Session 5: Sections 2-5 and 2-6 (Implicit Differentiation)

Specific Formulas for the Chain Rule

If g is a differentiable function, n is any real number, and b is any positive real number, then

- **Generalized Power Rule:** If $y = (g(x))^n$, then $y\,' = n(g(x))^{n-1} \cdot g\,'(x)$.
- **Generalized Exponential (base b) Rule:** If $y = b^{g(x)}$, then $y\,' = b^{g(x)}(\ln(b)) \cdot g\,'(x)$.
- **Generalized Logarithm (base b) Rule:** If $y = \log_b(g(x))$, then $y\,' = \dfrac{g\,'(x)}{(g(x))\,(\ln(b))}$.

1. Differentiate each of the following.

   (a) $f(x) = \left(5x^4 + 3x^2 + x\right)^{20}$

   (b) $g(x) = e^{\sqrt{x}+x^2-5}$

   (c) $t(x) = \log_5(2x^3 + e^x)$

   (d) $f(x) = e^{e^{2x}}$

(e) $y(x) = x^5 2^{x^2+5x} + e^{\pi^2}$

(f) $p(x) = \ln\left(\dfrac{2x^2+x}{x+1}\right)$

(g) $f(x) = \dfrac{\log_5(3x^4+7x)}{\sqrt{2x^3}}$

(h) $p(x) = \left[\log_2\left(x^2 + e^{3x^2}\right)\right]^4$

2. Find the $x-$values where the graph of $f(x) = x^3 e^{-x^2}$ has horizontal tangent lines.

3. Suppose $F(x) = g(f(x))$ and $f(2) = 3, f'(2) = -3, g(3) = 5$, and $g'(3) = 4$. Find $F'(2)$.

4. Determine an equation for the line tangent to $f(x) = \ln(3x)$ at $x = 2$.

Steps for using implicit differentiation to find $\frac{dy}{dx}$.

Step 1: Substitute $y(x)$ because we are assuming y is a function of x.

Step 2: Take the derivative of both sides of the equation with respect to x.

Step 3: Substitute y back in for $y(x)$ and write $y\,'(x)$ as $\frac{dy}{dx}$.

Step 4: Solve for $\frac{dy}{dx}$.

5. Use implicit differentiation to find $\frac{dy}{dx}$ for each of the following.

(a) $-6e^y + \frac{1}{3}y^3 - 2\ln(x) = -8$

(b) $x^2 + x + y^3 - 3y^2 = 45$

(c) $x^2 + xy^3 - 3y^2 = 45$

(d) $3x^4 - \ln\left(y^2\right) = \dfrac{4}{y^2}$

6. Find the slope of the line tangent to curve given by $2x^3 - 10\log_3(y^4) = \dfrac{120}{y} + 48$ at the point $(4, 3)$. If necessary, round to the nearest hundredth.

7. Suppose x and y are functions of t and are related by the equation $\ln\left(3x^2 - 5y\right) + 16 = x^4$. Given $\dfrac{dx}{dt} = 3$, $x = 2$, and $y = \dfrac{11}{5}$, use the information to find $\dfrac{dy}{dt}$.