



MATH 308: WEEK-IN-REVIEW 13
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Review for the Final Exam - Part 2

1. (Chapter 3) A spring is stretched 10cm by a force of 0.3N. A mass of 0.25kg is hung from the spring and is also attached to a viscous damper that exerts a force of 3N when the velocity of the mass is 6m/s. The mass is pulled 5cm below its equilibrium position and given an initial velocity of 10cm/s downward.
 - (a) Determine the position u of the mass as a function of time t
 - (b) Find the quasifrequency of the motion.
 - (c) If the system is also subjected to an external force of $2 \cos(4t)$, find $u(t)$, and the amplitude, period, and phase of the steady-state solution.



2. (Chapter 6) Use the definition to find the Laplace transform of $f(t) = \begin{cases} 5 - t, & 0 \leq t < 2, \\ 3t, & t \geq 2. \end{cases}$

3. Find the inverse Laplace transform of the function $F(s) = \frac{2s - 3}{s^2 + 2s + 10}$



4. (Chapter 6) Find the solution of the initial value problem

(a) $y'' + y = \delta(t - 2\pi) \cos t, \quad y(0) = 0, \quad y'(0) = 1.$

(b) $y'' + 3y' + 2y = \begin{cases} 1, & 0 \leq t < 10, \\ 0, & t \geq 10 \end{cases} \quad y(0) = y'(0) = 0.$



5. (Chapter 5) For the equation $y'' + xy' + 2y = 0$
- (a) Seek its power series solution about $x_0 = 0$, find the recurrence relation.
 - (b) Find the general term of each solution $y_1(x)$ and $y_2(x)$.
 - (c) Find the first four terms in each of the two solutions $y_1(x)$ and $y_2(x)$.
Show that $W[y_1, y_2](0) \neq 0$.



6. (Chapter 7) Solve the initial value problem

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}' = \begin{pmatrix} 1 & -5 \\ 1 & -3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \quad x_1(0) = 1, \quad x_2(0) = 1.$$



7. (Chapter 7) Find the general solution of

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}' = \begin{pmatrix} 1 & -5 \\ 1 & -3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ t \end{pmatrix}, \quad x_1(0) = 1, \quad x_2(0) = 1.$$