TEXAS A&M UNIVERSITY
Math Learning Center

Math 251 - Fall 2024
"Hands On Grades Up"
Exam 2 Review
Thursday, Sept 26 and
Thursday, October 3,
6:30-8:30 PM
ZACH 340/353

Exam 2 Review: Covering sections 14.1, 14.3-14.8

PLEASE SCAN THE QR CODE BELOW

We will begin at 6:30 PM. A problem will be displayed on the wall monitors. Collaborate with your table on how to solve each problem. If you have a question, raise your hand. At the end of a predetermined number of minutes, the solutions will be displayed on the table monitors. Feel free to take a picture of the solution, as the solutions are not posted.

(1) Shade the domain of $f(x, y) = \ln(36 - 9x^2 - 4y^2) + \sqrt{x - y}$ in the xy-plane. Clearly indicate, using dashes, whether a boundary curve is or is not included.

(2) Consider the level curves for $f(x, y) = xy$. Sketch and label the level curves for $k = 1, -1, 0$.

(3) If $f(x, y) = \sin(x^2 + y^2)$, find $\mathrm{f}_{xy}(\sqrt{\pi}, 0)$.

(4) If $z = f(x, y) = 2x^2 + y^2$:

a.) Find the tangent plane at the point $(1, 3)$.

b.) Use this plane to approximate $2(1.01)^2 + (2.98)^2$.

(5) Find the tangent plane and normal line to the surface $2xy + 3yz + 7xz = -9$ at the point $(1, 2, -1)$.

(6) The height of a cone was measured to be 3 cm with a maximum error of 0.1 cm and the radius of the cone is measured to be 2 cm with a maximum error of 0.2 cm. Use differentials to estimate the maximum error in the calculated volume of the cone.

(7) Use differentials to approximate $f(x, y, z) = x^2 + y^3 + z^4$ at the point $(3.01, 2.1, 0.08)$.

(8) For $w = x^2 - y^2$, $x = s\cos t$, $y = s\sin t$, find $\dfrac{\partial w}{\partial s}$ and $\dfrac{\partial w}{\partial t}$ when $s = 3$ and $t = \dfrac{\pi}{4}$.

(9) If $e^y \sin x = x + xy$, find $\dfrac{dy}{dx}$.

(10) The radius and height of a circular cylinder change with time. When the height is 1 meter and the radius is 2 meters, the radius is increasing at a rate of 4 meters per second and the height is decreasing at a rate of 2 meters per second. At that same instant, find the rate at which the volume is changing.

(11) Let $f(x, y) = \sqrt{xy}$.

a.) Find the directional derivative of f at the point $P(4, 1)$ in the direction from P to $Q(6, 2)$.

b.) What is the maximum rate of change of f at the point $P(4, 1)$?

c.) Find the directional derivative of f at the point $P(4, 1)$ in the direction of the angle corresponding to $\theta = \dfrac{4\pi}{3}$.

(12) Consider $f(x, y, z) = \ln(2x + 3y + 4z)$.

a.) At the point $P(4, 1, 2)$, in what direction does f increase the fastest and what is the largest rate of increase of f?

b.) At the point $P(4, 1, 2)$, in what direction does f decrease the fastest and what is the largest rate of decrease of f?

(13) For the $f(x, y) = 2x^3 - xy^2 + 5x^2 + y^2 + 5$:

a.) Find all critical points of $f(x, y)$.

b.) Classify these critical points as local minima, maxima, or saddle points of $f(x, y)$.

(14) Find the absolute maximum and minimum values of $f(x, y) = 7 + xy - x - 2y$ over the closed triangular region with verticies $(1, 0)$, $(5, 0)$, $(1, 4)$. Your solution must include an analysis of f on the boundary curve(s).

(15) Use the method of Lagrange to find the maximum and minimum values of $f(x, y) = y^2 - x^2$ subject to the constraint $\frac{1}{4}x^2 + y^2 = 25$.