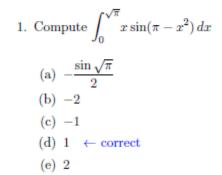


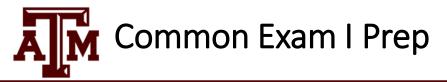
Week in Review Math 152

Week 03 Common Exam I Prep (5.5 - 7.1)

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2. Compute $\int_{1}^{2} x \ln(x^{2}) dx$. (a) $\frac{\ln 4}{2}$ (b) $\ln 4$ (c) $4 \ln 4 - 3$ (d) $\frac{3}{2}$ (e) $\ln 16 - \frac{3}{2} \leftarrow \text{correct}$



3. Which of the following gives the area of the region bounded by $y = |x^2 - 1|$ and x-axis on [-2, 2].

(a)
$$\int_{-2}^{2} (x^{2} - 1) dx$$

(b)
$$\int_{-2}^{1} (x^{2} - 1) dx + \int_{1}^{2} - (x^{2} - 1) dx$$

(c)
$$\int_{-2}^{-1} (x^{2} - 1) dx + \int_{-1}^{2} - (x^{2} - 1) dx$$

(d)
$$\int_{-2}^{-1} - |x^{2} - 1| dx + \int_{-1}^{1} |x^{2} - 1| dx + \int_{1}^{2} - |x^{2} - 1| dx$$

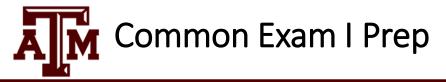
(e)
$$\int_{-2}^{-1} (x^{2} - 1) dx + \int_{-1}^{1} - (x^{2} - 1) dx + \int_{1}^{2} (x^{2} - 1) dx \quad \leftarrow \text{ correct}$$



4. Which of the following integrals gives the area of the region bounded by the curves $x = y^2$ and x = 6 - y?

(a)
$$\int_{-3}^{2} (6 - y - y^2) dy \quad \leftarrow \text{ correct}$$

(b) $\int_{-3}^{2} (y^2 - 6 + y) dy$
(c) $\int_{4}^{9} (6 - x - \sqrt{x}) dy$
(d) $\int_{4}^{9} (\sqrt{x} - 6 + x) dy$
(e) $\int_{4}^{9} (6 - y - y^2) dy$



5. The region bounded by $y = e^x$ and the x-axis on the interval [0, 2] is rotated about the x-axis. Find the volume of the resulting solid.

(a)
$$\frac{\pi e^4}{2}$$

(b) $\frac{\pi e^2}{2}$
(c) $\frac{\pi}{2}(e^4 - 1) \leftarrow \text{correct}$
(d) $\frac{\pi}{2}(e^2 - 1)$
(e) $2\pi(e^4 - 1)$



6. Consider the region bounded by the curves $x = y^2 - 2y$ and the y-axis. Which of the following represents the volume of solid formed when the region is rotated about y = 4?

(a)
$$\int_{0}^{2} 2\pi y (y^{2} - 2y) dy$$

(b)
$$\int_{0}^{2} 2\pi y (2y - y^{2}) dy$$

(c)
$$\int_{0}^{2} 2\pi (4 - y) (y^{2} - 2y) dy$$

(d)
$$\int_{0}^{2} \pi (y - 4) (4y^{2} - y^{4}) dy$$

(e)
$$\int_{0}^{2} 2\pi (4 - y) (2y - y^{2}) dy \quad \leftarrow \text{ correct}$$



7. Consider the region bounded by the two curves $y = \cos x$, $y = \sin x$ and the two lines x = 0 and $x = \frac{\pi}{4}$. Which of the following represents the volume of this region being rotated about the line x = -1?

(a)
$$\int_{0}^{\frac{\pi}{4}} 2\pi (x+1)(\cos x - \sin x) dx \leftarrow \text{correct}$$

(b) $\int_{0}^{\frac{\pi}{4}} 2\pi (x+1)(\sin x - \cos x) dx$
(c) $\int_{-1}^{\frac{\pi}{4}} 2\pi (x+1)(\cos x - \sin x) dx$
(d) $\int_{0}^{\frac{\pi}{4}} 2\pi (x+1)(\cos^2 x - \sin^2 x) dx$
(e) $\int_{0}^{\frac{\pi}{4}} \pi (\cos^2 x - \sin^2 x) dx$



8. Find the area of the region determined by the curve $f(x) = x \sin x$ and the x-axis on the interval $[0, \pi]$.

(a) 1 (b) $\pi \leftarrow \text{correct}$ (c) $\frac{\pi}{2}$ (d) $\pi - 1$

(e) -π



9. Which of the following integrals gives the volume of the solid obtained by rotating the region bounded by $y = 5 - x^2$ and y = 1 about the x-axis.

(a)
$$\pi \int_{-2}^{2} (1 - (5 - x^2)^2) dx$$

(b) $\pi \int_{-2}^{2} (4 - x^2)^2 dx$
(c) $2\pi \int_{-2}^{2} x(4 - x^2) dx$
(d) $\pi \int_{-2}^{2} ((5 - x^2)^2 - 1) dx \quad \leftarrow \text{ correct}$
(e) $2\pi \int_{-2}^{2} x(x^2 - 4) dx$



10. Find the volume of the solid obtained by rotating the region bounded by $x = y^2$ and $x = y^3$ around the y-axis.

(a)
$$\frac{\pi}{35}$$

(b) $\frac{\pi}{10}$
(c) $\frac{\pi}{12}$
(d) $\frac{2\pi}{35} \leftarrow \text{correct}$
(e) $\frac{\pi}{105}$



11. An ideal spring has a natural length of 10 meters. The work done in stretching the spring from 14 meters to 18 meters is 24J. Determine the spring constant k.

(a) $k = \frac{1}{2}$ N/m (b) $k = \frac{3}{8}$ N/m (c) k = 1 N/m \leftarrow correct (d) k = 3 N/m (e) k = 6 N/m



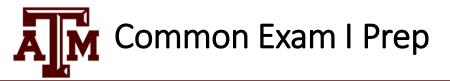
- 12. A 90 ft cable weighing 10 lb is hanging down the side of a 200 ft building. How much work is required to pull the rope 30 feet up the side of the building?
 - (a) 6000 ft-lb
 - (b) 1500 ft-lb
 - (c) 250 ft-lb \leftarrow correct
 - (d) 300 ft-lb
 - (e) 50 ft-lb

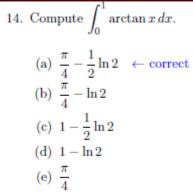


13. The solid S has a triangular base with vertices (-1,0), (1,0), and (0,2). Cross sections perpendicular to the x-axis are squares. Find the volume of S.

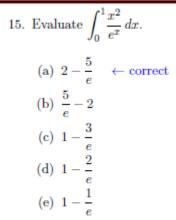
(a)
$$\frac{4}{3}$$

(b) $\frac{8}{3} \leftarrow \text{correct}$
(c) $\frac{1}{3}$
(d) $\frac{2}{3}$
(e) $\frac{5}{3}$



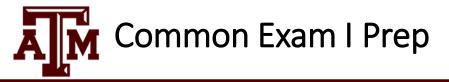




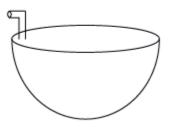


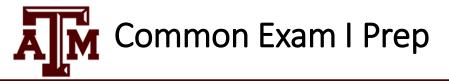


16. (10 points) Consider the solid whose base is the upper half of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$. Cross sections perpendicular to the y axis are semicircles. Find the volume of the solid.

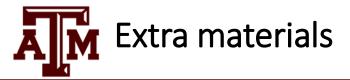


17. (10 points) A hemispherical tank has the shape shown below. The tank has a radius of 10 meters with a 2 meter spout at the top of the tank. The tank is filled with water to a depth of 7 meters. The weight density of water is $\rho g = 9800$ N/m³. Suppose we want to find the work required to pump the water through the spout





18. (7 points) Compute $\int \! x^5 e^{x^3} \, dx$





1. The region bounded by the curves $y = x^2$, y = 4 and x = 0 is rotated about the line y = 4. Which of the following gives the volume of the resulting solid?

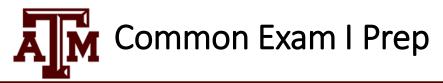
(a)
$$\int_{0}^{4} \pi (4 - x^{2})^{2} dx$$

(b)
$$\int_{0}^{2} \pi (4 - x^{2})^{2} dx$$

(c)
$$\int_{0}^{1} 2\pi x (x^{2} - 4) dx$$

(d)
$$\int_{0}^{2} 2\pi (4 - x) (x^{2} - 4) dx$$

(e)
$$\int_{0}^{4} \pi (4 - x) (x^{2} - 4)^{2} dx$$



2. Evaluate $\int x^5 \sqrt{x^3 + 1} \, dx$ (a) $C + \frac{2}{15} (x^3 + 1)^{5/2} - \frac{2}{9} (x^3 + 1)^{3/2}$ (b) $C + \frac{1}{6} x^6 \left(\frac{1}{4} x^4 + x\right)^{1/2}$ (c) $C + 5x^4 (x^3 + 1)^{1/2} + \frac{3}{2} x^7 (x^3 + 1)^{-1/2}$ (d) $C + \frac{2}{15} x^{15/2} + \frac{1}{6} x^6$ (e) $C + \frac{6}{19} (x^3 + 1)^{19/6} - \frac{2}{3} (x^3 + 1)^{3/2}$



4. The region bounded by the curves $y = e^x$, y = 0, x = 0 and x = 3 is rotated about the x-axis. Find the volume of the resultant solid.

(a)
$$\frac{\pi}{2}(e^6 - 1)$$

(b) $\frac{\pi}{6}(e^9 - 1)$
(c) $2\pi(e^6 - 1)$
(d) $\pi(e^6 - 1)$

(e)
$$\frac{\pi}{2}(e^3 - 1)$$



5. Evaluate
$$\int_{0}^{2} x^{3} e^{x^{2}} dx$$
(a) e^{4}
(b) $\frac{1}{2}(3e^{4}+1)$
(c) $\frac{1}{2}(3e^{4}-1)$
(d) $\frac{1}{2}(5e^{4}-1)$
(e) $2e^{4}-6$



6. The region bounded by the curves $y = 6x - x^2$ and y = 5 is rotated about the y-axis. Which of the following integrals gives the volume of the resulting solid?

(a)
$$2\pi \int_{1}^{5} x(6x - x^2 - 5) dx$$

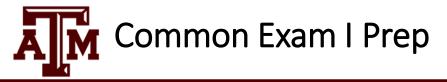
(b) $2\pi \int_{0}^{6} x(5 - 6x + x^2) dx$
(c) $\pi \int_{0}^{6} (x - 5)(6x - x^2)^2 dx$
(d) $\pi \int_{5}^{9} (6x - x^2 - 5)^2 dx$
(e) $2\pi \int_{1}^{5} (5 - x)(6x - x^2 - 5) dx$



7. Compute $\int \cos^3(x) \sin^2(x) dx$

(a)
$$C - \frac{\cos^5(x)}{5} + \frac{\cos^3(x)}{3}$$

(b) $C + \frac{\cos^3(x)\sin^3(x)}{3} + \frac{\cos^4(x)\sin^2(x)}{4}$
(c) $C - \frac{\sin^5(x)}{5} + \frac{\sin^3(x)}{3}$
(d) $C + \frac{\sin^4(x)}{4} + \frac{\sin^2(x)}{2}$
(e) $C - \frac{\sin^6(x)}{6} + \frac{\sin^4(x)}{4} - \frac{\sin^2(x)}{2}$

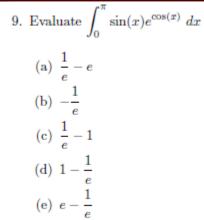


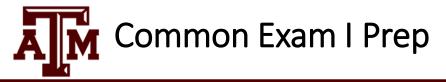
8. The region bounded in the first quadrant by the curves $y = x^2$ and 4x - y = 0 is rotated about the line x = 10. Which of the following integrals gives the volume of the resulting solid?

(a)
$$\pi \int_{0}^{4} \left[(10 - 4x)^{2} - (10 - x^{2})^{2} \right] dy$$

(b) $\pi \int_{0}^{4} (4x - x^{2})^{2} dy$
(c) $\pi \int_{0}^{16} \left[\left(10 - \frac{y}{4} \right)^{2} - (10 - \sqrt{y})^{2} \right] dy$
(d) $\pi \int_{0}^{16} \left[(\sqrt{y})^{2} - \left(\frac{y}{4} \right)^{2} \right] dy$
(e) $2\pi \int_{0}^{16} \left[\left(16 - \frac{y}{4} \right)^{2} - (16 - \sqrt{y})^{2} \right] dy$







- 10. A uniform cable hangs over the side of a building that is 150 feet tall. The cable is 80 feet long, weighs 240 pounds and is attached to a 50 pound weight at the bottom. How much work is done to pull 10 feet of rope up to the top of the building?
 - (a) 650 ft-lb
 - (b) 1350 ft-lb
 - (c) 860 ft-lb
 - (d) 2750 ft-lb
 - (e) 11550 ft-lb



11. Which of the following gives the area of the region bounded by the curves $x = y^2$ and x + y = 6.

(a)
$$\int_{-3}^{2} (y^2 - 6 + y) dy$$

(b) $\int_{-3}^{2} (6 - y - y^2) dy$
(c) $\int_{4}^{9} (6 - x - \sqrt{x}) dx$
(d) $\int_{4}^{9} (\sqrt{x} - 6 + x) dx$
(e) $\int_{-3}^{2} (\sqrt{x} - 6 + x) dy$

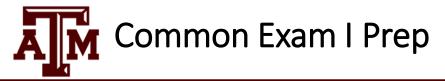


12. The base of a solid is a triangle with vertices (0,0), (1,1) and (1,-1). The cross sections perpendicular to the x-axis are squares. What is the volume of the solid?

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{4}{3}$ (d) $\frac{16}{3}$ (e) $\frac{32}{3}$



13. Compute $\int \cos^2(2x) dx$ (a) $C + \frac{1}{2}x + \frac{1}{4}\sin(2x)$ (b) $C + \frac{1}{2}x + \frac{1}{8}\sin(4x)$ (c) $C + \frac{1}{2}x - \frac{1}{4}\sin(2x)$ (d) $C + \frac{1}{3}\sin^3(2x)$ (e) $C + \frac{1}{2}x - \frac{1}{8}\sin(4x)$



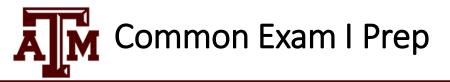
- 14. If the work required to stretch a spring from its natural length to 4 m beyond its natural length is 16 J, then how much force would be needed to stretch the spring 6 m beyond its natural length?
 - (a) 12 N.
 - (b) 18 N.
 - (c) 24 N.
 - (d) 36 N.
 - (e) 72 N.



15. Evaluate $\int_{1}^{e} x^{2} \ln x \, dx$. (a) $\frac{2}{9}e^{3} + \frac{1}{9}$ (b) $\frac{2}{9}e^{3} - \frac{1}{9}$ (c) 1 - e(d) $e^{2} - \frac{1}{9}e^{3} + \frac{1}{9}$ (e) None of these



16. (8 points) Evaluate $\int_0^{\sqrt{3}} \arctan(x) \, dx$.



17. (10 points) Compute $\int 5x^2 \sin(3x) dx$.



- 18. (12 points) Consider the region bounded by the curves $x = 6y y^2$ and y-axis.
 - (a) Set up an integral to find the volume of the solid formed by rotating this region about the line y = 10. Do not evaluate your integral.
 - (b) Set up an integral to find the volume of the solid formed by rotating this region about the line x = -5. Do not evaluate your integral.