $f(x) = 2x^2 - 5x + 7$

$$\frac{f(x+h)-f(x)}{h} = \frac{\left(2(x+h)^2-5(x+h)+7\right)-\left(2x^2-5x+7\right)}{h}$$

$$= \frac{2(x^2+2xh+h^2)-5x-5h+7-2x^2+5x-7}{h}$$

$$= \text{------}$$

(b) $g(x) = \dfrac{9}{1-x}$

$$\frac{g(x+h)-g(x)}{h} = \frac{\frac{9}{1-(x+h)}-\frac{9}{1-x}}{h} = \frac{\frac{9}{1-x-h}-\frac{9}{1-x}}{h}\cdot\frac{(1-x-h)(1-x)}{(1-x-h)(1-x)}$$

$$= \frac{\frac{9\cancel{(1-x-h)}(1-x)}{\cancel{1-x-h}}-\frac{9(1-x-h)\cancel{(1-x)}}{\cancel{1-x}}}{h\cdot(1-x-h)(1-x)} = \frac{\overbrace{9(1-x)-9(1-x-h)}^{\text{multiply out \& simplify...}}}{h(1-x-h)(1-x)}$$

$$= \text{-----}$$

(c) $h(x) = \sqrt{8x+3}$

$$\frac{h(x+h)-h(x)}{h} = \frac{\sqrt{8(x+h)+3}-\sqrt{8x+3}}{h}\cdot\frac{\overbrace{\sqrt{8(x+h)+3}+\sqrt{8x+3}}^{\text{conjugate of orange numerator}}}{\sqrt{8(x+h)+3}+\sqrt{8x+3}}$$

$$= \frac{\left(\sqrt{8(x+h)+3}\right)^2-\left(\sqrt{8x+3}\right)^2}{h\left(\sqrt{8(x+h)+3}+\sqrt{8x+3}\right)} = \frac{8(x+h)+3-(8x+3)}{h\left(\sqrt{8(x+h)+3}+\sqrt{8x+3}\right)}$$

$$= \text{-----}$$

25. Rewrite $j(x) = 7|8-3x|+2$ as a piecewise function.

$|8-3x| = 0$

$-3x = -8$

$x = 8/3$ (split value)

$$j(x) = \begin{cases} 7\cdot(8-3x)+2, & x < 8/3 \\ 7\cdot -(8-3x)+2, & x \geq 8/3 \end{cases}$$

$+$ $\quad$ $-$

$x = 0$ $\quad$ $8/3$ $\quad$ $x = 3$

$8-3(0) = 8 > 0$ $\quad$ $8-3(3) = -1 < 0$

so $$j(x) = \begin{cases} 7(8-3x)+2, & x < 8/3 \\ -7(8-3x)+2, & x \geq 8/3 \end{cases}$$

26. Fully simplify the given expression.

$$\frac{125^x \cdot 2^{3x-11}}{5^{2-x} \cdot 16^{x-2}}$$

$125 = 5^3$

$16 = 2^4$

$$\frac{(5^3)^x \cdot 2^{3x-11}}{5^{2-x} \cdot (2^4)^{x-2}} \rightarrow \frac{5^{3x} \cdot 2^{3x-11}}{5^{2-x} \cdot 2^{4x-8}}$$

$$\rightarrow 5^{(3x-(2-x))} \cdot 2^{((3x-11)-(4x-8))}$$

$$\rightarrow \boxed{5^{4x-2} \cdot 2^{-x-3}}$$

27. Algebraically solve the given equation.

$$27^{2x-1} = \frac{81}{3^{-5x}}$$

$27 = 3^3$

$81 = 3^4$

$$(3^3)^{2x-1} = \frac{3^4}{3^{-5x}}$$

$$3^{6x-3} = 3^{4+5x}$$

$$6x-3 = 4+5x$$

$$\boxed{x = 7}$$

28. Write the function $h(x)$ that is the parent function $f(x) = \sqrt[3]{x}$ with the following transformations:

- shift left 5 units $\sqrt[3]{x+5}$
- vertical compression by a factor of $\frac{5}{3}$ $\frac{3}{5} \cdot \sqrt[3]{x+5}$
- shift down 4 units

$$\frac{3}{5} \cdot \sqrt[3]{x+5} - 4$$

$$\boxed{h(x) = \frac{3}{5} \cdot \sqrt[3]{x+5} - 4}$$

29. Use the given functions to determine the following.

$$f(x) = 2\sqrt{4x+1} \qquad g(x) = \frac{x+3}{x-1}$$

(a) $(f+g)(2)$ $= f(2) + g(2)$

$f(2) = 2\sqrt{4(2)+1} = 2\sqrt{9} = 6$

$g(2) = \frac{2+3}{2-1} = 5$

$(f+g)(2) = 6 + 5 = \boxed{11}$

(b) $\left(\frac{f}{g}\right)(0)$ $= \frac{f(0)}{g(0)}$

$f(0) = 2\sqrt{4(0)+1} = 2\sqrt{1} = 2$

$g(0) = \frac{0+3}{0-1} = -3$

$\left(\frac{f}{g}\right)(0) = \boxed{\frac{2}{-3}}$

(c) $f(g(-1))$

$g(-1) = \frac{-1+3}{-1-1} = \frac{2}{-2} = -1$

$f(g(-1)) = f(-1) = 2\sqrt{4(-1)+1} = 2\sqrt{-3}$ $\boxed{\text{DNE}}$

30. Write the given expression as a single logarithmic term.

$$4 + 9\log_3(x) - 2\log_3(x+7) + \log_3(x-5)$$

$4 = \log_3(3^4)$

$\log_3(\underbrace{3^4}_{top}) + \log_3(\underbrace{x^9}_{top}) - \log_3\underbrace{(x+7)^2}_{bottom} + \log_3\underbrace{(x-5)}_{top}$

$$\boxed{\log_3\left(\frac{81x^9(x+7)^2}{x-5}\right)}$$

31. Algebraically solve the given equation.

$$\log_3(x^2-3) = 1 + \log_3(x-1)$$

$\log_3(x^2-3) - \log_3(x-1) = 1$

$\log_3\left(\frac{x^2-3}{x-1}\right) = 1$

$\cancel{3^{\log_3}}\left(\frac{x^2-3}{x-1}\right) = 3^1$

$\frac{x^2-3}{x-1} = 3$

$x^2 - 3 = 3x - 3$

$x^2 - 3x = 0$

$x(x-3) = 0$

$\underbrace{x = 0,\ x = 3}_{\text{must check}}$

$\log_3(\cancel{0^2-3}) = 1 + \log_3(\cancel{0-1})$

$\log_3(3^2-3) = 1 + \log_3(3-1)$

$\boxed{x = 3 \text{ only}}$

32. For the given functions, determine the domain.

(a) $f(x) = \sqrt[3]{5x+2} - \dfrac{x+2}{x-1} + \ln(x+11)$

odd root (ok)

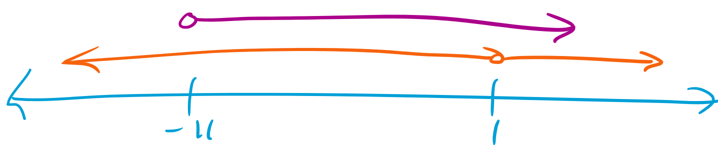
denom: $x - 1 \neq 0$

$x \neq 1$

log: $x + 11 > 0$

$x > -11$

$\boxed{D: (-11, 1) \cup (1, \infty)}$



(b) $g(x) = \dfrac{5\sqrt[4]{-5x+28}}{3\log_6(2x+9)}$

even root: $-5x + 28 \geq 0$

$-5x \geq -28$

$x \leq 28/5$

denom: $3\log_6(2x+9) \neq 0$

$\log_6(2x+9) \neq 0$

$2x + 9 \neq 1$

$2x \neq -8$

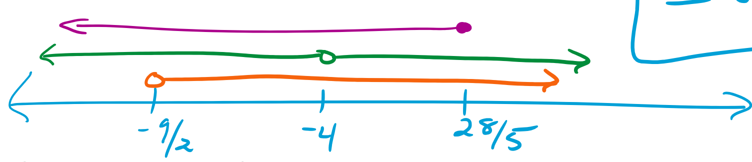
$x \neq -4$

log: $2x + 9 > 0$

$2x > -9$

$x > -9/2$

$\boxed{D: (-9/2, -4) \cup (-4, \tfrac{28}{5}]}$



33. How long will it take \$5,000 to grow to \$12,000 as a one-time investment at an annual interest rate of 4.3% compounded continuously?

$A = Pe^{rt}$

$A = 12000$

$P = 5000$

$r = 0.043$

$12000 = 5000e^{0.043t}$

$\dfrac{12000}{5000} = \dfrac{12}{5} = e^{0.043t}$

$\ln(12/5) = \cancel{\ln}(e^{0.043t})$

$\ln(12/5) = 0.043t$

$t = \dfrac{\ln(12/5)}{0.043}$

$\boxed{t \approx 20.4 \text{ years}}$

34. You borrow \$2,500 as a short-term simple interest loan for 25% interest to be paid back in 30 months. How much will you have to pay back in 30 months?

$A = P + I$ & $I = Prt$

$P = 2500$

$r = .25$

$t = \frac{30}{12}$ years

$A = 2500 + (2500)(.25)(30/12)$

$\boxed{A = \$4,062.50 \text{ in } 30 \text{ months}}$

35. From the following accounts, which would be the best for a $\boxed{\text{loan?}}$ ← smallest effective rate

- Account A: 3.8% annual interest, compounded monthly $m=12$

$Eff(3.8, 12) = 3.8668\ldots\%$

- Account B: 3.72% annual interest, compounded weekly $m=52$

$Eff(3.72, 52) = 3.78867\ldots\%$

- Account A: 3.85% annual interest, compounded continuously

$Eff = e^r - 1 = e^{0.0385} - 1 = 0.03925\ldots$

so $3.925\ldots\%$

$\boxed{\text{Account B b/c smallest effective rate}}$

36. How much would you need to deposit in a savings account if the account has a 5.2% annual interest rate compounded monthly and you want to have $25,000 after 7 years?

N = (7)(12)
I% = 5.2
PV = ?
PMT = 0
FV = 25000
P/Y = 12
C/Y = 12

$17,385.95 should be deposited

37. To help with your eventual retirement, you open an account with an iniital deposit of $2,700 when you are 25 years old. You then make deposits of $150 in the account each month until you are 50 years old. How much money is in the account at that point? How much did you earn in interest from the account?

★ there should be "at an annual interest rate of 2.3% compounded monthly"

N = (25)(12)
I% = 2.3
PV = -2700
PMT = -150
FV = ?
P/Y = 12
C/Y = 12

there is $65,538.00 in the account

total paid in = (PMT)(N) + PV
= (150)(300) + 2700
= $47,700

Interest = $65,538 − $47,700
= $17,838

38. Your business wants to purchase an office building that costs \$850,000. The bank is willing to finance your business for 82% of the cost at an annual interest rate of 3.3% compounded quarterly for 10 years. If your business takes out the maximum loan possible, what will be the monthly payment (to the nearest cent) your business will have to make to the bank each quarter? What will be the total paid for the building at the end of the loan?

$N = (10)(4)$

$I = 3.3$

$PV = (.82)(850000)$

$PMT = ?$

$FV = 0$

$P/Y = 4$

$C/Y = 4$

quarterly payments are \$20,529.11

total paid = (PMT)(N) + (Down Payment)

= (20529.11)(40) + (.18)(850000)

total paid for building = \$974,164.40