Example 1. *True/False.*

(a) The curve $\mathbf{r}(t) = \langle t^2, 2t+1, t\rangle$ *lies on the plane* $y - 2z = 1$.

True *False*

(b) The curvature of a straight line is zero.

True *False*

(c) Let $\mathbf{u}, \mathbf{v}$, *and* $\mathbf{w}$ *be orthogonal to each other. Then no planes can contain all three vectors.*

True *False*

(d) The line $x = 1 + 2t$, $y = 1 + t$, $z = 3 - 3t$ *is orthogonal to the plane* $2x + y + 2z = 1$.

True *False*

(e) Let $\mathbf{a}$ *and* $\mathbf{b}$ *be two nonzero vectors. Then the vectors* $\text{proj}_{\mathbf{a}}\mathbf{b}$ *and* $\mathbf{a}$ *are parallel.*

True *False*

Example 2 (12.1)**.** *Find the intersection of the sphere* $x^2+y^2+z^2-4x+6y-10z+29=0$ *with*

(a) the xy*-plane.*

(b) the plane $z = 8$*.*

Example 3 (12.2)**.** *Consider two vectors* $\mathbf{v} = \mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$ *and* $\mathbf{w} = 2\mathbf{i} + \mathbf{k}$*. If a vector* $\mathbf{a}$ *is in the direction of* $\mathbf{v} - \mathbf{w}$ *and has magnitude* 3 *units, find the components of the vector* $\mathbf{a}$*.*

Example 4 (12.3)**.** *Find the work done by a force* $\mathbf{F} = 3\mathbf{i} + 2\mathbf{j} - 5\mathbf{k}$ *that moves an object from the point* $A(1, 0, 2)$ *along a straight line to the point* $B(2, 4, 3)$*. Also, find the angle between the displacement and force vectors.*

Example 5 (12.4)**.** *Determine whether or not the points* $A(0, 0, 1)$, $B(1, 2, 1)$, $C(1, 0, -1)$, *and* $D(3, 2, 1)$ *lie in the same plane.*

Example 6 (12.5). *Suppose a line L_1 passes through the point $P(1, 4, -2)$ and is orthogonal to the plane $3x - 2y + z = 35$.*

(a) Determine symmetric equations of the line L_1.

(b) Find the point of intersection of the line L_1 and the plane.

Example 7 (12.5)**.** *Determine whether the lines* L_1 *and* L_2 *are parallel, intersecting, or skew.*

$$L_1: \quad \frac{x+2}{3} = \frac{y-4}{-2} = z$$

$$L_2: \quad \frac{x+1}{2} = \frac{y+3}{3} = \frac{z-1}{-2}.$$

Example 8 (12.5). *Find an equation of the plane that passes through the point* $P(1, -2, 3)$ *and contains the line* $x = 3 + t, y = 5, z = 2 - 5t$.

Example 9 (12.5). *Consider that a plane P contains the triangle ABC with vertices $A(1,2,2)$, $B(1,3,3)$, and $C(3,1,0)$.*

(a) Determine an equation of the plane P.

(b) Which of the following lines is orthogonal to the plane P?

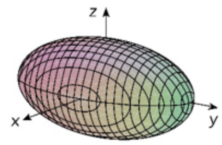
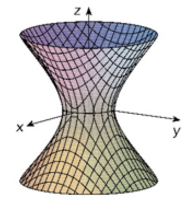
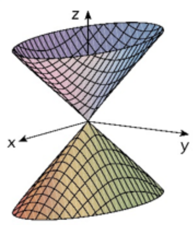
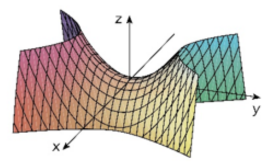
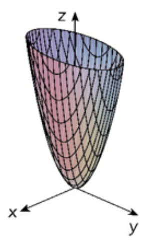
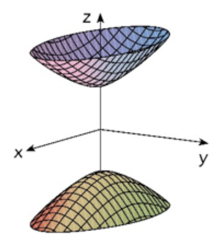
$$L_1: \quad x = 3 + t, \quad y = 4 + 2t, \quad z = 3 + 2t$$

$$L_2: \quad x = 1 - t, \quad y = 4 - 2t, \quad z = 3 - 2t$$

$$L_3: \quad x = -1 + 3t, \quad y = 2 + 2t, \quad z = -2 - 2t$$

$$L_4: \quad x = 1 - 3t, \quad y = 5 + 6t, \quad z = 7 - 6t$$

None of the above lines.

Ellipsoid 	$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ "A bunch of ellipses stacked together" Special case: If $a=b=c$, we have a sphere	Hyperboloid of One Sheet 	$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$ In the xy plane, the traces are ellipses. In the xz or yz planes, the traces are hyperbolas. *Whichever variable is negative corresponds to the axis of symmetry
Cone 	$\frac{z^2}{c^2}=\frac{x^2}{a^2}+\frac{y^2}{b^2}$ In the xy plane, the traces are ellipses. In the xz or yz planes, the traces are hyperbolas, except when $x=0$ or $y=0$, then the traces are pairs of lines	Hyperbolic Paraboloid 	$\frac{z}{c}=\frac{x^2}{a^2}-\frac{y^2}{b^2}$ In the xy plane, the traces are hyperbolas. In the xz or yz plane, the traces are parabolas.
Elliptic Paraboloid 	$\frac{z}{c}=\frac{x^2}{a^2}+\frac{y^2}{b^2}$ In the xy plane, the traces are ellipses. In the xz or yz planes, the traces are parabolas.	Hyperboloid of Two Sheets 	$-\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ In the xy plane, the traces are ellipses if $z>c$ or $z<-c$ In the xz or yz planes, the traces are hyperbolas.

Example 10 (12.6). *Identify and sketch the following quadric surfaces.*

$$(y-2)^2-(x+1)^2-z^2+4y+2x-6=0$$

Example 11 (13.1)**.** *Find all the points of intersection of the curve* $\mathbf{r}(t) = \left\langle t, 2t, t^2 + 4 \right\rangle$ *and the plane* $x + 2y - z = 0$.

Example 12 (13.2)**.** *Compute the integral* $\displaystyle\int_0^1 \mathbf{r}(t)$, *where* $\mathbf{r}(t) = te^{-t}\mathbf{i} + \dfrac{1}{t+1}\mathbf{j} + 3t^2\mathbf{k}$.

Example 13 (13.1)**.** *Consider the vector function* $\mathbf{r}(t) = \left\langle e^{-6t}, \ln(2t+1), \dfrac{t^2-9}{t-3} \right\rangle$.

(a) Find the domain of $\mathbf{r}$.

(b) Find $\lim_{t\to 3} \mathbf{r}(t)$.

Example 14 (13.1)**.** *Find parametric equations for the curve of intersection of the paraboloid* $z = \dfrac{1}{2}(x^2+y^2)$ *and the plane* $z = x$.

Example 15 (13.2). *Find parametric equations for the tangent line to the curve given by the vector function* $\mathbf{r}(t) = \left\langle \ln(t+1), t\sin 2t, e^{-2t} \right\rangle$ *at the point* $(0, 0, 1)$.

Definition: *The* ***curvature*** *of a curve given by the vector valued function* $\mathbf{r}$ *is*

$$\kappa(t) = \left|\frac{d\mathbf{T}}{ds}\right| = \frac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|} = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3},$$

where $\mathbf{T}$ *is the unit tangent vector.*

Example 16 (13.3). *Consider a curve given by the vector function*

$$\mathbf{r}(t) = \left\langle t, , \frac{1}{t}, \sqrt{2}\ln t \right\rangle.$$

(a) Find the length of the curve from $(1, 1, 0)$ *to* $\left(e, \frac{1}{e}, \sqrt{2}\right)$.

(b) Find the unit tangent vector $\mathbf{T}(t)$ *and the unit normal vector* $\mathbf{N}(t)$ *at the point* $(1, 1, 0)$

(c) Find the curvature of the curve at the point $(1, 0, 1)$.

Example 17 (13.4). *The position function of a moving particle in space is given by* $\mathbf{r}(t) = \langle \sin t, 2t + 1, \cos t \rangle$. *Find its velocity, speed, and acceleration at time* $t = \pi$.

Example 18 (13.4). *Find the velocity and position vector of a particle such that*

$$\mathbf{a}(t) = (-\cos t)\mathbf{i} + 2\mathbf{j} + 4e^{-2t}\mathbf{k},\ \ \mathbf{v}(0) = -2\mathbf{k},\ \ \mathbf{r}(0) = \mathbf{i} + 3\mathbf{j} + \mathbf{k}.$$

