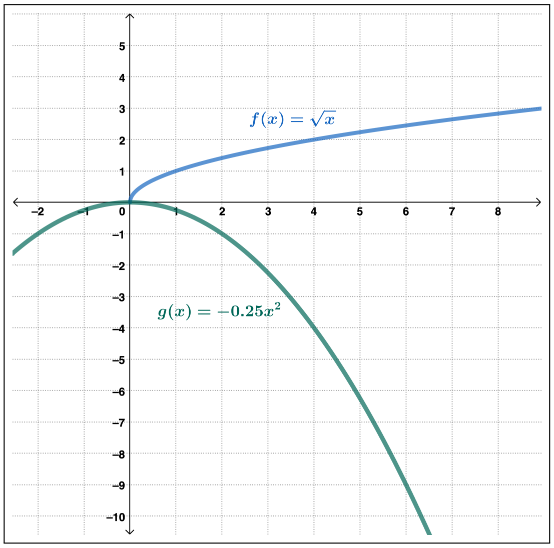
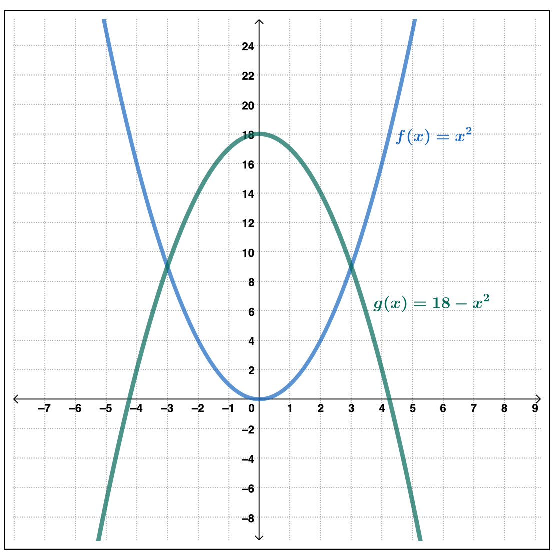
Session 1: Review for Final Exam

Section 4.6

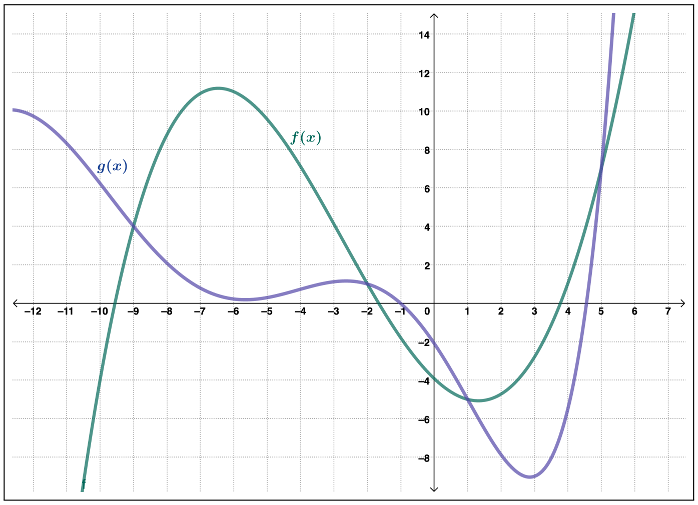
1. Find the area between the two curves $f(x) = \sqrt{x}$ and $g(x) = -0.25x^2$ on the interval $[0, 4]$.



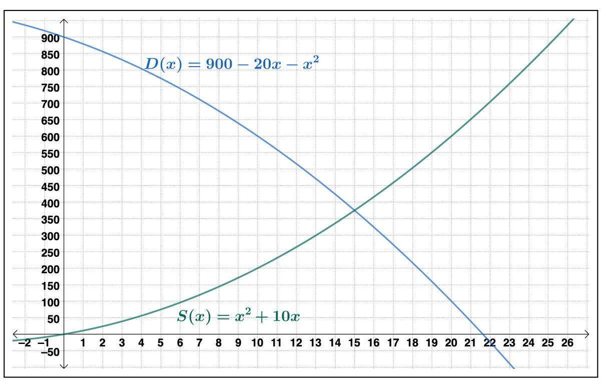
2. Find the area between the two curves $f(x) = x^2$ and $g(x) = 18 - x^2$.



3. Given the graphs of $f(x)$ and $g(x)$ below, find the integral that represents the area between the two curves.



4. Suppose the supply function of a certain item is given by $S(x) = x^2 + 10x$ (in dollars) and the demand function is given by $D(x) = 900 - 20x - x^2$ (in dollars) where x is the number of items produced and sold.



(a) Find the consumers' surplus.

(b) Find the producers' surplus.

Concepts from Exam 1

5. Given $h(x)$ below, answer the questions that follow.

$$h(x) = \begin{cases} 2e^{x+5} & x < -5 \\ \dfrac{-20}{x-5} & -5 \le x < 4 \\ \dfrac{(3x-2)(x-8)}{x-8} & 4 \le x \le 10 \end{cases}$$

(a) $\lim_{x \to -5^+} h(x)$.

(b) $\lim_{x \to -5^-} h(x)$

(c) $\lim_{x \to 4} h(x)$

(d) the intervals on which $h(x)$ is continuous.

6. Given the information about $p(x)$ below, find the equation of the line tangent to $p(x)$ at $x = 2$.

- $p'(x) = \sqrt{2x + 5}$
- $p''(x) = (2x + 5)^{-1/2}$
- $p(-2.5) = 3$
- $p(2) = 12.$

7. Find the limits specified below. If the limit does not exist but has infinite behavior, use limit notation to describe the infinite behavior.

(a) $\lim_{x \to 3} \frac{2x^2 - 18}{4(x - 3)(x - 2)}$

(b) $\lim_{x \to \infty} \frac{x^4 + x^2 + 1}{3x^3 + x - 3}$

8. Find the horizontal asymptotes for $f(x) = \dfrac{4 + e^{x} - 5e^{-x}}{3e^{-x} + 8}$.

9. Use the limit definition of the derivative to find $f'(x)$ given $f(x) = x^2 + 1$.

Concepts from Exam 2

10. Find the derivatives of the following functions. You do not need to simplify your answers. Apply the correct derivative rules and leave in unsimplified form.

    (a) $q(x) = \log_4\left(x^2 + e^{3x}\right)$

    (b) $f(x) = 4^{2x^3+4x-1}$

    (c) $r(x) = \dfrac{(x^2+3)^4}{e^{3x^2}}$

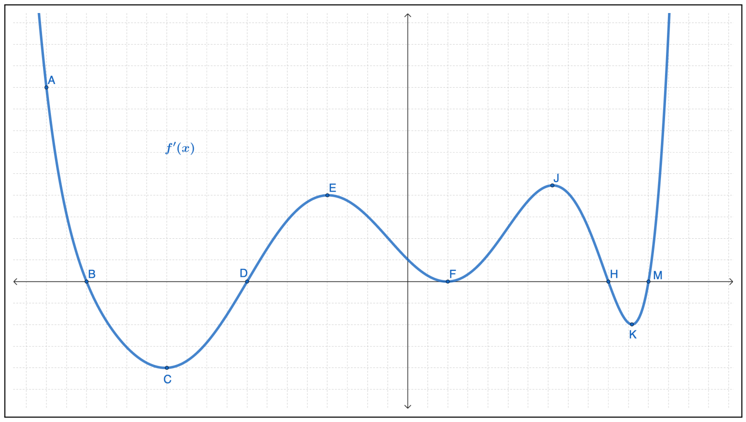
    (d) $t(x) = 2^{5x^3}\left(\ln(2x^2 + 4x - 1)\right)$

11. Given $16x^2 + \frac{1}{2}y^2 - 3xy = 16$, find $\frac{dy}{dx}$ and then use that to find the equation of the line tangent to the given curve at the point $(1, 6)$.

12. Given $h(x) = \dfrac{(f(x))^2}{3e^x}$, $f(0) = 5$ and $f'(0) = -1$ find $h'(0)$.

13. The cost equation (in dollars) for a certain company to produce x items is given by $C(x) = \sqrt{x}(x + 12) + 50$ for $0 \leq x \leq 500$. Find the ***approximate*** cost of producing the 36th item.

14. A graph of $f\,'(x)$ is given below. Use the graph of $f\,'(x)$ to answer questions that follow.



(a) Determine the intervals where $f(x)$ is increasing and concave down.

(b) Determine the intervals where $f(x)$ is decreasing and concave up.

(c) Determine the x-value(s) at which $f(x)$ has local extrema.

(d) Determine the x-values at which $f(x)$ has points of inflection.

15. Given $g(x)$ is continuous on its domain of $(-\infty, 4) \cup (4, \infty)$, $g'(x) = \dfrac{2(x-2)(x+5)}{(x-4)^2}$ and $g''(x) = \dfrac{-22x+16}{(x-4)^3}$:

(a) Find all partition values for $g'(x)$.

(b) Find all critical values for $g(x)$.

(c) Find all intervals where $g(x)$ is increasing.

(d) Find all intervals where $g(x)$ is concave down.

(e) Find all x-values where $g(x)$ has point(s) of inflection.

(f) Find all x-values where $g(x)$ has local extrema. Be sure to specify the type of extrema in your answer.

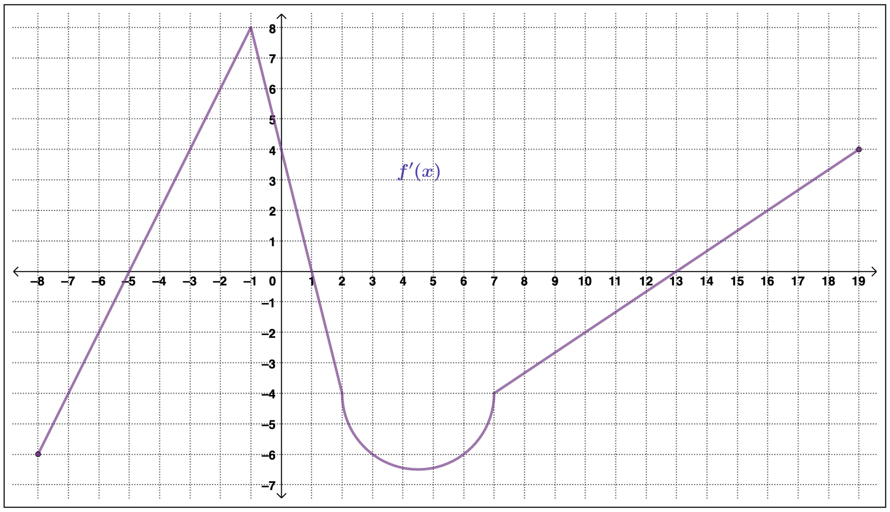
Concepts from Exam 3

16. Find $\int \frac{t^2+1}{\sqrt{t}}\, dt$

17. Evaluate $\int 3xe^{x^2+5}\, dx$.

18. Find $\int_1^b \left(\frac{2}{x^2}+6\right)\, dx$, where b is a real number and $b > 1$.

19. Use the graph of $f'(x)$ below to find $\displaystyle\int_{-3}^{7} f'(x)\,dx$.



20. Find all abosolute extrema of $g(x) = 2x^4 - 4x^2 + 1$ on $[-2, 3]$.

21. A small business sells wind chimes and has a revenue function (in dollars) of

$$R(x) = -\frac{1}{30}x^3 + 7x^2,$$

where x is the number of wind chimes produced and sold, and a cost function (in dollars) of

$$C(x) = 130x + 3000.$$

Use techniques of calculus to find the number of wind chimes the company would have to produce to **maximize profit** knowing the company will produce between 0 and 170 wind chimes.