Math 151 - Week-In-Review 5

Topics for the week:

3.1 Derivatives of Polynomials and Exponential Functions
3.2 The Product and Quotient Rules

3.1 Derivatives of Polynomials and Exponential Functions

1. Compute the derivative of $f(x) = 3x^5 - 4x^2 + 6$.

2. Compute $\dfrac{dg(t)}{dt}$ for $g(t) = \dfrac{1}{3}t^8 - \dfrac{2}{7t^4} + 6t^{-3}$.

3. For $y = \dfrac{1 + x - 4\sqrt{x}}{x}$, find $\dfrac{dy}{dx}$.

4. Compute the derivative of f with respect to t, $f(t) = \dfrac{7}{3x^2} - \dfrac{5}{2x} - \dfrac{3}{e^{-x}}$.

5. Compute $f'(x)$ for $f(x) = 11e^x + e^{11}x$.

6. Compute $g'(z)$ for $g(z) = \sqrt[5]{z} + 10\sqrt[6]{z^5}$.

7. Given a position function of $s(t) = \dfrac{(t^3+1)(t^2-t+1)}{t^4}$, determine the corresponding velocity and acceleration functions.

8. For $y = \frac{4}{9}x^3 z + 3x^2 z^7$, compute $\frac{dy}{dx}$ and $\frac{dy}{dz}$.

9. Write the equation for the line tangent to the curve $y = x\sqrt{x} - \frac{9}{2x}$ at $x = 4$.

10. Find the equation of the line perpendicular to the tangent to the curve $y = x^3 - 4x + 1$ at the point $(2, 1)$. What is the smallest slope on the curve?

11. Determine the values of a and b such that f is differentiable everywhere.

$$f(x) = \begin{cases} x^4 + bx + 2 & \text{for} \quad x \leq 0 \\ \frac{1}{2}e^x + a & \text{for} \quad x > 0 \end{cases}$$

3.2 The Product and Quotient Rules

12. For $y = \left(1 + x - 4\sqrt{x}\right)(e^x)$, find $\dfrac{dy}{dx}$.

13. Compute the derivative of $f(x) = ax^{1/3}(x+1) - x^{-3/4}$ with respect to x.

14. Suppose $f(2) = 3$ and $f'(2) = \frac{1}{4}$, determine $\frac{d}{dx}[xf(x)]$ at $x = 2$.

15. Compute $g'(p)$ for $g(p) = \frac{3e^p}{6p^2 - 8p}$.

16. Given $y = \frac{x^3 - 11}{1 - x^2}$, compute both the first and second derivative of y with respect to x.

17. Differentiate $f(x) = \dfrac{ax^2}{k^2 - x^2}$ with respect to x. Assume that a and k are positive constants. Identify the values of x for which $f(x)$ is not differentiable.

18. Write the equation of the line tangent to the curve $y = \dfrac{8}{x^2 + 4}$ at the point $(2, 1)$.

19. The functions f and g that satisfy the properties as shown in the table. Compute the indicated quantity.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	1	-3	3	5
1	2	9	7	11
2	-5	0	2	10
3	4	-1	-4	8

(a) $H'(3)$ if $H(x) = (x^3 + 2)g(x)$

(b) $\frac{d}{dx}\left(\frac{x^3}{f(x)}\right)\Bigg|_{x=1}$