MATH 140: WEEK-IN-REVIEW 1 (2.1 & 2.2)

1. Find the slope of the line that passes through the points $(-1, 4)$ and $(2, 8)$.

$$\text{slope} = m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8-4}{2-(-1)} = \frac{4}{3}$$

2. Find the equation of the line that passes through the point $(-2, 3)$ and

(a) has slope $-\frac{1}{2}$

point-slope formula: $y - y_1 = m(x - x_1)$

$m = \text{slope} = -\frac{1}{2}$

$(x_1, y_1) = (-2, 3)$

$$y - 3 = -\frac{1}{2}(x - (-2))$$

$$y - 3 = -\frac{1}{2}(x+2) \Rightarrow y = -\frac{1}{2}(x+2) + 3$$

(b) the slope is undefined

slope is undefined if line is <u>vertical</u>

i.e. $x = \text{const.}$

$$x = -2$$

(c) the slope is 0

slope is zero if line is <u>horizontal</u>

i.e. $y = \text{const.}$

$$y = 3$$

(d) also passes through the point $(1, -2)$.

* find slope * $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{3-(-2)}{-2-1} = \dfrac{-5}{3}$

* point-slope form * $y - y_1 = m(x - x_1)$

$$y - (-2) = -\frac{5}{3}(x-1)$$

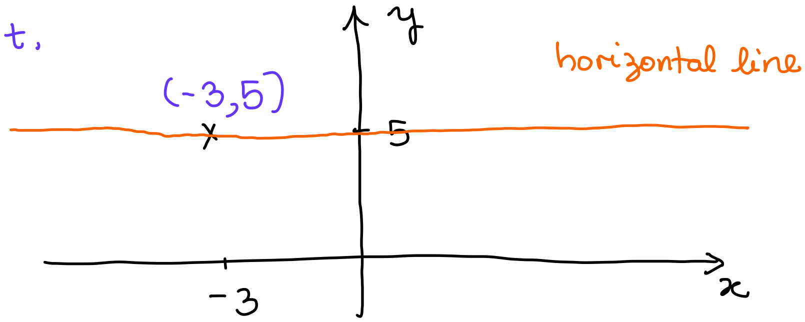
$$y + 2 = -\frac{5}{3}(x-1) \Rightarrow y = -\frac{5}{3}(x-1) - 2$$

3. Determine the equation of the line that passes through the point $(-3, 5)$ and is

(a) horizontal

$y = \text{const.}$

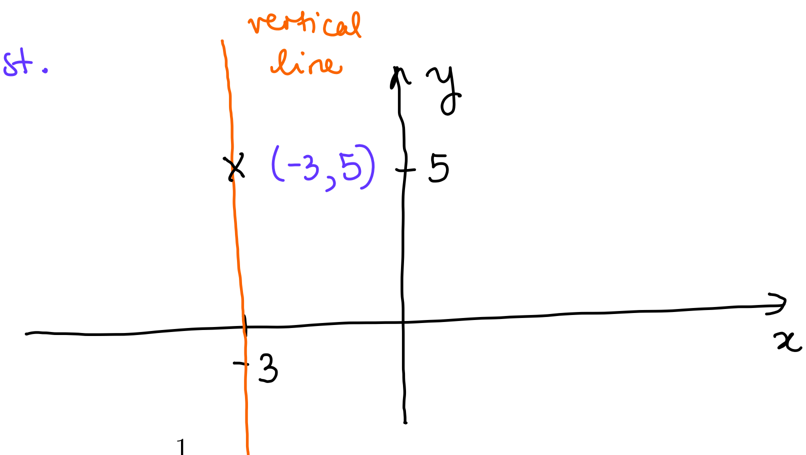
$\boxed{y = 5}$



(b) vertical

$x = \text{const.}$

$\boxed{x = -3}$



4. Determine the equation of the line with slope $\frac{1}{5}$ and x-intercept $(-1/2, 0)$. Leave your answer in standard form.

* point-slope formula * $\quad y - y_1 = m(x - x_1)$, $m = \frac{1}{5}$

$(x_1, y_1) = (-\frac{1}{2}, 0)$

$$y - 0 = \frac{1}{5}\left(x - \left(-\frac{1}{2}\right)\right) \Rightarrow y = \frac{1}{5}\left(x + \frac{1}{2}\right)$$

$$\Downarrow$$

$$5y = x + \frac{1}{2} \Rightarrow 10y = 2x + 1$$

$\boxed{-2x + 10y = 1}$ $\rightarrow$ standard form (integer coefficients)

5. Determine the equation of the line with slope -1 and y-intercept $(0, 5)$. Leave your answer in slope-intercept form.

* slope-intercept form * $\quad y = mx + b$

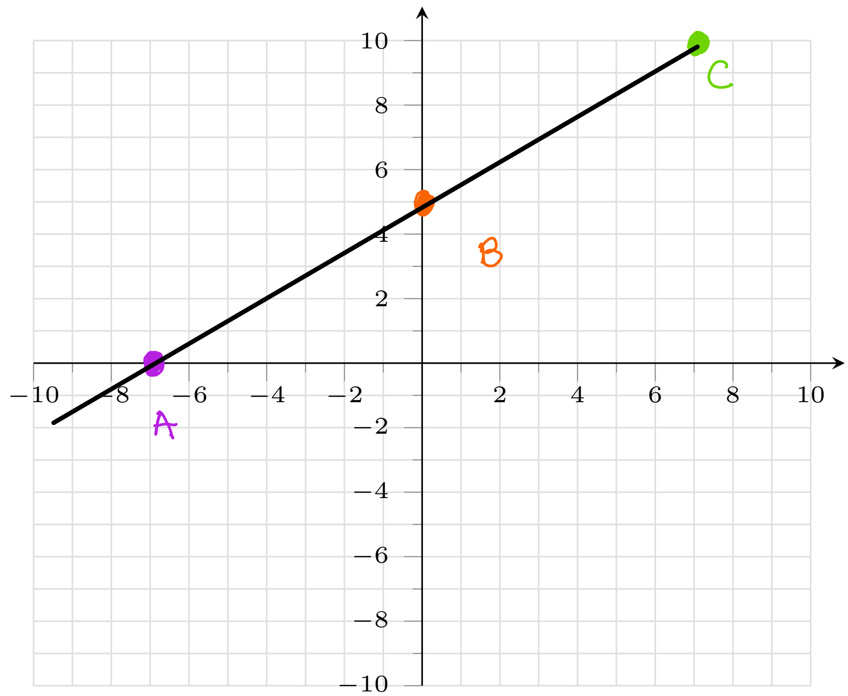
y-intercept: $(0, b)$

m = slope

$$y = (-1)x + 5$$

$\boxed{y = -x + 5}$

6. On the grid below, use intercepts to accurately graph the line $7y - 5x = 35$



x-intercept

$y = 0$

$7(0) - 5x = 35$

$-5x = 35$

$x = -7$

$A(-7, 0)$

y-intercept

$x = 0$

$7y - 5(0) = 35$

$7y = 35$

$y = \frac{35}{7} = 5$

$B: (0, 5)$

$x = 7$:

$7y - 5(7) = 35$

$7y = 35 + 35 = 70$

$y = 10$

$C: (7, 10)$

7. The equation of a line is given in standard form by $3y + 2x = -9$.

(a) Determine the slope of the line. * write in slope-intercept form *

$3y + 2x = -9 \Rightarrow 3y = -2x - 9$

$y = -\frac{2}{3}x - \frac{9}{3} = \frac{-2}{3}x - 3$

$y = \frac{-2}{3}x - 3$

slope $= -2/3$

(b) Determine the x-intercept of the line.

@ the x-intercept, $y = 0$.

$3(0) + 2x = -9 \Rightarrow 2x = -9 \Rightarrow x = -9/2$

x-intercept: $(-9/2, 0)$

(c) Determine the y-intercept of the line.

@ the y-intercept, $x = 0$

$3y + 2(0) = -9 \Rightarrow 3y = -9 \Rightarrow y = \frac{-9}{3} = -3$

y-intercept: $(0, -3)$

(d) If x increases, does y increase or decrease?

slope $= -2/3$ is NEGATIVE

if x increases, then y decreases (opposite change)

(e) Suppose $\underbrace{x \text{ increases by 6 units}}_{}$, what is the $\underbrace{\text{corresponding change in } y}_{}$?

$\Delta x = 6$ $\Delta y = ?$

$m = \frac{\Delta y}{\Delta x} \Rightarrow \Delta y = m\Delta x = (-2/3)(6) = -4$

$\Delta y = -4$

y decreases by 4 units

(f) If y decreases by 4 units, what is the corresponding change in x?

$\Delta y = -4$ $\Delta x = ?$

$m = \frac{\Delta y}{\Delta x} \Rightarrow \Delta x = \frac{\Delta y}{m} = \frac{-4}{(-2/3)} = 4 \cdot \frac{3}{2} = 6 \Rightarrow \Delta x = 6$

x increases by 6 units

$V(0)$

8. You buy an electronic gadget for \$3,000. Suppose that the value of the gadget depreciates by the same amount every year until the gadget reaches scrap value. After 5 years, the gadget is worth \$1,750.

   (a) Determine the value $V(t)$ of the electronic gadget, in dollars, after t years.

$V(t) = mt + b$, b = purchase price = 3000

$(t, V(t))$

$(0, 3000)$
$(5, 1750)$
$\Downarrow$

$m = \dfrac{3000-1750}{0-5} = -250$ ↳ $|m|$ is the rate of depreciation

$V(t) = -250t + 3000$ where $V(t)$ is the value (in dollars) at time t years

   (b) Determine the rate of depreciation of the gadget.

The rate of depreciation is \$250 per year.

↑ don't forget units!

   (c) Determine the number of years for the gadget to reach scrap value.

At scrap value, $V(t) = 0$. (assume scrap value is zero, since it is not provided)

$0 = -250t + 3000 \Rightarrow 250t = 3000$

$t = \dfrac{3000}{250} = 12$ years

   (d) Determine the value of the gadget after 90 months.

90 months $= \dfrac{90}{12}$ years $= 7.5$ years

$V(7.5) = -250(7.5) + 3000 = \$1,125$

   (e) Determine the value of the gadget after 15 years.

Time to reach $\underbrace{\text{scrap value}}_{\$0}$ = 12 years

$V(15) = 0$, \$0 after 15 years

9. A machine purchased in 2015 was worth \$8,000 in 2020. According to the manufacturer, the machine reaches a scrap value of \$1,000 after 12 years of service. Assuming the machine depreciates by the same amount each year, $t=0$ (2015), $t=5$ (2020)

 (a) What was the purchase price of the machine ?

$(t, V(t))$

$(5, 8000)$

$(12, 1000)$

scrap value

$V(t) = mt + b$ find b = purchase price

$$m = \frac{\Delta V}{\Delta t} = \frac{8000-1000}{5-12} = \frac{7000}{-7} = -1000$$

$$V - 1000 = -1000(t - 12)$$

$$V - 1000 = -1000t + 12000$$

$$V = -1000t + 13000$$ purchase price

The machine was bought for \$13000 in 2015

 (b) Determine the value of the machine in 2028.

In 2028, $t = 2028 - 2015 = 13$ years > 12 years (time to reach scrap value)

$V(13) = 1000$ scrap value

In 2028, the machine is worth the scrap value of \$1,000.

10. A company making shoes has fixed costs of \$50,000 each month. The cost of making one pair of shoes is \$15 and each pair of shoes sells for \$70.

(a) Determine the linear cost, revenue, and profit functions for the shoe company.

Cost

$C(x) = mx + F$ (fixed costs) $= 15x + 50000$

production cost per unit

$C(x) = 15x + 50000$

Revenue

$R(x) = px$ (selling price)

$= 70x$

$R(x) = 70x$

Profit

$P(x) = R(x) - C(x)$

$= 70x - (15x + 50000)$

$= 55x - 50000$

$P(x) = 55x - 50000$

(b) What is the company's profit if 2000 pairs of shoes are made and sold?

* Find $P(x)$ when $x = 2000$ *

$P(2000) = 55(2000) - 50000$

$= 60000$

* When 2000 pairs of shoes are produced and sold, the profit made is \$60,000 *

(c) How many shoes need to be sold in order for the company to reach a profit of \$5,000?

* Find x when $P(x) = 5000$ *

$5000 = 55x - 50000$

$55x = 55000$

$x = 1000$

* To reach a profit of \$5,000 the company needs to sell 1000 pairs of shoes *

11. The total cost of producing 500 items is \$12,500. If no items are produced, \$7,500 is still spent on fixed costs. Each item sells for \$25 dollars.

(a) What is the production cost for each item?

$(x, C(x))$: $(500, 12500)$, $(0, 7500)$

$m = \frac{\Delta C}{\Delta x} = \frac{12500 - 7500}{500 - 0} = \frac{5000}{500} = 10$

$C(x) = mx + F$ ← fixed costs

m: production cost per unit

* The production cost per unit, m, is \$10

(b) Determine the linear cost, revenue, and profit functions.

Cost

$C(x) = mx + F$

$= 10x + 7500$

$C(x) = 10x + 7500$

Revenue

$R(x) = px$ ← selling price

$= 25x$

$R(x) = 25x$

Profit

$P(x) = R(x) - C(x)$

$= 25x - (10x + 7500)$

$= 15x - 7500$

$P(x) = 15x - 7500$

Where $C(x), R(x), P(x)$ are the linear cost, revenue, and profit functions in dollars, respectively

(c) How many gadgets need to be sold for the revenue to equal the total costs?

* Find x such that $R(x) = C(x)$ *

$R(x) = C(x)$

$25x = 10x + 7500$

$15x = 7500$

$x = \frac{7500}{15} = 500$

OR

$P(x) = 0$

$15x - 7500 = 0$

$15x = 7500$

$x = \frac{7500}{15} = 500$

For the revenue to equal total costs, 500 items need to be sold.

12. A leather belt manufacturer has production costs of \$20 for each belt produced, and incurs \$144,000 in fixed costs each year.

$C(x) = mx + F$ → fixed costs
↳ production cost per unit

(a) At the end of the year, the manufacturer has a profit of \$36,000 after selling 6000 belts. What is the selling price of the belts?

$m = 20,\ F = 144\,000 \Rightarrow C(x) = 20x + 144\,000$

Let the selling price be p. Then $R(x) = px$.

$$P(x) = R(x) - C(x) = px - (20x + 144000)$$
$$= (p-20)x - 144\,000$$

When $P(6000) = 36\,000$, then

$$36\,000 = (p-20)(6000) - 144\,000$$
$$6\,000p = 36\,000 + 144\,000 + (20)(6000)$$
$$= 300\,000$$
$$p = \frac{300\,000}{6\,000} = 50$$

Selling price = \$50 per belt

(b) What is the profit function for the company?

$$P(x) = (p-20)x - 144\,000, \quad p = 50$$
$$= (50-20)x - 144\,000$$
$$= 30x - 144\,000$$

The profit function is $P(x) = 30x - 144000$

13. Suppose $x =$ number of widgets supplied or demanded per month and $p =$ the unit price for each widget (in dollars). Equation 1 is $2p - 3x = 12$ and Equation 2 is $2p + x = 20$

(a) Which of the two equations is the demand equation? Explain.

Slope intercept form : Equation 1 : $2p = 3x + 12 \Rightarrow p = \frac{3}{2}x + 6$

Equation 2 : $2p = -x + 20 \Rightarrow p = -\frac{1}{2}x + 10$

The demand equation has negative slope. So Equation 2 is the demand equation, and Equation 1 is the supply.

(b) How many units will consumers demand if the widgets are free?

$D(x) = p(x) = 0$ (if free)

$0 = -\frac{1}{2}x + 10 \Rightarrow \frac{1}{2}x = 10 \Rightarrow x = 20$ widgets

(c) What is the highest price consumers are willing to pay for the widgets?

The highest price consumers are willing to pay is the p-intercept of the demand equation (when $x = 0$)

$p = -\frac{1}{2}(0) + 10 = 10$.

Consumers are willing to pay a maximum of \$10 per widget

(d) Suppliers will only provide the widgets if the price is above what value?

The minimum price for suppliers to provide the widgets is the p-intercept of the supply eqn, (when $x = 0$)

$p = \frac{3}{2}(0) + 6 = 6$

Suppliers will provide widgets if the price per widget is \$6 or more

S(x): (800, 20)

14. If the price per book is \$20, publishers will market 800 books. Consumers will take 9000 books if they were offered for free. If the book price is increased by \$1, consumers will buy 300 less books, while publishers will sell 100 more books.

D(x): (9000, 0)   S(x): (900, 21)   D(x): (8700, 1)

(a) Determine the linear supply and demand equations, $p(x)$, where x is the number of books provided or bought at a price of p dollars.

Supply

$S(x) = p(x) = mx + b$

$(x, S(x))$: $(800, 20)$, $(900, 21)$

$$m = \frac{\Delta p}{\Delta x} = \frac{21-20}{900-800} = \frac{1}{100}$$

$$p - 20 = \frac{1}{100}(x - 800) = \frac{1}{100}x - 8$$

$$p = \frac{1}{100}x + 12$$

Demand

$D(x) = p(x) = mx + b$

$(x, D(x))$: $(9000, 0)$, $(8700, 1)$

$$m = \frac{\Delta p}{\Delta x} = \frac{1-0}{8700-9000} = \frac{-1}{300}$$

$$p - 0 = \frac{-1}{300}(x - 9000)$$

$$p = -\frac{1}{300}x + 30$$

(b) At what price will 2325 books be supplied?

Supply equation:

$$p(2325) = \frac{1}{100} \cdot 2325 + 12$$
$$= 23.25 + 12$$
$$= 35.25$$

At a price of \$35.25 per book, publishers will supply 2325 books.

(c) How many books will be demanded at a price of \$15 per book?

Demand Equation

* Find x when $p(x) = 15$

$$p(x) = -\frac{1}{300}x + 30$$

$$15 = -\frac{1}{300}x + 30$$

$$\frac{1}{300}x = 30 - 15 = 15 \Rightarrow x = 15 * 300 = 4500$$

At a price of \$15 per book, 4500 books will be demanded.