Week in Review
Math 152

Week 03

Common Exam I Prep

(5.5 - 7.1)

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1. Compute $\int_0^{\sqrt{\pi}} x \sin(\pi - x^2)\, dx$

   (a) $-\dfrac{\sin\sqrt{\pi}}{2}$

   (b) -2

   (c) -1

   (d) 1 ← correct

   (e) 2

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2. Compute $\int_1^2 x \ln(x^2)\, dx$.

   (a) $\dfrac{\ln 4}{2}$

   (b) $\ln 4$

   (c) $4 \ln 4 - 3$

   (d) $\dfrac{3}{2}$

   (e) $\ln 16 - \dfrac{3}{2}$ ← correct

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3. Which of the following gives the area of the region bounded by $y = |x^2 - 1|$ and x-axis on $[-2, 2]$.

(a) $\int_{-2}^{2} (x^2 - 1)\, dx$

(b) $\int_{-2}^{1} (x^2 - 1)\, dx + \int_{1}^{2} -(x^2 - 1)\, dx$

(c) $\int_{-2}^{-1} (x^2 - 1)\, dx + \int_{-1}^{2} -(x^2 - 1)\, dx$

(d) $\int_{-2}^{-1} -|x^2 - 1|\, dx + \int_{-1}^{1} |x^2 - 1|\, dx + \int_{1}^{2} -|x^2 - 1|\, dx$

(e) $\int_{-2}^{-1} (x^2 - 1)\, dx + \int_{-1}^{1} -(x^2 - 1)\, dx + \int_{1}^{2} (x^2 - 1)\, dx$ ← correct

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4. Which of the following integrals gives the area of the region bounded by the curves $x = y^2$ and $x = 6 - y$?

   (a) $\int_{-3}^{2} (6 - y - y^2)\, dy$ ← correct

   (b) $\int_{-3}^{2} (y^2 - 6 + y)\, dy$

   (c) $\int_{4}^{9} (6 - x - \sqrt{x})\, dy$

   (d) $\int_{4}^{9} (\sqrt{x} - 6 + x)\, dy$

   (e) $\int_{4}^{9} (6 - y - y^2)\, dy$

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5. The region bounded by $y = e^x$ and the x-axis on the interval $[0, 2]$ is rotated about the x-axis. Find the volume of the resulting solid.

   (a) $\frac{\pi e^4}{2}$

   (b) $\frac{\pi e^2}{2}$

   (c) $\frac{\pi}{2}(e^4 - 1)$ ← correct

   (d) $\frac{\pi}{2}(e^2 - 1)$

   (e) $2\pi(e^4 - 1)$

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6. Consider the region bounded by the curves $x = y^2 - 2y$ and the y-axis. Which of the following represents the volume of solid formed when the region is rotated about $y = 4$?

(a) $\int_0^2 2\pi y(y^2 - 2y)\,dy$

(b) $\int_0^2 2\pi y(2y - y^2)\,dy$

(c) $\int_0^2 2\pi(4 - y)(y^2 - 2y)\,dy$

(d) $\int_0^2 \pi(y - 4)(4y^2 - y^4)\,dy$

(e) $\int_0^2 2\pi(4 - y)(2y - y^2)\,dy$ ← correct

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7. Consider the region bounded by the two curves $y = \cos x$, $y = \sin x$ and the two lines $x = 0$ and $x = \dfrac{\pi}{4}$. Which of the following represents the volume of this region being rotated about the line $x = -1$?

(a) $\displaystyle\int_0^{\frac{\pi}{4}} 2\pi(x+1)(\cos x - \sin x)\,dx$ ← correct

(b) $\displaystyle\int_0^{\frac{\pi}{4}} 2\pi(x+1)(\sin x - \cos x)\,dx$

(c) $\displaystyle\int_{-1}^{\frac{\pi}{4}} 2\pi(x+1)(\cos x - \sin x)\,dx$

(d) $\displaystyle\int_0^{\frac{\pi}{4}} 2\pi(x+1)(\cos^2 x - \sin^2 x)\,dx$

(e) $\displaystyle\int_0^{\frac{\pi}{4}} \pi(\cos^2 x - \sin^2 x)\,dx$

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8. Find the area of the region determined by the curve $f(x) = x \sin x$ and the x-axis on the interval $[0, \pi]$.

(a) 1

(b) π ← correct

(c) $\frac{\pi}{2}$

(d) $\pi - 1$

(e) $-\pi$

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9. Which of the following integrals gives the volume of the solid obtained by rotating the region bounded by $y = 5 - x^2$ and $y = 1$ about the x-axis.

(a) $\pi \int_{-2}^{2} \left(1 - (5 - x^2)^2\right) dx$

(b) $\pi \int_{-2}^{2} (4 - x^2)^2 \, dx$

(c) $2\pi \int_{-2}^{2} x(4 - x^2) \, dx$

(d) $\pi \int_{-2}^{2} \left((5 - x^2)^2 - 1\right) dx$ ← correct

(e) $2\pi \int_{-2}^{2} x(x^2 - 4) \, dx$

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10. Find the volume of the solid obtained by rotating the region bounded by $x = y^2$ and $x = y^3$ around the y-axis.

(a) $\frac{\pi}{35}$

(b) $\frac{\pi}{10}$

(c) $\frac{\pi}{12}$

(d) $\frac{2\pi}{35}$ ← correct

(e) $\frac{\pi}{105}$

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11. An ideal spring has a natural length of 10 meters. The work done in stretching the spring from 14 meters to 18 meters is 24J. Determine the spring constant k.

(a) $k = \frac{1}{2}\,\mathrm{N/m}$

(b) $k = \frac{3}{8}\,\mathrm{N/m}$

(c) $k = 1\,\mathrm{N/m}$ ← correct

(d) $k = 3\,\mathrm{N/m}$

(e) $k = 6\,\mathrm{N/m}$

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12. A 90 ft cable weighing 10 lb is hanging down the side of a 200 ft building. How much work is required to pull the rope 30 feet up the side of the building?

(a) 6000 ft-lb

(b) 1500 ft-lb

(c) 250 ft-lb ← correct

(d) 300 ft-lb

(e) 50 ft-lb

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13. The solid S has a triangular base with vertices $(-1, 0)$, $(1, 0)$, and $(0, 2)$. Cross sections perpendicular to the x-axis are squares. Find the volume of S.

(a) $\frac{4}{3}$

(b) $\frac{8}{3}$ ← correct

(c) $\frac{1}{3}$

(d) $\frac{2}{3}$

(e) $\frac{5}{3}$

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14. Compute $\int_0^1 \arctan x \, dx$.

(a) $\frac{\pi}{4} - \frac{1}{2}\ln 2$ ← correct

(b) $\frac{\pi}{4} - \ln 2$

(c) $1 - \frac{1}{2}\ln 2$

(d) $1 - \ln 2$

(e) $\frac{\pi}{4}$

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15. Evaluate $\int_0^1 \frac{x^2}{e^x}\,dx$.

(a) $2 - \frac{5}{e}$ ← correct

(b) $\frac{5}{e} - 2$

(c) $1 - \frac{3}{e}$

(d) $1 - \frac{2}{e}$

(e) $1 - \frac{1}{e}$

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16. (10 points) Consider the solid whose base is the upper half of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$. Cross sections perpendicular to the y axis are semicircles. Find the volume of the solid.

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17. (10 points) A hemispherical tank has the shape shown below. The tank has a radius of 10 meters with a 2 meter spout at the top of the tank. The tank is filled with water to a depth of 7 meters. The weight density of water is $\rho g = 9800\text{N/m}^3$. Suppose we want to find the work required to pump the water through the spout

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18. (7 points) Compute $\int x^5 e^{x^3}\, dx$

Extra materials

1. The region bounded by the curves $y = x^2$, $y = 4$ and $x = 0$ is rotated about the line $y = 4$. Which of the following gives the volume of the resulting solid?

   (a) $\displaystyle\int_0^4 \pi(4 - x^2)^2 dx$

   (b) $\displaystyle\int_0^2 \pi(4 - x^2)^2 dx$

   (c) $\displaystyle\int_0^1 2\pi x(x^2 - 4) dx$

   (d) $\displaystyle\int_0^2 2\pi(4 - x)(x^2 - 4) dx$

   (e) $\displaystyle\int_0^4 \pi(4 - x)(x^2 - 4)^2 dx$

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2. Evaluate $\int x^5\sqrt{x^3+1}\,dx$

(a) $\mathrm{C}+\frac{2}{15}(x^3+1)^{5/2}-\frac{2}{9}(x^3+1)^{3/2}$

(b) $\mathrm{C}+\frac{1}{6}x^6\left(\frac{1}{4}x^4+x\right)^{1/2}$

(c) $\mathrm{C}+5x^4(x^3+1)^{1/2}+\frac{3}{2}x^7(x^3+1)^{-1/2}$

(d) $\mathrm{C}+\frac{2}{15}x^{15/2}+\frac{1}{6}x^6$

(e) $\mathrm{C}+\frac{6}{19}(x^3+1)^{19/6}-\frac{2}{3}(x^3+1)^{3/2}$

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4. The region bounded by the curves $y = e^x$, $y = 0$, $x = 0$ and $x = 3$ is rotated about the $x-$axis. Find the volume of the resultant solid.

   (a) $\frac{\pi}{2}(e^6 - 1)$

   (b) $\frac{\pi}{6}(e^9 - 1)$

   (c) $2\pi(e^6 - 1)$

   (d) $\pi(e^6 - 1)$

   (e) $\frac{\pi}{2}(e^3 - 1)$

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5. Evaluate $\int_0^2 x^3 e^{x^2}\, dx$

(a) e^4

(b) $\frac{1}{2}(3e^4 + 1)$

(c) $\frac{1}{2}(3e^4 - 1)$

(d) $\frac{1}{2}(5e^4 - 1)$

(e) $2e^4 - 6$

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6. The region bounded by the curves $y = 6x - x^2$ and $y = 5$ is rotated about the $y-$axis. Which of the following integrals gives the volume of the resulting solid?

(a) $2\pi \int_1^5 x(6x - x^2 - 5)\, dx$

(b) $2\pi \int_0^6 x(5 - 6x + x^2)\, dx$

(c) $\pi \int_0^6 (x - 5)(6x - x^2)^2\, dx$

(d) $\pi \int_5^9 (6x - x^2 - 5)^2\, dx$

(e) $2\pi \int_1^5 (5 - x)(6x - x^2 - 5)\, dx$

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7. Compute $\int \cos^3(x)\sin^2(x)\,dx$

(a) $C - \frac{\cos^5(x)}{5} + \frac{\cos^3(x)}{3}$

(b) $C + \frac{\cos^3(x)\sin^3(x)}{3} + \frac{\cos^4(x)\sin^2(x)}{4}$

(c) $C - \frac{\sin^5(x)}{5} + \frac{\sin^3(x)}{3}$

(d) $C + \frac{\sin^4(x)}{4} + \frac{\sin^2(x)}{2}$

(e) $C - \frac{\sin^6(x)}{6} + \frac{\sin^4(x)}{4} - \frac{\sin^2(x)}{2}$

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8. The region bounded in the first quadrant by the curves $y = x^2$ and $4x - y = 0$ is rotated about the line $x = 10$. Which of the following integrals gives the volume of the resulting solid?

(a) $\pi \int_0^4 \left[(10 - 4x)^2 - (10 - x^2)^2\right]\, dy$

(b) $\pi \int_0^4 (4x - x^2)^2\, dy$

(c) $\pi \int_0^{16} \left[\left(10 - \frac{y}{4}\right)^2 - (10 - \sqrt{y})^2\right]\, dy$

(d) $\pi \int_0^{16} \left[(\sqrt{y})^2 - \left(\frac{y}{4}\right)^2\right]\, dy$

(e) $2\pi \int_0^{16} \left[\left(16 - \frac{y}{4}\right)^2 - (16 - \sqrt{y})^2\right]\, dy$

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9. Evaluate $\int_0^{\pi} \sin(x)e^{\cos(x)}\,dx$

(a) $\frac{1}{e} - e$

(b) $-\frac{1}{e}$

(c) $\frac{1}{e} - 1$

(d) $1 - \frac{1}{e}$

(e) $e - \frac{1}{e}$

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10. A uniform cable hangs over the side of a building that is 150 feet tall.The cable is 80 feet long, weighs 240 pounds and is attached to a 50 pound weight at the bottom. How much work is done to pull 10 feet of rope up to the top of the building?

    (a) 650 ft-lb
    (b) 1350 ft-lb
    (c) 860 ft-lb
    (d) 2750 ft-lb
    (e) 11550 ft-lb

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11. Which of the following gives the area of the region bounded by the curves $x = y^2$ and $x + y = 6$.

(a) $\int_{-3}^{2} (y^2 - 6 + y)\ dy$

(b) $\int_{-3}^{2} (6 - y - y^2)\ dy$

(c) $\int_{4}^{9} (6 - x - \sqrt{x})\ dx$

(d) $\int_{4}^{9} (\sqrt{x} - 6 + x)\ dx$

(e) $\int_{-3}^{2} (\sqrt{x} - 6 + x)\ dy$

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12. The base of a solid is a triangle with vertices $(0,0)$, $(1,1)$ and $(1,-1)$. The cross sections perpendicular to the $x-$axis are squares. What is the volume of the solid?

(a) $\frac{1}{3}$

(b) $\frac{2}{3}$

(c) $\frac{4}{3}$

(d) $\frac{16}{3}$

(e) $\frac{32}{3}$

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13. Compute $\int \cos^2(2x)\, dx$

(a) $C + \frac{1}{2}x + \frac{1}{4}\sin(2x)$

(b) $C + \frac{1}{2}x + \frac{1}{8}\sin(4x)$

(c) $C + \frac{1}{2}x - \frac{1}{4}\sin(2x)$

(d) $C + \frac{1}{3}\sin^3(2x)$

(e) $C + \frac{1}{2}x - \frac{1}{8}\sin(4x)$

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14. If the work required to stretch a spring from its natural length to 4 m beyond its natural length is 16 J, then how much force would be needed to stretch the spring 6 m beyond its natural length?

    (a) 12 N.
    (b) 18 N.
    (c) 24 N.
    (d) 36 N.
    (e) 72 N.

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15. Evaluate $\int_1^e x^2 \ln x \, dx$.

(a) $\frac{2}{9}e^3 + \frac{1}{9}$

(b) $\frac{2}{9}e^3 - \frac{1}{9}$

(c) $1 - e$

(d) $e^2 - \frac{1}{9}e^3 + \frac{1}{9}$

(e) None of these

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16. (8 points) Evaluate $\int_0^{\sqrt{3}} \arctan(x)\, dx$.

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17. (10 points) Compute $\int 5x^2 \sin(3x)\, dx$.

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18. (12 points) Consider the region bounded by the curves $x = 6y - y^2$ and $y-$axis.

(a) **Set up** an integral to find the volume of the solid formed by rotating this region about the line $y = 10$. **Do not evaluate your integral.**

(b) **Set up** an integral to find the volume of the solid formed by rotating this region about the line $x = -5$. **Do not evaluate your integral.**