Math 150 - Week-In-Review 10

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Problem Statements, sections 7.6, 8.1, 8.2 and 8.3

Fundamental Trigonometric Identities

Reciprocal Identities

$$\csc(\theta) = \frac{1}{\sin(\theta)} \qquad \sec(\theta) = \frac{1}{\cos(\theta)} \qquad \cot(\theta) = \frac{1}{\tan(\theta)}$$

Quotient Identities

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} \qquad \cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$$

Pythagorean Identities

$$\sin^2(\theta) + \cos^2(\theta) = 1$$

$$1 + \cot^2(\theta) = \csc^2\theta$$

$$\tan^2(\theta) + 1 = \sec^2\theta$$

Cofunctions

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta \qquad \cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta \qquad \cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$$

$$\sec\left(\frac{\pi}{2} - \theta\right) = \csc\theta \qquad \csc\left(\frac{\pi}{2} - \theta\right) = \sec\theta$$

Even/Odd Identities

$$\sin(-\theta) = -\sin\theta \qquad \cos(-\theta) = \cos\theta \qquad \tan(-\theta) = -\tan\theta$$

$$\csc(-\theta) = -\csc\theta \qquad \sec(-\theta) = \sec\theta \qquad \cot(-\theta) = -\cot\theta$$

Sum and Difference Identities

$$\sin(u+v) = \sin(u)\cos(v) + \cos(u)\sin(v) \qquad \sin(u-v) = \sin(u)\cos(v) - \cos(u)\sin(v)$$

$$\cos(u+v) = \cos(u)\cos(v) - \sin(u)\sin(v) \qquad \cos(u-v) = \cos(u)\cos(v) + \sin(u)\sin(v)$$

$$\tan(u+v) = \frac{\tan(u) + \tan(v)}{1 - \tan(u)\tan(v)} \qquad \tan(u-v) = \frac{\tan(u) - \tan(v)}{1 + \tan(u)\tan(v)}$$

Double Angle Identities

$$\sin(2u) = 2\sin(u)\cos(u)$$

$$\begin{aligned}\cos(2u) &= \cos^2(u) - \sin^2(u)\\ &= 2\cos^2(u) - 1\\ &= 1 - 2\sin^2(u)\end{aligned}$$

1. Given $\cos(\theta) = \frac{4}{5}$ and $\csc(\theta) < 0$, find the value of $\cos(2\theta)$, $\sin(2\theta)$ and $\tan(2\theta)$ using the identities.

2. State the domain and range of $y = \text{arcsec}(x)$

3. State the domain and range of $y = \arccos(x)$

4. Simplify the expression $\dfrac{2\tan(7^\circ)}{1-\tan^2(7^\circ)}$

5. Find the exact value of $2\cos^2(22.5^\circ)-1$

6. Write the following expression as product of trig functions.

$$\cos(9x) + \cos(2x)$$

7. Find the exact value of $\sin(\frac{5\pi}{12})$

8. Find the exact value of $\cos(\frac{19\pi}{8})$

9. Find the exact value of $\arccos(-\frac{1}{2})$

10. A 30-ft ladder is leaning against a building. If the base of the ladder is 6 ft from the base of the building, what is the angle of the elevation of the ladder? How high does the ladder reach on the building?

11. Simplify each composition, if possible.

$\tan\left[\arctan\left(\frac{\sqrt{3}}{3}\right)\right] =$ _____

$\arcsin\left[\sin\left(\frac{5\pi}{4}\right)\right] =$ _____

$\arccos\left[\tan(0)\right] =$ _____

$\cot\left(\text{arcsec}(1)\right) =$ _____

$$\tan\left[\arccos\left(\frac{2x}{\sqrt{4x^2+25}}\right)\right] = \_\_\_\_\_$$

$$\sec\left[\arcsin\left(\frac{x}{b}\right)\right] = \_\_\_\_\_$$

12. Find all solutions to $2\sin(3\theta) - 1 = 0$, then state the solutions in the interval $[0, 2\pi)$.

13. Find all solutions to $\sin(2\theta - \pi)\cos(\theta) - 0.65\sin(2\theta - \pi) - \cos(\theta) + 0.65 = 0$, then state the solutions in the interval $[0, 2\pi)$.

14. Solve the equation $\sin(2x) + \cos(x) = 0$, then state the solutions in the interval $[0, 2\pi)$.

15. Find all solutions to the equation $\dfrac{\cos(2x)}{\cos^2 x} = 1$.

16. Solve the equation $5\sin(\theta)\cot(\theta) + 4\cot(\theta) = 0$

17. Find all solutions for $\tan(2x) + \tan x = 0$ on $[0, 2\pi)$

18. Simplify $\tan(\arctan(3) - \arctan(2))$

19. Solve the trigonometric equation $\arcsin(x) = \arccos(2x)$

20. Verify the following identities.
    (a) $\sec^4(x) - \tan^4(x) = \sec^2(x) + \tan^2(x)$

    (b) $\dfrac{\cos(\theta)}{1 - \sin(\theta)} = \dfrac{\sin(\theta) - \csc(\theta)}{\cos(\theta) - \cot(\theta)}$

(c) $\dfrac{1-\cos(x)}{\sin(x)} + \dfrac{\sin(x)}{1-\cos(x)} = 2\csc(x)$

(d) $\dfrac{\sin(3x)+\cos(3x)}{\cos(x)-\sin(x)} = 1 + 4\sin(x)\cos(x)$