Math 151 - Week-In-Review 4

Topics for the week:

2.7 Derivatives and Rates of Change
2.8 The Derivative as a Function
J.1 through 2.8 Exam Review

2.7 Derivatives and Rates of Change

1. Write the equation of the line tangent to the graph of $h(x) = 5x - x^2$ at the point $(-1, -6)$.

2. The displacement, in feet, of a particle moving in a straight line is given by $y = \sqrt{10 - 3t}$, where t is measured in minutes.

   (a) Compute the average velocity over the interval $[2, 3]$.

   (b) Compute the instantaneous velocity when $t = 2$.

2.8 The Derivative as a Function

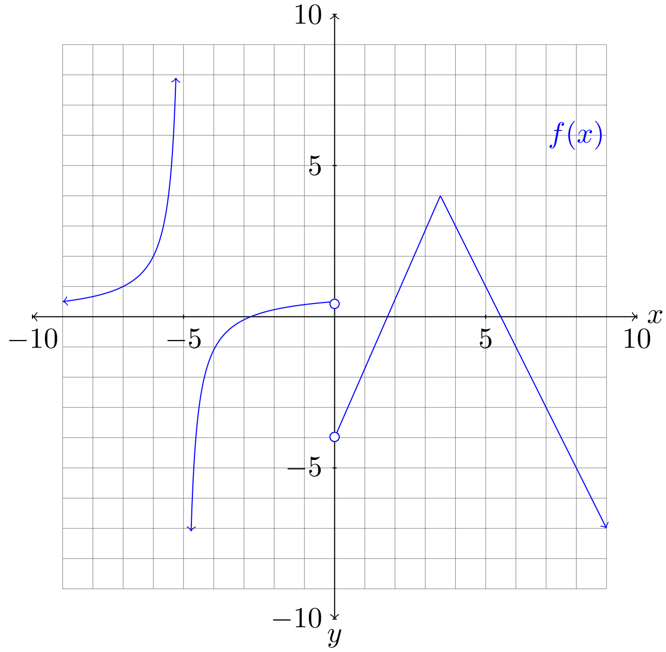
3. Compute the derivative of the function $g(x) = \dfrac{x}{x-4}$, using the definition of the derivative. Then state the domain of both $g(x)$ and $g'(x)$.

4. Given $y = 4x^2 - 11x + 25$,

   (a) show $\dfrac{dy}{dx} = 8x - 11$.

   (b) Is there any point along the curve where the tangent line is horizontal?

5. Given the graph of $f(x)$ below,



(a) Sketch the graph of $f'(x)$ using $f(x)$.

(b) State the values of x at which $f(x)$ is not differentiable.

Exam Review (J.1 - 2.8)

6. Simplify the expression $\tan\left(\arcsin\left(\frac{5x}{8}\right)\right)$

7. Given the points $J(0,5)$ and $K(-2,0)$, compute a vector of length $\frac{1}{2}$ that is in the same direction as $\overrightarrow{JK}$.

8. Two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ act on an object. The force $\mathbf{F}_1$ has a magnitude of 32 lbs and a direction of $60°$ counterclockwise from the positive x-axis, and $\mathbf{F}_2$ has a magnitude of 45 lbs and a direction of $120°$ counterclockwise from the positive x-axis. State the resultant force $\mathbf{F}$.

9. A force is given by a vector $\mathbf{F} = 2\mathbf{i} + 5\mathbf{j}$ and moves an object from the point $M(4, 2)$ to the point $N(7, 6)$. Compute the work done.

10. Compute the angle between the vectors $\mathbf{a} = \langle -2, 9 \rangle$ and $\mathbf{b} = \langle 8, 4 \rangle$.

11. Write a parametric equation of the line passing through the points $(12, 5)$ and $(9, -2)$.

12. Determine the parametric equations for the line that passes through the point (3,-1) and is

    (a) is parallel to the vector $\langle -5, -4 \rangle$.

    (b) is perpendicular to the vector $\langle -5, -4 \rangle$.

13. State the slope of the line with corresponding vector equation $\mathbf{r}(t) = \langle 5 - 2t, -8 + 7t \rangle$.

14. Determine whether the lines, $L_1 = \mathbf{r}(t) = (-6 + 2t)\mathbf{i} + (7 - 6t)\mathbf{j}$ and $L_2 = \mathbf{r}(s) = \left(5 + \frac{1}{2}s\right)\mathbf{i} + \left(-8 + \frac{3}{2}s\right)\mathbf{j}$, are parallel, perpendicular, or neither.

15. Evaluate each limit:

(a) $\lim\limits_{x\to 6^+}\left(\dfrac{x+1}{x-6}\right)$

(b) $\lim\limits_{x\to 2\pi^-}\left(x\csc(x)\right)$

(c) $\lim\limits_{x\to 5}\left(\dfrac{5-x}{x^2-25}\right)$

16. Evaluate the limit $\lim_{x\to7^+}\left(\frac{|7-x|}{x-7}\right)$

17. Show that $f(x)=\begin{cases}\sin(x) & x<\frac{\pi}{2}\\ \frac{x}{2}-\frac{\pi}{4} & x\geq\frac{\pi}{2}\end{cases}$ is not continuous at $x=\frac{\pi}{2}$.

18. Determine the values of a and b such that $f(x)$ is continuous over the real numbers.

$$f(x) = \begin{cases} ax^2 - 5x + b & \text{if } x < -1 \\ -\frac{65}{2}x - \frac{39}{2} & \text{if } -1 \leq x < 1 \\ bx^3 - 9ax & \text{if } x \geq 1 \end{cases}$$

19. Evaluate the limit, $\lim\limits_{x \to \infty} \dfrac{3e^{2x}}{2e^{2x} - e^{x}}$, if possible.

20. Determine the left end-behavior (as $x \to -\infty$) of the function $g(x) = \dfrac{-\sqrt{3x^4 + 2x^2} + 5}{4x^4 - x^2}$.

21. Evaluate the limit, $\lim\limits_{x \to \infty} \left[\ln(3 + 6x^2) - \ln(x^3 + x - 1)\right]$, if possible.