SESSION 7: REVIEW EXAM #2

1. Find the derivatives of the following functions. You do not need to simplify your answers.

   (a) $f(x) = (2x+1)\sqrt{x^2+1}$ $= (2x+1)(x^2+1)^{1/2}$

   $f'(x) = \frac{d}{dx}(2x+1)(x^2+1)^{1/2} + (2x+1)\frac{d}{dx}\left((x^2+1)^{1/2}\right)$

   $= (2)(x^2+1)^{1/2} + (2x+1)\left[\frac{1}{2}(x^2+1)^{-1/2}(2x)\right]$

   (b) $f(x) = \dfrac{1}{e^x + e^{-x}}$ $= (e^x + e^{-x})^{-1}$

   Using Product Rule

   $f'(x) = -1(e^x + e^{-x})^{-2}(e^x + e^{-x}(-1))$

   $= -(e^x + e^{-x})^{-2}(e^x - e^{-x})$

   OR

   Using Quotient Rule

   $\dfrac{(e^x + e^{-x})\frac{d}{dx}(1) - (1)\frac{d}{dx}(e^x + e^{-x})}{(e^x + e^{-x})^2}$

   $= \dfrac{(e^x + e^{-x})(0) - (e^x + e^{-x}(-1))}{(e^x + e^{-x})^2}$

   $= \dfrac{-(e^x - e^{-x})}{(e^x + e^{-x})^2}$

   (c) $f(x) = e^{2x}\ln\left(2x^3 + x\right)$

   $f'(x) = e^{2x}\frac{d}{dx}\left(\ln(2x^3+x)\right) + \frac{d}{dx}(e^{2x})\ln(2x^3+x)$

   $= e^{2x}\left(\dfrac{6x^2+1}{2x^3+x}\right) + e^{2x}(2)\ln(2x^3+x)$

(d) $f(x) = 4^{x^4+5}$

$$f'(x) = 4^{x^4+5}\,(4x^3)\,(\ln 4)$$

(e) $f(x) = \dfrac{\sqrt{x^2+4x-e}}{3^{2x^4+x}}$

$$f'(x) = \frac{3^{2x^4+x}\,\frac{d}{dx}\left((x^2+4x-e)^{1/2}\right) - (x^2+4x-e)^{1/2}\,\frac{d}{dx}\left(3^{2x^4+x}\right)}{\left(3^{2x^4+x}\right)^2}$$

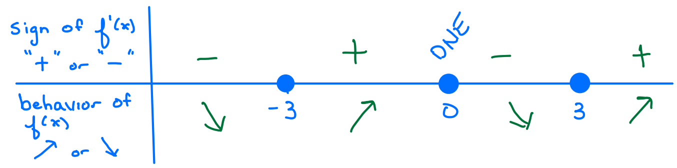
$$= \frac{3^{2x^4+x}\left(\frac{1}{2}(x^2+4x-e)^{-1/2}(2x+4)\right) - (x^2+4x-e)^{1/2}\left(3^{2x^4+x}(8x^3+1)(\ln 3)\right)}{\left(3^{2x^4+x}\right)^2}$$

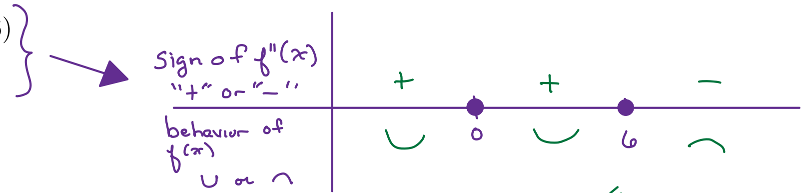
(f) $f(x) = \left(x^2+6x+1\right)^4 + e^{2\pi} - \log_4\left(2x^3 - \sqrt{x}\right)$

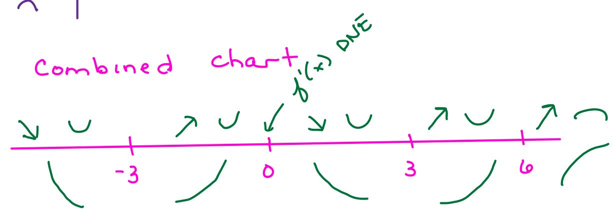
$$f'(x) = 4(x^2+6x+1)^3(2x+6) + 0 - \frac{6x^2 - \frac{1}{2}x^{-1/2}}{(2x^3 - x^{1/2})(\ln 4)}$$

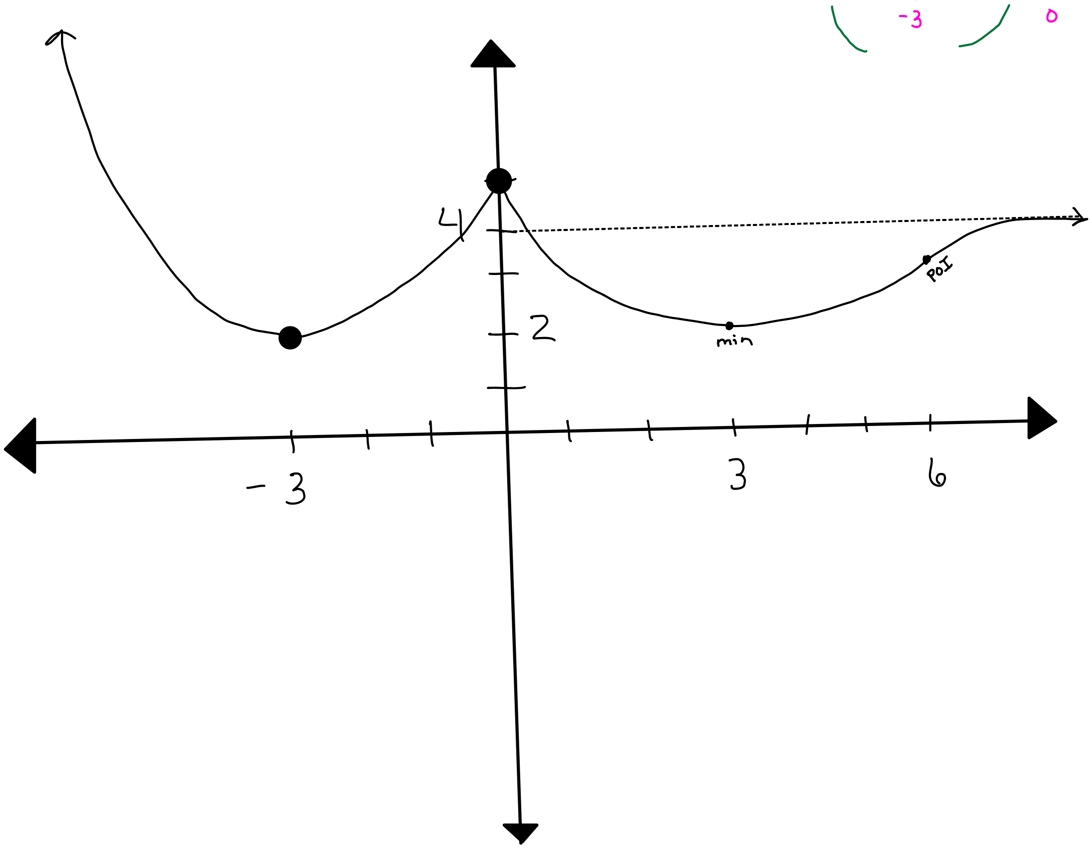
2. Sketch a graph of a function that satisfies the following conditions.

- Domian: $(-\infty, \infty)$
- Range: $[2, \infty)$
- Continuous on $(-\infty, \infty)$
- $f'(x) > 0$ on $(-3, 0) \cup (3, \infty)$
- $f'(x) < 0$ on $(-\infty, -3) \cup (0, 3)$
- $f'(0)$ is undefined
- $f''(x) > 0$ on $(-\infty, 0) \cup (0, 6)$
- $f''(x) < 0$ on $(6, \infty)$
- $\lim_{x \to \infty} f(x) = 4$
- $\lim_{x \to -\infty} f(x) \to \infty$
- $f(-3) = 2, \; f(0) = 5$









3. Given a function $f(x)$ with domain $(-\infty, 2) \cup (2, \infty)$, $f'(x) = \dfrac{(x-7)(x+3)}{(x-2)^2}$, and $f''(x) = \dfrac{50}{(x-2)^3}$, find all intervals where $f(x)$ is

(a) decreasing and concave down.

$(-3, 2)$

(b) decreasing and concave up.

$(2, 7)$

(c) increasing and concave down.

$(-\infty, -3)$

(d) increasing and concave up.

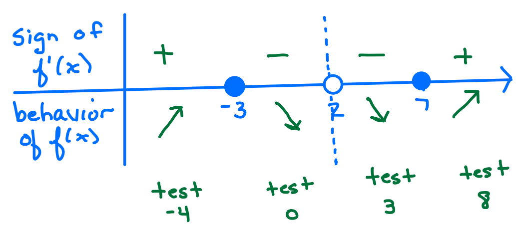
$(7, \infty)$

Partition numbers for $f'(x)$

$f'(x)$ DNE $x = 2$

$f'(x) = 0$ $x = 7, -3$

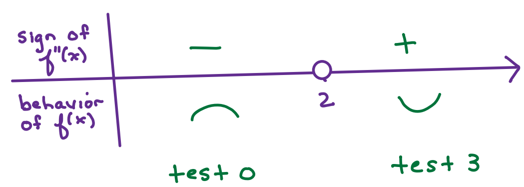
critical values: $x = -3, 7$

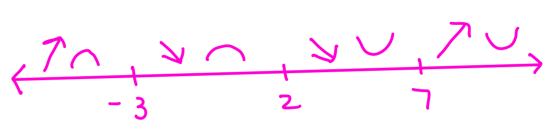


Partition numbers for $f''(x)$

$f''(x)$ DNE $x = 2$

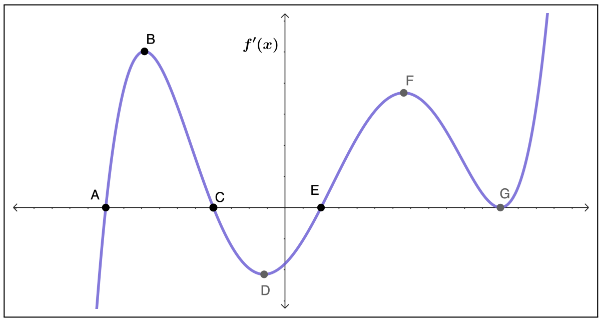
$f''(x)$ will never be zero





assume domain $(-\infty, \infty)$

4. A graph of $f'(x)$ is given below.



above or below x-axis

sign of $f'(x)$	$-$		$+$		$-$		$+$		$+$
behavior of $f(x)$	↘	A	↗	C	↘	E	↗	G	↗
		min		max		min			

$f'(x)$ ↗ ↘	↗		↘		↗		↘		↗
$f(x)$ ∪ ∩	∪	B	∩	D	∪	F	∩	G	∪

Use the graph to find:

(a) the x-values for all local extrema of $f(x)$.

local max at $x = C$ and local min at $x = A, E$

(b) determine the x-values at which $f(x)$ has points of inflection.

$x = B, D, F, G$

(c) determine the intervals where $f(x)$ is concave down.

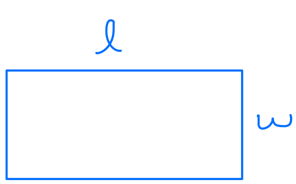
(B, D), (F, G)

helpful chart

f	+	$-$	↗	↘	∪	∩				
f'			+	$-$	↗	↘	∪	∩		
f''					+	$-$	↗	↘	∪	∩

5. The length of a rectangle is increasing at a rate of 3 centimeters per second, and the width is increasing at a rate of 2 centimeters per second. How fast is the area of the rectangle increasing when the length is 10 centimeters and the width is 5 centimeters?

Step 1:



Step 2: Know: $\frac{dl}{dt} = 3 \text{ cm/sec}$, $\frac{dw}{dt} = 2 \text{ cm/sec}$

TBD: $\frac{dA}{dt}$ when $l = 10$, $w = 5$

Step 3: $A = l \cdot w$

Step 4: $\frac{dA}{dt} = l \frac{dw}{dt} + w \frac{dl}{dt}$

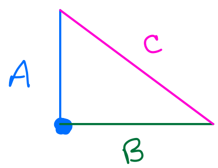
Step 5: $\frac{dA}{dt} = 10(2) + 5(3)$
$= 20 + 15$
$= 35$

The area of the rectangle is increasing at $35 \text{ cm}^2/\text{sec}$.

6. Two cars start from the same point. One car travels north at 60 kilometers per hour, and the other car travels east at 80 kilometers per hour. How fast is the distance between the two cars increasing 2 hours later?

A B

Step 1:



A $\longrightarrow$ distance 1st car is from point
B $\longrightarrow$ distance 2nd car is from point
C $\longrightarrow$ distance between A and B

Step 2:

Know: $\frac{dA}{dt} = 60 \text{ Km/hr}$ $\quad \frac{dB}{dt} = 80 \text{ Km/hr}$

TBD: $\frac{dC}{dt}$ when $\underbrace{A = 120 \text{ Km and } B = 160 \text{ Km}}$

$120^2 + 160^2 = 40000$

$\sqrt{40000} = 200 = C$

Step 3:

$$A^2 + B^2 = C^2$$

Step 4:

$$2A\frac{dA}{dt} + 2B\frac{dB}{dt} = 2C\frac{dC}{dt}$$

Step 5:

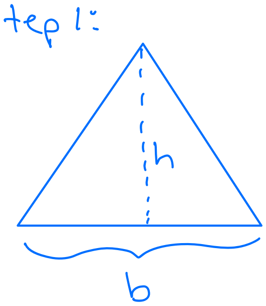
$$2(120)(60) + 2(160)(80) = 2(200)\frac{dC}{dt}$$

$$100 = \frac{dC}{dt}$$

$\boxed{\text{The distance between the two cars is increasing at 100 Km/hr.}}$

7. A triangle has a height that is increasing at a rate of 2 cm/second, and its area is increasing at a rate of 4 cm^2/second. Find the rate at which the length of the base of the triangle is changing when the height of the triangle is 4 cm and its area is 20 cm^2.

Step 1:



Step 2:

Know:

$\frac{dh}{dt} = 2 \text{ cm/sec}$

$\frac{dA}{dt} = 4 \text{ cm}^2\text{/sec}$

TBD:

$\frac{db}{dt}$ when $h = 4$ and $A = 20$

$20 = \frac{1}{2}b(4)$

$20 = 2b$

$10 = b$

Step 3 + 4:

$$A = \frac{1}{2}bh$$

$$\frac{dA}{dt} = \frac{1}{2}b\frac{dh}{dt} + \frac{1}{2}\frac{db}{dt}\cdot h$$

Step 5:

$$4 = \frac{1}{2}(10)(2) + \frac{1}{2}\frac{db}{dt}(4)$$

$$4 = 10 + 2\frac{db}{dt}$$

$$-\frac{6}{2} = \frac{db}{dt}$$

$$-3 = \frac{db}{dt}$$

$\boxed{\text{The base is decreasing at a rate of 3 cm/sec.}}$

for $f'(x)$

8. Given $f(x) = x^{2/3} - x^2$, find all (a) partition numbers, (b) critical values, (c) the intervals where $f(x)$ is increasing, (d) the intervals where $f(x)$ is decreasing, and (e) all local extrema. Round to 2nd decimal place if necessary.

Domain $(-\infty, \infty)$

$f'(x) = \frac{2}{3}x^{-1/3} - 2x$

$= \frac{2}{3\sqrt[3]{x}} - 2x$

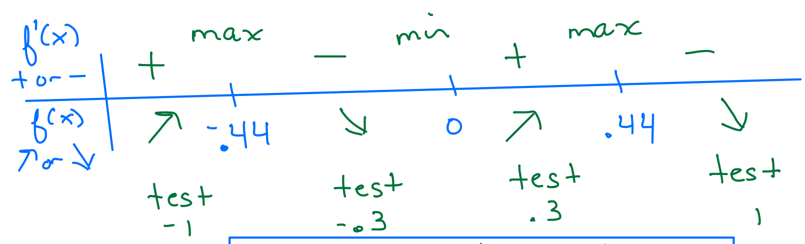
$f'(x)$ DNE when $x = 0$

$f'(x) = 0$ when

$0 = \frac{2}{3x^{1/3}} - 2x$

$\frac{2x}{1} \cancel{=} \frac{2}{3x^{1/3}} \longrightarrow 6x^{4/3} = 2$

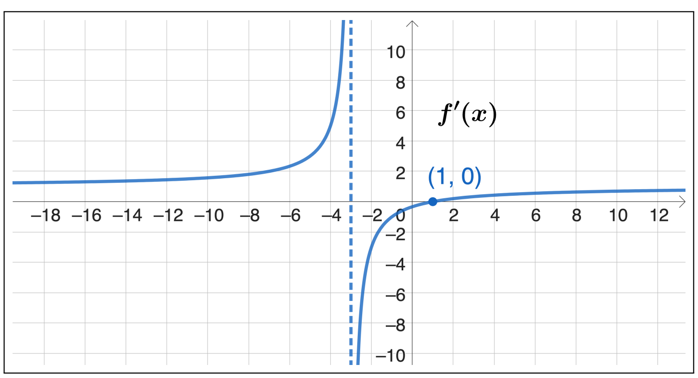
$\left(x^{4/3}\right)^{3/4} = \left(\frac{2}{6}\right)^{3/4} \approx \pm .44$



(a) $x = 0, .44, -.44$
(b) $x = 0, .44, -.44$
(c) $(-\infty, -.44), (0, .44)$
(d) $(-.44, 0), (.44, \infty)$
(e) local max $x = \pm .44$
local min $x = 0$

9. Given the graph of $f'(x)$ below and that the domain of $f(x)$ is $(-\infty, -3) \cup (-3, \infty)$, find all (a) partition numbers, (b) critical values, (c) the intervals where $f(x)$ is increasing, (d) the intervals where $f(x)$ is decreasing, and (e) all local extrema.

for $f'(x)$



Domain: $x \neq -3$

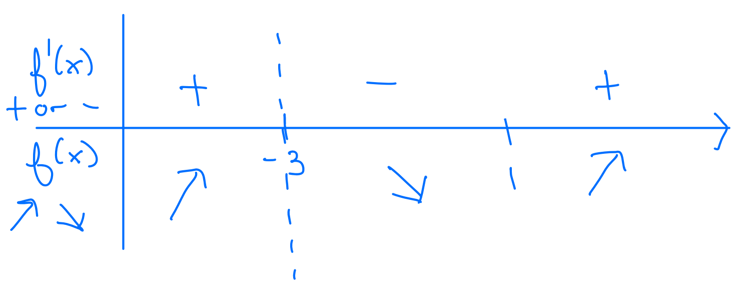
(a) $x = -3, 1$

(b) $x = 1$

(c) $(-\infty, -3), (1, \infty)$

(d) $(-3, 1)$

(e) local min at $x = 1$



Round any answers to two decimals

10. Given $f(x) = (x^2 + 3x)^{2/3}$, find:

   (a) the equation of the tangent line to $f(x)$ at $x = -2$.

   (b) all critical values for $f(x)$.

   (c) all values of x where the line tangent to $f(x)$ is horizontal.

$$f'(x) = \frac{2}{3}(x^2 + 3x)^{-1/3}(2x+3)$$

$f'(x)$ DNE when $x^2 + 3x = 0$

$$x(x+3) = 0$$

$$x = 0, -3$$

$f'(x) = 0$ when $0 = \dfrac{2(2x+3)}{3(x^2+3x)^{1/3}}$

$$0 = 4x + 6$$

$$-\frac{3}{2} = \frac{-6}{4} = x$$

(a) $f'(-2) = \frac{2}{3}(-2)^{-1/3}(-1)$

$= \dfrac{-2}{3\sqrt[3]{-2}} \approx .53$

$f(-2) = 1.59$

$y - 1.59 = .53(x+2)$

(b) $x = 0, -3, -3/2$

(c) $x = -\frac{3}{2}$

11. Find the ^approximate cost of producing the 50^{th} item if a company's cost function is $C(x) = \sqrt{x}(x-10)$ where $C(x)$ is in dollars and x is the number of items produced.

Use nderiv

$c'(49) = 9.79$

$\boxed{\$9.79}$

12. Use implicit differentiation to find the slope of the line tangent to the graph of $19 - 3x^2 + 9^x = 4\sqrt{y}$ at $(0, 25)$.

$$19 - 3x^2 + 9^x = 4y^{1/2}$$

$$0 - 6x + 9^x \ln 9 = 2y^{-1/2}\frac{dy}{dx}$$

when $x = 0$, $y = 25$

$$-6(0) + 9^0 \ln 9 = 2(25)^{-1/2}\frac{dy}{dx}$$

$$\ln 9 = \frac{2}{5}\frac{dy}{dx}$$

$$\boxed{\frac{5}{2}\ln 9 = \frac{dy}{dx}}$$

13. Find $\frac{dy}{dx}$ for the curve $5e^{xy} - 6x^2 = 8y^3 + 7$.

Product rule

$$5e^{xy} - 6x^2 = 8y^3 + 7$$

$$5e^{xy}\left(y + x\frac{dy}{dx}\right) - 12x = 24y^2\frac{dy}{dx} + 0$$

$$5e^{xy}y + 5xe^{xy}\frac{dy}{dx} - 12x = 24y^2\frac{dy}{dx}$$

$$5xe^{xy}\frac{dy}{dx} - 24y^2\frac{dy}{dx} = -5e^{xy}y + 12x$$

$$\frac{dy}{dx} = \frac{-5e^{xy}y + 12x}{5xe^{xy} - 24y^2}$$

14. Suppose that $x = x(t)$ and $y = y(t)$ are both functions of t. If $x^2 + y^2 = 40$, and $\frac{dx}{dt} = 3$ when $x = 2$ and $y = 6$, what is $\frac{dy}{dt}$?

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$2(2)(3) + 2(6)\frac{dy}{dt} = 0$$

$$12 + 12\frac{dy}{dt} = 0$$

$$12\frac{dy}{dt} = -12$$

$$\frac{dy}{dt} = -1$$