Week in Review – Stat 201
Session 8 – Wednesday March 27th, 2024
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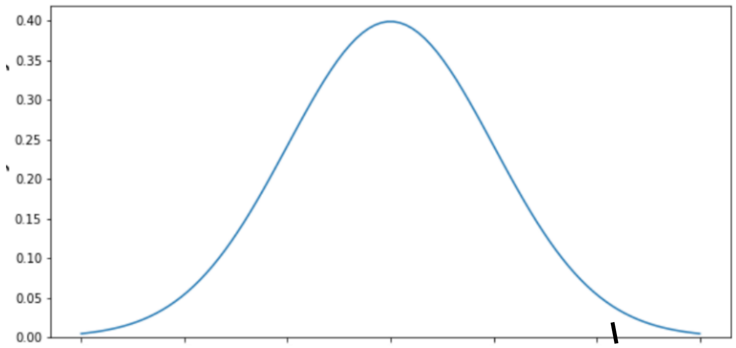
TEXAS A&M UNIVERSITY
Math Learning Center

Problem 1

Suppose X is a continuous random variable. If $P(X \geq 16) = 0.03$, then

1. $P(X < 16) = 1 - 0.03 = 0.97$
2. $P(X \leq 16) = P(X < 16) = 0.97$
3. $P(X > 16) = P(X \geq 16) = 0.03$
4. $P(X=16)=0$



Problem 2

Topics: continuous random variables, Normal distribution, empirical rule

Given an approximately normal distribution with a mean of 175 and a standard deviation of 37.

5. Draw a normal curve and label 1, 2, and 3 standard deviations on both sides on the mean.
6. What percent of values are within the interval (138, 212)?
   $P(138 < X < 212) = P\left(\frac{138-175}{37} < Z < \frac{212-175}{37}\right) = P(-1 < Z < 1) = .68$ 68%
7. What percent of values are within the interval (64, 286)?
   $P(64 < X < 286) = P\left(\frac{64-175}{37} < Z < \frac{286-175}{37}\right) = P(-3 < Z < 3) = .997$ 99.7%

Problem 3

Topics: continuous random variables, Normal distribution, empirical rule

It is known that when a specific type of radish is grown in a certain manner without fertilizer the weights of the radishes produced are normally distributed with a mean of 40g and a standard deviation of 10g.

Determine the proportion of radishes grown:

8. Without fertilizer with weights less than 50 grams.
   $P(X < 50) = P\left(Z < \frac{50-40}{10}\right) = P(Z < 1) = .84$

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9. Without fertilizer with weights between 20 and 60 grams.

   $P(20 < X < 60) = P(\frac{20-40}{10} < Z < \frac{60-40}{10}) = P(-2 < Z < 2) = .95$

10. Without fertilizer that will have weights greater than or equal to 60 grams.

    $P(X > 60) = P(Z > \frac{60-40}{10}) = P(Z > 2) = .025$

Problem 4

Topics: continuous random variables, Normal distribution, empirical rule

11. Which of the following would indicate that a dataset is **not** bell-shaped[3]?
    a. The range is equal to 5 standard deviations.
    b. The range is larger than the interquartile range.
    c. The mean is much smaller than the median.
    d. There are no outliers.
    e. None of the above

Problem 5

12. What is the z-score of x = 5 if it is 1.8 standard deviations below the mean?

    Z-score = -1.8 (it is negative because it is below the mean, z = 0)

Problem 6

Topics: continuous random variable, standard normal distribution, probability, use of the Z table

What percent of a standard normal distribution N(μ =0, σ= 1) is found in each region? Be sure to draw a graph

THE BLUE NUMBERS ARE DIRECTLY FROM Z TABLE

13. Z < 1.35 P(Z < 1.35) = 91.15%
14. Z > 1.48 P(Z > 1.48) = 1 - P(Z < 1.48) = 1 - 0.9306 = 6.94%
15. 0.4 < Z < 1.5 P(.4 < Z < 1.5) = P(Z < 1.5) - P(Z < .4) = .9332 - .6554 = 27.78%
16. Z < -20.92 or Z > 20.97 P(|Z| > 20.92) = 2 x P(Z < -20.92) = 2 x 0 = 0%

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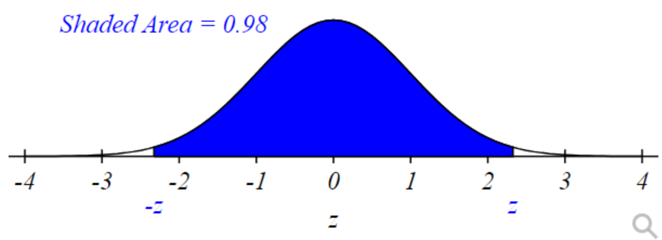
Problem 7

17. Using the standard normal distribution, find the two z-scores that form the middle shaded region. The shaded region is symmetric about z = 0, Round your z-scores to two decimal places.



Z-scores: ±2.326
Need to find z -score with area = .99 *to the left* of it. Search z-table for .9900 and record z-score. Since it is symmetric, the other value is the same but negative.

Problem 8

Topics: histogram, Normal approximation to data, Normal probability plot, Q-Q plot

18. Can we approximate poker winnings by a normal distribution? We consider the poker winnings of an individual over 50 days. A histogram and normal probability plot of these data are shown in the following figure[4]:

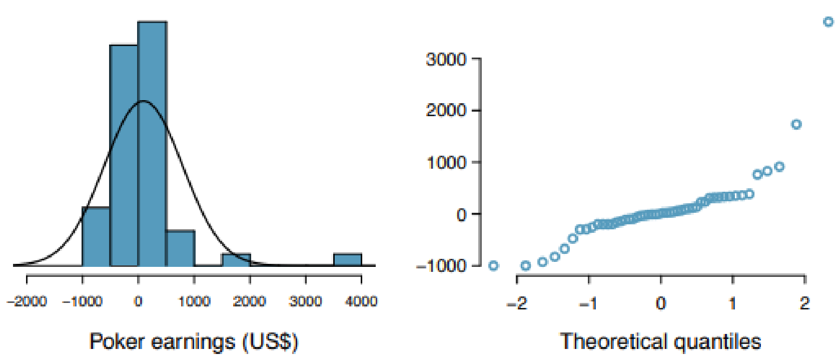


Figure 3.13: A histogram of poker data with the best fitting normal plot and a normal probability plot.

Answer: No, both the histogram and the QQ plot show that the distribution is skewed to the right.

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Problem 9

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19. Overweight baggage. Suppose weights of the checked baggage of airline passengers follow a nearly normal distribution with mean 45 pounds and standard deviation 3.2 pounds. Most airlines charge a fee for baggage that weigh in excess of 50 pounds[4]. Determine what percent of airline passengers incur this fee.

    $P(X > 50) = P(Z > \frac{50-45}{3.2}) = P(Z > 1.56) = 1 - P(Z < 1.56) = 1 - .9406 = 0.0594$

Problem 10

THE BLUE NUMBERS ARE DIRECTLY FROM Z TABLE

The cholesterol content of large chicken eggs is normally distributed with a mean of 200 milligrams and standard deviation 15 milligrams.

20. What is the probability that the cholesterol content of a random egg is less than 205 milligrams?

    $P(X < 205) = P(Z < \frac{205-200}{15}) = P(Z < .3333) = 0.6293$

21. In sixty-seven percent of the eggs, the cholesterol content is less than a certain value "C".

    Find the value of "C".

    a) 0.33
    b) 206.6
    c) 210
    d) 0.44
    e) 193.4

P(Z < ?) = .6700 (use z table to solve for ?), find that ? = .44

BUT WE'RE NOT DONE YET! We have to convert it back to X using the z-transformation formula:

$$Z = \frac{X - MEAN}{SD} \quad => \quad .44 = \frac{C-200}{15} \quad => \quad C = .44(15) + 200 = 206.6$$

Problem 11

Topics: Normal distribution, parameters of the normal distribution, z-score, quartiles, use of the Z table

THE BLUE NUMBERS ARE DIRECTLY FROM Z TABLE

Auto insurance premiums. Suppose a newspaper article states that the distribution of auto insurance premiums for residents of California is approximately normal with a mean of $1,650. The article also states that 25% of California residents pay more than $1,800.

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22. What is the z-score that corresponds to the top 25% of the standard normal distribution?
    P(Z > ?) = .25, then P(Z < ?) = .75, using the z table we get ? = .674

23. What is the mean insurance cost? What is the cutoff for the 75th percentile?
    The 75th percentile is the value where 75% of the data lies below it and 25% of the data lies above it. This value is given: $1,800

24. Identify the standard deviation of insurance premiums in LA.
    We have enough information to use the z-transformation formula:
    $Z = \frac{X - MEAN}{SD}$ => $.674 = \frac{1800 - 1650}{SD}$ => $SD = \frac{1800 - 1650}{.674}$ = \$222.55

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