Week in Review

Math 152

Week 03

Common Exam I Prep

(5.5 - 7.1)

Common Exam I Prep

1. Compute $\int_0^{\sqrt{\pi}} x\sin(\pi - x^2)\,dx$

   (a) $-\frac{\sin\sqrt{\pi}}{2}$
   (b) -2
   (c) -1
   (d) 1 ← correct
   (e) 2

Rewrite : $\int_0^{\sqrt{\pi}} \sin(\pi - x^2)\,(xdx)$

Id f and g'

- $f =$
- $g =$
- $g' =$

$f = \sin x$
$g = \pi - x^2$
$g' = -2x$

u-sub

$u = \pi - x^2$
$du = -2xdx$
$\Rightarrow xdx = -\frac{1}{2}du$

complete substitution for the limits

$\int_{x=0}^{x=\sqrt{\pi}} \Rightarrow \int_{\pi-0^2}^{\pi-\sqrt{\pi}^2}$

$\int_a^b f(x)dx = -\int_b^a f(x)dx$

Evaluate the integral:

$\int_\pi^0 \sin u\left(-\frac{1}{2}du\right)$
$= \frac{1}{2}\int_0^\pi \sin u\,du$
$= \frac{1}{2}[-\cos u]_0^\pi$
$= \frac{1}{2}[-\cos\pi + \cos 0] = 1$

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2. Compute $\int_1^2 x \ln(x^2)\, dx$.

(a) $\frac{\ln 4}{2}$

(b) $\ln 4$

(c) $4 \ln 4 - 3$

(d) $\frac{3}{2}$

(e) $\ln 16 - \frac{3}{2}$ $\leftarrow$ correct

u –sub : $u = x^2 \quad \Rightarrow du = 2x dx$

$$= \frac{1}{2} \int_1^4 \ln u \, du$$

$$= \frac{1}{2} [u \ln|u| - u]_1^4$$

$$= \frac{1}{2} [4 \ln 4 - (4 - 1)]$$

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3. Which of the following gives the area of the region bounded by $y = |x^2 - 1|$ and x-axis on $[-2, 2]$.

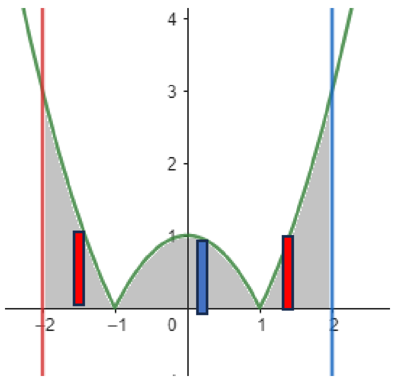
(a) $\displaystyle\int_{-2}^{2} (x^2 - 1)\, dx$

(b) $\displaystyle\int_{-2}^{1} (x^2 - 1)\, dx + \int_{1}^{2} -(x^2 - 1)\, dx$

(c) $\displaystyle\int_{-2}^{-1} (x^2 - 1)\, dx + \int_{-1}^{2} -(x^2 - 1)\, dx$

(d) $\displaystyle\int_{-2}^{-1} -|x^2 - 1|\, dx + \int_{-1}^{1} |x^2 - 1|\, dx + \int_{1}^{2} -|x^2 - 1|\, dx$

(e) $\displaystyle\int_{-2}^{-1} (x^2 - 1)\, dx + \int_{-1}^{1} -(x^2 - 1)\, dx + \int_{1}^{2} (x^2 - 1)\, dx$ ← correct



$(1 - x^2)$

dx

$A(\;) = (1 - x^2)dx$

$(x^2 - 1)$

dx

$A(\;) = (x^2 - 1)dx$

$$A = \int_{-2}^{-1}(x^2 - 1)dx + \int_{-1}^{1}(1 - x^2)dx + \int_{1}^{2}(x^2 - 1)dx$$

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4. Which of the following integrals gives the area of the region bounded by the curves $x = y^2$ and $x = 6 - y$?

(a) $\int_{-3}^{2} (6 - y - y^2)\,dy$ ← correct

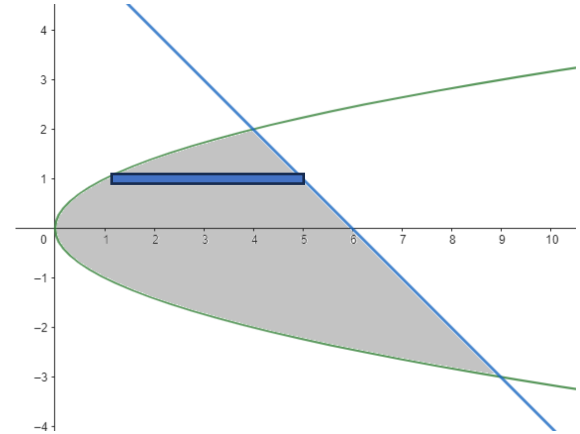
(b) $\int_{-3}^{2} (y^2 - 6 + y)\,dy$

(c) $\int_{4}^{9} (6 - x - \sqrt{x})\,dy$

(d) $\int_{4}^{9} (\sqrt{x} - 6 + x)\,dy$

(e) $\int_{4}^{9} (6 - y - y^2)\,dy$

Plot



Slice

$A(\,===\,) = [(6 - y) - (y^2)]dy$

Intersections

$6 - y = y^2$

$y^2 + y - 6 = 0$

$(y - 2)(y + 3) = 0$

$y = -3, 2$

Area between curve

$\int_{-3}^{2} (6 - y - y^2)dy$

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5. The region bounded by $y = e^x$ and the x-axis on the interval $[0, 2]$ is rotated about the x-axis. Find the volume of the resulting solid.

   (a) $\frac{\pi e^4}{2}$

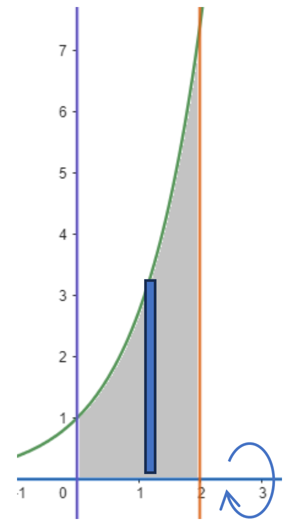
   (b) $\frac{\pi e^2}{2}$

   (c) $\frac{\pi}{2}(e^4 - 1)$ ← correct

   (d) $\frac{\pi}{2}(e^2 - 1)$

   (e) $2\pi(e^4 - 1)$

Plot



Slice

$$V(\ ) = \pi(e^x)^2 dx$$

$$\pi e^{2x} dx$$

limits

$$dx \in [0,2]$$

Volume

$$\int_0^2 \pi e^{2x} dx$$

$$= \pi \left[\frac{1}{2} e^{2x}\right]_0^2$$

$$= \frac{\pi}{2}(e^4) - 1$$

6. Consider the region bounded by the curves $x = y^2 - 2y$ and the y-axis. Which of the following represents the volume of solid formed when the region is rotated about $y = 4$?

(a) $\int_0^2 2\pi y(y^2 - 2y)\,dy$

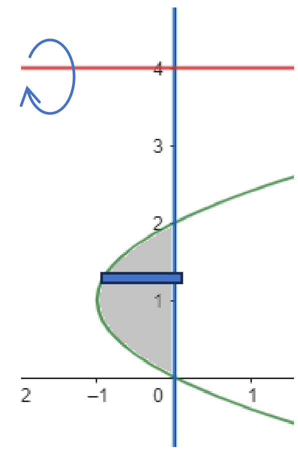
(b) $\int_0^2 2\pi y(2y - y^2)\,dy$

(c) $\int_0^2 2\pi(4 - y)(y^2 - 2y)\,dy$

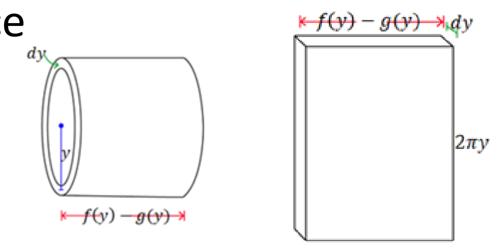
(d) $\int_0^2 \pi(y - 4)(4y^2 - y^4)\,dy$

(e) $\int_0^2 2\pi(4 - y)(2y - y^2)\,dy$ ← correct

Plot



Slice



$V(\;) = 2\pi(4 - y)(0 - [y^2 - 2y])dy$

limit

$dy \in [0,2]$

Volume

$\int_0^2 2\pi(4 - y)(2y - y^2)dy$

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7. Consider the region bounded by the two curves $y = \cos x$, $y = \sin x$ and the two lines $x = 0$ and $x = \frac{\pi}{4}$. Which of the following represents the volume of this region being rotated about the line $x = -1$?

(a) $\int_0^{\frac{\pi}{4}} 2\pi(x+1)(\cos x - \sin x)\,dx$ ← correct

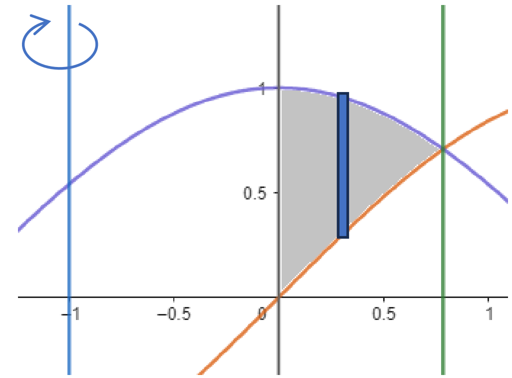
(b) $\int_0^{\frac{\pi}{4}} 2\pi(x+1)(\sin x - \cos x)\,dx$

(c) $\int_{-1}^{\frac{\pi}{4}} 2\pi(x+1)(\cos x - \sin x)\,dx$

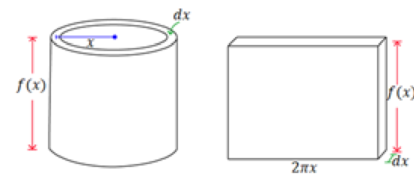
(d) $\int_0^{\frac{\pi}{4}} 2\pi(x+1)(\cos^2 x - \sin^2 x)\,dx$

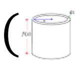
(e) $\int_0^{\frac{\pi}{4}} \pi(\cos^2 x - \sin^2 x)\,dx$

Plot



Slice



$V($  $) = 2\pi(x-(-1))(\cos x - \sin x)dx$

limit

$dx \in \left[0, \frac{\pi}{4}\right]$

Volume

$\int_0^{\pi/4} 2\pi(x+1)(\cos x - \sin x)dx$

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8. Find the area of the region determined by the curve $f(x) = x\sin x$ and the x-axis on the interval $[0, \pi]$.

(a) 1

(b) π ← correct

(c) $\frac{\pi}{2}$

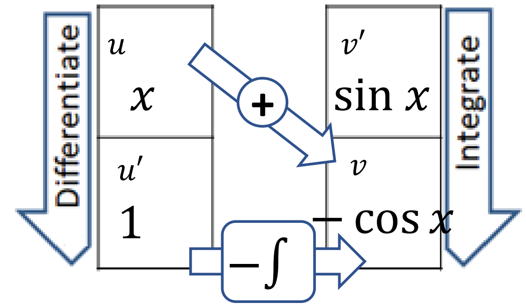
(d) $\pi - 1$

(e) $-\pi$

$$\int_0^{\pi} |x \sin x| dx$$
$$= \int_0^{\pi} x \sin x \, dx$$

u----**L I A T E**----v'

x $\sin x$



$$= -[x\cos x]_0^{\pi} + \int_0^{\pi} \cos x \, dx$$
$$= -[\pi \cos \pi - 0]_0^{\pi} + [\sin \pi]_0^{\pi}$$
$$= \pi$$

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9. Which of the following integrals gives the volume of the solid obtained by rotating the region bounded by $y = 5 - x^2$ and $y = 1$ about the x-axis.

(a) $\pi \int_{-2}^{2} \left(1 - (5 - x^2)^2\right) dx$

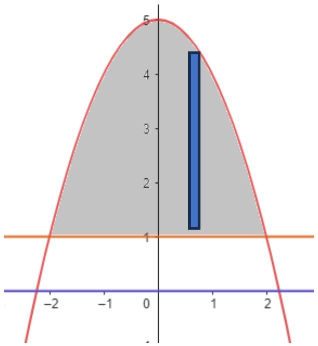
(b) $\pi \int_{-2}^{2} (4 - x^2)^2 \, dx$

(c) $2\pi \int_{-2}^{2} x(4 - x^2) \, dx$

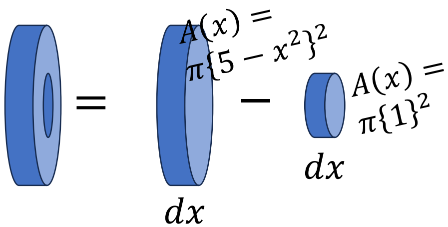
(d) $\pi \int_{-2}^{2} \left((5 - x^2)^2 - 1\right) dx$ ← correct

(e) $2\pi \int_{-2}^{2} x(x^2 - 4) \, dx$

Plot



Slice



$V($  $) = \pi(5 - x^2)^2 dx - \pi(1)^2 dx$

$\pi[(5 - x^2)^2 - 1]dx$

Limit

$5 - x^2 = 1 \quad \Rightarrow x^2 = 4 \quad \Rightarrow x = \pm 2$

$dx \in [-2,2]$

Volume

$\int_{-2}^{2} \pi[(5 - x^2)^2 - 1]dx$

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10. Find the volume of the solid obtained by rotating the region bounded by $x = y^2$ and $x = y^3$ around the y-axis.

(a) $\frac{\pi}{35}$

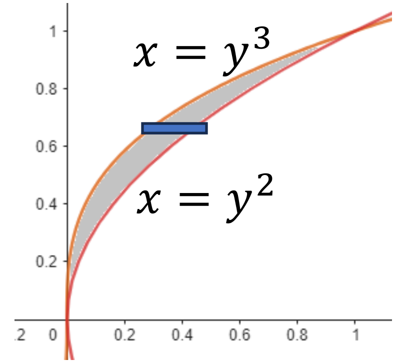
(b) $\frac{\pi}{10}$

(c) $\frac{\pi}{12}$

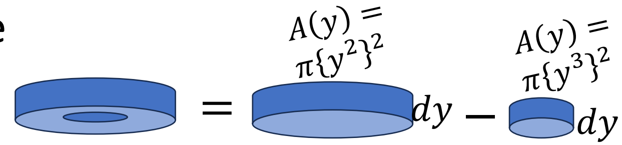
(d) $\frac{2\pi}{35}$ ← correct

(e) $\frac{\pi}{105}$

Plot



Slice



$V($  $) = \pi(y^2)^2dy - \pi(y^3)^2dy$

$\pi[y^4 - y^6]dy$

Limit

$y^2 = y^3 \quad \Rightarrow x = 0,1$

$dy \in [0,1]$

Volume

$\int_0^1 \pi(y^4 - y^6)dy$

$= \pi\left[\frac{1}{5}y^5 - \frac{1}{7}y^7\right]_0^1 = \pi\left[\frac{1}{5} - \frac{1}{7}\right] = \frac{2\pi}{35}$

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11. An ideal spring has a natural length of 10 meters. The work done in stretching the spring from 14 meters to 18 meters is 24J. Determine the spring constant k.

(a) $k = \frac{1}{2}\,\mathrm{N/m}$

(b) $k = \frac{3}{8}\,\mathrm{N/m}$

(c) $k = 1\,\mathrm{N/m}$ $\leftarrow$ correct

(d) $k = 3\,\mathrm{N/m}$

(e) $k = 6\,\mathrm{N/m}$

$F(x) = kx$

$dW = F(x)dx = kxdx$

Work done from x_0 to x_1 (from resting length)

$W = \int_{x_0}^{x_1} kxdx = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_0^2$

Work done from 14 to 18 (spring length)

Work done from 4 to 8 (from resting length)

$$24 = \frac{1}{2}k8^2 - \frac{1}{2}k4^2$$
$$= \frac{k}{2}(8-4)(8+4)$$
$$= \frac{k}{2}4 \cdot 12 = k24$$
$$k = 1$$

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12. A 90 ft cable weighing 10 lb is hanging down the side of a 200 ft building. How much work is required to pull the rope 30 feet up the side of the building?

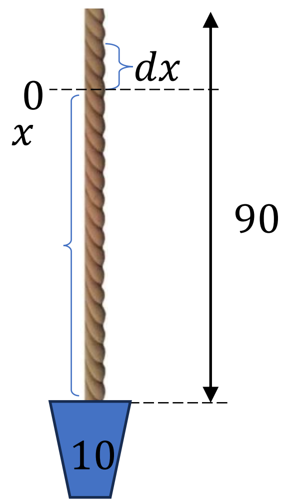
(a) 6000 ft-lb

(b) 1500 ft-lb

(c) 250 ft-lb ← correct

(d) 300 ft-lb

(e) 50 ft-lb



Work = (Force)x(distance)

=(weight)x(distance)

Force (weight) of the remaining cable(length $= x$)

$$\frac{10}{90} = \frac{w}{x} \Rightarrow w = \frac{1}{9}x$$

Work needed to lift the remaining cable(length $= x$) by dx

$$dW = \left(\frac{1}{9}x\right)dx$$

Work needed to lift the cable by 30 ft: $x \in [60,90]$

$$W = \int_{60}^{90}\left(\frac{1}{9}x\right)dx = \left[\frac{x^2}{18}\right]_{60}^{90}$$

$$= \frac{90^2-60^2}{2\cdot 9} = \frac{(90-60)(90+60)}{2\cdot 9}$$

$$= \frac{30\cdot 150}{2\cdot 9} = \frac{3\cdot 10\cdot 3\cdot 50}{2\cdot 9} = 250$$

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13. The solid S has a triangular base with vertices $(-1,0)$, $(1,0)$, and $(0,2)$. Cross sections perpendicular to the x-axis are squares. Find the volume of S.

(a) $\frac{4}{3}$

(b) $\frac{8}{3}$ $\leftarrow$ correct

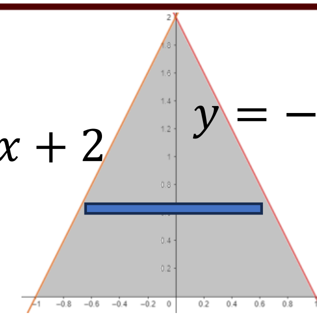
(c) $\frac{1}{3}$

(d) $\frac{2}{3}$

(e) $\frac{5}{3}$

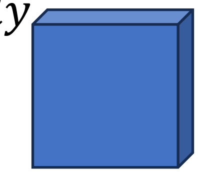
Plot

$y = 2x + 2$ $\quad$ $y = -2x + 2$



Slice

dy



$\frac{(2-y)}{2} - \frac{y-2}{2}$

$\frac{(2-y)}{2} - \frac{y-2}{2}$

$V($$) = (y-2)^2 dy$

limit

$dy \in [0,2]$

Volume

$\int_0^2 (y-2)^2 dy$

$= \frac{1}{3}[(y-2)^3]_0^2$

$= \frac{1}{3}[0 - (-2)^3]$

$= \frac{8}{3}$

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14. Compute $\int_0^1 \arctan x\, dx$.

(a) $\frac{\pi}{4} - \frac{1}{2}\ln 2$ ← correct

(b) $\frac{\pi}{4} - \ln 2$

(c) $1 - \frac{1}{2}\ln 2$

(d) $1 - \ln 2$

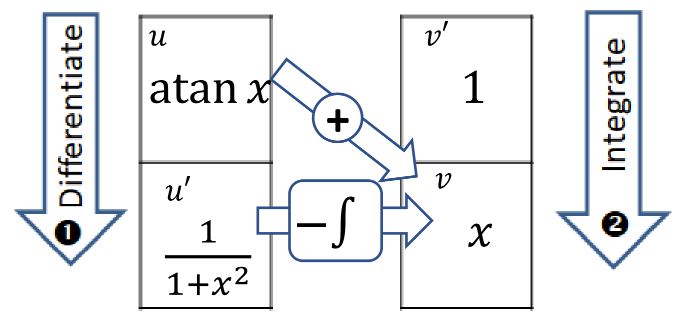
(e) $\frac{\pi}{4}$

Evaluate $\int \tan^{-1} x\, dx$ by the tabular method

Hint: $\int \tan^{-1} x\, dx = \int 1 \cdot \tan^{-1} x\, dx$

u----**L I A T E**----v'

$\tan^{-1} x \quad 1$



$$\int_0^1 \tan^{-1} x = [x\tan^{-1} x]_0^1 - \int_0^1 \frac{x}{1+x^2}\, dx$$

$$= [x\tan^{-1} x]_0^1 - \frac{1}{2}[\ln(1+x^2)]_0^1 \text{ (u-sub)}$$

$$= (\tan^{-1}(1) - 0) - \frac{1}{2}(\ln 2 - \ln 1)$$

$$= \frac{\pi}{4} - \frac{1}{2}\ln 2$$

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15. Evaluate $\int_0^1 \frac{x^2}{e^x}\,dx$.

(a) $2 - \frac{5}{e}$ ← correct

(b) $\frac{5}{e} - 2$

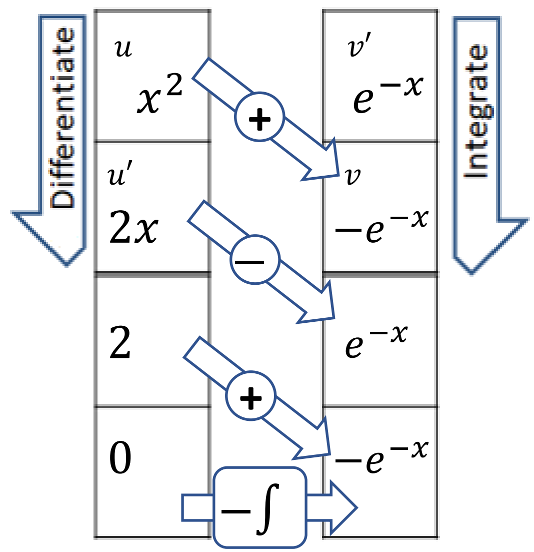
(c) $1 - \frac{3}{e}$

(d) $1 - \frac{2}{e}$

(e) $1 - \frac{1}{e}$

u----L I **A** T **E**----v'

x^2 e^{-x}



$$\int_0^1 \frac{x^2}{e^2}\,dx = [-x^2e^{-x} - 2xe^{-x} - 2e^{-x}]_0^1$$
$$= (-e^{-1} - 2e^{-1} - 2e^{-1}) - (-2)$$
$$= 2 - 5e^{-1}$$

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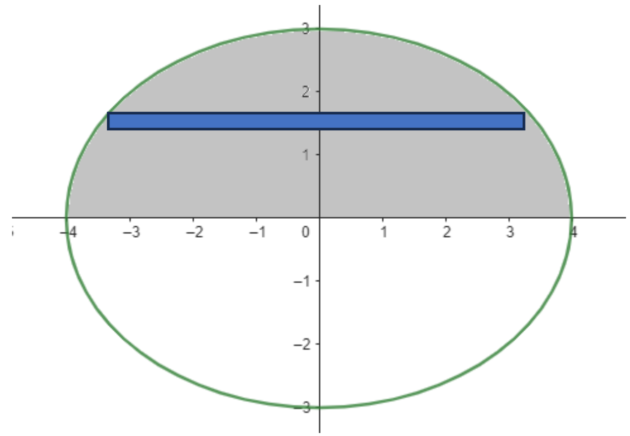
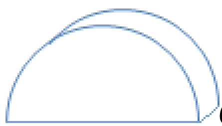
16. (10 points) Consider the solid whose base is the upper half of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$. Cross sections perpendicular to the y axis are semicircles. Find the volume of the solid.

Plot

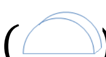


Slice



dy

$$A(y)=\frac{\pi}{2}[x(y)]^2$$

$V($  $) = \frac{\pi}{2}\left[16\left(1-\frac{y^2}{9}\right)\right]dy = 8\pi\left(1-\frac{y^2}{9}\right)dy$

limit

$dy \in [0,3]$

Volume

$$\int_0^3 8\pi\left(1-\frac{y^2}{9}\right)dy$$

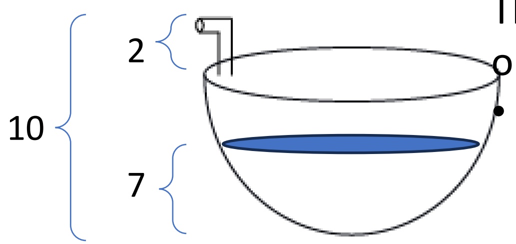
$$=8\pi\left[y-\frac{1}{27}y^3\right]_0^3$$

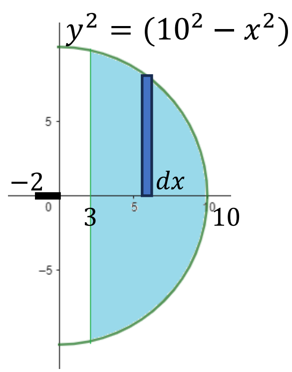
$$=8\pi[3-1]$$

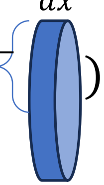
$$=16\pi$$

Common Exam I Prep

17. (10 points) A hemispherical tank has the shape shown below. The tank has a radius of 10 meters with a 2 meter spout at the top of the tank. The tank is filled with water to a depth of 7 meters. The weight density of water is $\rho g = 9800\text{N}/\text{m}^3$. Suppose we want to find the work required to pump the water through the spout





$V(\sqrt{10^2 - x^2}$  $) = \pi r^2 dx$

$= \pi(10^2 - x^2)dx$

The tank shown is full of water. Find the work required to pump the water out of the spout. (Use 9800 N/m^3 as water density)

- **Step 1** : plot a graph in the coordinate system (tank shape vs depth): Set the top of the tank = 0
- **Step 2:** Slicing the tank by dx height (Set the top = 0) and consider a slab at location x (to be lifted by x)
  - Find the volume of the disc at x
    - $dv = \pi(10^2 - x^2)dx$
- **Step 3:** Find the weight of water within the disc (=force, F)
  - **water weight = (water volume)x(weight density)**
    - $dF = \rho dv = 9800\pi(10^2 - x^2)dx$
- **Step 4.** Find the work done by pumping the water disc dF lb by a length of $x + 2$fts (due to spout).
  - $dW = (dF)x = [9800\pi(10^2 - x^2)dx](x + 2)$
    $= 9800\pi(10^2 - x^2)(x + 2)dx$
- **Step 5.** Find the total work by integrating dW (Limit ??)
  - $W = 9800\pi \int_3^{10} (10^2 - x^2)(x + 2)dx$

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18. (7 points) Compute $\int x^5 e^{x^3}\, dx$

Rewrite

$\int x^5 e^{x^3} dx = \int x^3 e^{x^3} (x^2 dx)$

u −sub

$u = x^3 \;\Rightarrow du = 3x^2 dx \quad \Rightarrow x^2 dx = \frac{1}{3} du$

$\int x^3 e^{x^3} (x^2 dx) = \int u e^u \left(\frac{1}{3} du\right)$

$= \frac{1}{3} \int u e^u du$

$= \frac{1}{3} [u e^u - e^u] + C$

$= \frac{1}{3} x^3 e^{x^3} - e^{x^3} + C$

Evaluate $\int x e^x dx$

u----**L**ogarithmic, **I**nverse trigonometric, **A**lgebraic, **T**rigonometric, **E**xponential----v'

x $\qquad$ e^x

u'-- 1 $\qquad$ e^x--v

$\underset{\int uv'dx =}{\int x e^x dx =} \quad \underset{uv}{x e^x} \quad \underset{-\int u'v dx}{-\int 1 \cdot e^x dx}$

$= x e^x - e^x + C$

Extra materials

1. The region bounded by the curves $y = x^2$, $y = 4$ and $x = 0$ is rotated about the line $y = 4$. Which of the following gives the volume of the resulting solid?

   (a) $\displaystyle\int_0^4 \pi(4 - x^2)^2 dx$

   (b) $\displaystyle\int_0^2 \pi(4 - x^2)^2 dx$

   (c) $\displaystyle\int_0^1 2\pi x(x^2 - 4) dx$

   (d) $\displaystyle\int_0^2 2\pi(4 - x)(x^2 - 4) dx$

   (e) $\displaystyle\int_0^4 \pi(4 - x)(x^2 - 4)^2 dx$

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2. Evaluate $\int x^5\sqrt{x^3+1}\,dx$

(a) $C+\frac{2}{15}(x^3+1)^{5/2}-\frac{2}{9}(x^3+1)^{3/2}$

(b) $C+\frac{1}{6}x^6\left(\frac{1}{4}x^4+x\right)^{1/2}$

(c) $C+5x^4(x^3+1)^{1/2}+\frac{3}{2}x^7(x^3+1)^{-1/2}$

(d) $C+\frac{2}{15}x^{15/2}+\frac{1}{6}x^6$

(e) $C+\frac{6}{19}(x^3+1)^{19/6}-\frac{2}{3}(x^3+1)^{3/2}$

4. The region bounded by the curves $y = e^x$, $y = 0$, $x = 0$ and $x = 3$ is rotated about the $x-$axis. Find the volume of the resultant solid.

   (a) $\frac{\pi}{2}(e^6 - 1)$
   (b) $\frac{\pi}{6}(e^9 - 1)$
   (c) $2\pi(e^6 - 1)$
   (d) $\pi(e^6 - 1)$
   (e) $\frac{\pi}{2}(e^3 - 1)$

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5. Evaluate $\int_0^2 x^3 e^{x^2}\, dx$

   (a) e^4
   (b) $\frac{1}{2}(3e^4+1)$
   (c) $\frac{1}{2}(3e^4-1)$
   (d) $\frac{1}{2}(5e^4-1)$
   (e) $2e^4-6$

6. The region bounded by the curves $y = 6x - x^2$ and $y = 5$ is rotated about the y–axis. Which of the following integrals gives the volume of the resulting solid?

(a) $2\pi \int_1^5 x(6x - x^2 - 5)\, dx$

(b) $2\pi \int_0^6 x(5 - 6x + x^2)\, dx$

(c) $\pi \int_0^6 (x - 5)(6x - x^2)^2\, dx$

(d) $\pi \int_5^9 (6x - x^2 - 5)^2\, dx$

(e) $2\pi \int_1^5 (5 - x)(6x - x^2 - 5)\, dx$

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7. Compute $\int \cos^3(x)\sin^2(x)\,dx$

(a) $C - \dfrac{\cos^5(x)}{5} + \dfrac{\cos^3(x)}{3}$

(b) $C + \dfrac{\cos^3(x)\sin^3(x)}{3} + \dfrac{\cos^4(x)\sin^2(x)}{4}$

(c) $C - \dfrac{\sin^5(x)}{5} + \dfrac{\sin^3(x)}{3}$

(d) $C + \dfrac{\sin^4(x)}{4} + \dfrac{\sin^2(x)}{2}$

(e) $C - \dfrac{\sin^6(x)}{6} + \dfrac{\sin^4(x)}{4} - \dfrac{\sin^2(x)}{2}$

8. The region bounded in the first quadrant by the curves $y = x^2$ and $4x - y = 0$ is rotated about the line $x = 10$. Which of the following integrals gives the volume of the resulting solid?

(a) $\pi \int_0^4 \left[(10 - 4x)^2 - (10 - x^2)^2\right] \, dy$

(b) $\pi \int_0^4 (4x - x^2)^2 \, dy$

(c) $\pi \int_0^{16} \left[\left(10 - \frac{y}{4}\right)^2 - (10 - \sqrt{y})^2\right] \, dy$

(d) $\pi \int_0^{16} \left[(\sqrt{y})^2 - \left(\frac{y}{4}\right)^2\right] \, dy$

(e) $2\pi \int_0^{16} \left[\left(16 - \frac{y}{4}\right)^2 - (16 - \sqrt{y})^2\right] \, dy$

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9. Evaluate $\int_0^{\pi} \sin(x)e^{\cos(x)}\,dx$

(a) $\frac{1}{e} - e$

(b) $-\frac{1}{e}$

(c) $\frac{1}{e} - 1$

(d) $1 - \frac{1}{e}$

(e) $e - \frac{1}{e}$

10. A uniform cable hangs over the side of a building that is 150 feet tall. The cable is 80 feet long, weighs 240 pounds and is attached to a 50 pound weight at the bottom. How much work is done to pull 10 feet of rope up to the top of the building?

(a) 650 ft-lb

(b) 1350 ft-lb

(c) 860 ft-lb

(d) 2750 ft-lb

(e) 11550 ft-lb

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11. Which of the following gives the area of the region bounded by the curves $x = y^2$ and $x + y = 6$.

(a) $\displaystyle\int_{-3}^{2} \left(y^2 - 6 + y\right)\, dy$

(b) $\displaystyle\int_{-3}^{2} \left(6 - y - y^2\right)\, dy$

(c) $\displaystyle\int_{4}^{9} \left(6 - x - \sqrt{x}\right)\, dx$

(d) $\displaystyle\int_{4}^{9} \left(\sqrt{x} - 6 + x\right)\, dx$

(e) $\displaystyle\int_{-3}^{2} \left(\sqrt{x} - 6 + x\right)\, dy$

12. The base of a solid is a triangle with vertices $(0, 0)$, $(1, 1)$ and $(1, -1)$. The cross sections perpendicular to the $x-$axis are squares. What is the volume of the solid?

(a) $\frac{1}{3}$

(b) $\frac{2}{3}$

(c) $\frac{4}{3}$

(d) $\frac{16}{3}$

(e) $\frac{32}{3}$

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13. Compute $\int \cos^2(2x)\,dx$

(a) $C + \frac{1}{2}x + \frac{1}{4}\sin(2x)$

(b) $C + \frac{1}{2}x + \frac{1}{8}\sin(4x)$

(c) $C + \frac{1}{2}x - \frac{1}{4}\sin(2x)$

(d) $C + \frac{1}{3}\sin^3(2x)$

(e) $C + \frac{1}{2}x - \frac{1}{8}\sin(4x)$

14. If the work required to stretch a spring from its natural length to 4 m beyond its natural length is 16 J, then how much force would be needed to stretch the spring 6 m beyond its natural length?

(a) 12 N.
(b) 18 N.
(c) 24 N.
(d) 36 N.
(e) 72 N.

15. Evaluate $\int_1^e x^2 \ln x \, dx$.

(a) $\frac{2}{9}e^3 + \frac{1}{9}$

(b) $\frac{2}{9}e^3 - \frac{1}{9}$

(c) $1 - e$

(d) $e^2 - \frac{1}{9}e^3 + \frac{1}{9}$

(e) None of these

16. (8 points) Evaluate $\int_0^{\sqrt{3}} \arctan(x)\, dx$.

17. (10 points) Compute $\int 5x^2 \sin(3x)\, dx$.

18. (12 points) Consider the region bounded by the curves $x = 6y - y^2$ and y−axis.

(a) **Set up** an integral to find the volume of the solid formed by rotating this region about the line $y = 10$. **Do not evaluate your integral.**

(b) **Set up** an integral to find the volume of the solid formed by rotating this region about the line $x = -5$. **Do not evaluate your integral.**