Math 152 - Week-in-Review 9

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Find the Radius and the Interval of Convergence for the following power series.

1. $\displaystyle\sum_{n=2}^{\infty} \frac{3^n(x-1)^n}{n\ln(n)}$.

2. $\displaystyle\sum_{n=0}^{\infty} \frac{2^n(x-3)^n}{n!}$.

3. If the power series given by $\sum_{n=0}^{\infty} C_n(x-2)^n$ coverges at $x = 5$ and diverges at $x = -4$, what can we say about the following?

(a) $\sum_{n=0}^{\infty} C_n$

(b) $\sum_{n=0}^{\infty} C_n(-3)^n$

(c) $\sum_{n=0}^{\infty} C_n 9^n$

(d) $\sum_{n=0}^{\infty} C_n(-5)^n$

(e) $\sum_{n=0}^{\infty} C_n(-2)^n$

(f) $\sum_{n=0}^{\infty} C_n 4^n$

Let's recap the Power Series essentials:

Find a Power Series representation for the following functions. Give the radius and interval of convergence.

4. $f(x) = \dfrac{x}{25x^2 - 4}$

5. $f(x) = \dfrac{1}{(2-x)^2}$

6. $f(x) = \dfrac{x^2}{(2-x)^2}$

7. $f(x) = \ln(1 + 5x^2)$

8. $f(x) = \arctan(2x)$

Evaluate the following integrals as Power Series.

9. $f(x) = \int \frac{1}{1+x^4}\, dx$

10. $f(x) = \int x^2 \arctan(3x^2)\, dx$

11. If $f(x) = \sum_{n=0}^{\infty} \frac{(3x)^n}{n!}$, find the power series for $f'(x)$ and $\int f(x)dx$. Identify $f(x)$.

12. Find the 25^{th} derivative for the function $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{3^n(n+2)} x^n$ centered at $x = 0$.

Find the Taylor Series Representions for the following functions

13. $f(x) = e^{3x}$ centered at $x = 5$

14. $f(x) = \ln(x)$ centered at $a = 2$

Find the Maclaurin Series Representation for the following functions.

15. $f(x) = \int_0^x e^{-t^2}\ dt.$

16. $f(x) = x^3 \cos(x)$

Find the sum of the following series.

17. $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n (3)^n (\pi^n)}{n!}$

18. $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n 3^{2n+1}}{(2n+1)!}$

19. $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n 3^{2n+1}}{(2n)!}$

20. $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n 3^{2n+1}}{2^{2n+1}(2n)!}$