Math 152/172

WEEK in REVIEW 10

Spring 2025.

- 1. Find $f^{(152)(0)}$, the 152nd derivative for the function $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{3^n (n+2)}$
- 2. Find the Taylor series expansion for the following functions
 - (a) xe^{3x} centered at x = 5
 - (b) $\ln(1+x)$ centered at x=2
- 3. Find the Maclaurin series for f(x).
 - (a) $f(x) = x^2 \cos(2x)$
 - (b) $f(x) = xe^{-x^2}$
 - (c) $f(x) = \sqrt[3]{8+x}$
- 4. Evaluate the integral

(a)
$$\int 5x^2 \arctan(7x^3) dx$$

(b)
$$\int_0^x e^{-t^2} dt$$

5. Find the sum of the following series

(a)
$$f(x) = \sum_{n=2}^{\infty} \frac{(-1)^n 3^n \pi^n}{n!}$$

(b) $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n 3^{2n+1}}{(2n+1)!}$
(c) $f(x) = \sum_{n=2}^{\infty} \frac{(-1)^n \pi^{2n}}{6^{2n}(2n)!}$

- 6. Find the third degree Taylor polynomial for $f(x) = \sqrt{x}$ at x = 4.
- 7. Find the second degree Taylor polynomial for $f(x) = \arctan(x)$ at x = 1.
- 8. Find the radius and interval of convergence for the series $\sum_{n=1}^{\infty} \frac{3^n (x-5)^n}{n^2+1}$
- 9. If the power series $\sum_{n=0}^{\infty} c_n (x-2)^n$ converges at x = 4 and diverges at x = -2, which of the following series will also converge?

(a)
$$\sum_{n=0}^{\infty} c_n (-1)^n 2^n$$

(b)
$$\sum_{n=0}^{\infty} c_n 7^n$$

(c)
$$\sum_{n=0}^{\infty} c_n (-1)^n 2^{-n}$$

- 10. Find the power series representation for the function $f(x) = \frac{x^3}{(5-3x^2)^2}$
- 11. Which of the following series converge. State the test you have used.

(a)
$$\sum_{n=2}^{\infty} \frac{n^2 - 2n - 1}{n^2 + 4n}$$

(b) $\sum_{n=0}^{\infty} \frac{1}{n^2 + 2n + 4}$
(c) $\sum_{n=1}^{\infty} ne^{-n^2}$
(d) $\sum_{n=0}^{\infty} \frac{1}{n!}$
(e) $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{n}}$

12. Which of the following series converges absolutely?

(a)
$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$$

(b)
$$\sum_{n=2}^{\infty} \frac{(-1)^n \ln n}{n}$$

(c)
$$\sum_{n=2}^{\infty} \frac{(-1)^n}{n^4}$$

13. How many terms is needed to approximate the sum of the series $\sum_{n=1}^{\infty} \frac{1}{n^5}$ within 2×10^{-9} ?

14. Use the fifth partial sum to approximate the sum of the series $\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{(n+3)!}$. Find the upper bound for the error in the estimate.