Section 1.2: Matrix Multiplication

Pr 1. An online streaming service records the number of downloads of movies and series based upon which studio produced the movie or series. During the month of January 3000 animated series, 6500 animated movies, 6200 live action series, 5000 live action movies, 1200 documentary series, and 6800 documentary movies were downloaded, while in February the downloads were 3800, 2900, 2600, 5100, 6500, and 9500 respectively.

a. The streaming service is considering charging per film or series download, instead of the traditional subscription service. If the online streaming service charges $.99 per movie download and $1.99 per series download, write a matrix equation that would allow the service to compute how much they make for each studio. → animation / live action / documentary

stick w/ Jan

Series   Movies

$$A \cdot B = C \qquad 3\times 2 \quad 2\times 1 \quad 3\times 1$$

$$\begin{array}{c|cc} & S & M \\ \hline \text{animated} \; A & 3000 & 6500 \\ \text{live action} \; L & 6200 & 5000 \\ \text{docum.} \; D & 1200 & 6800 \end{array} = J \qquad \begin{array}{c} \$ \\ \begin{bmatrix} 1.99 \\ .99 \end{bmatrix} \end{array} \begin{array}{c} S \\ M \end{array} = C \qquad C = \begin{bmatrix} \$ \\ \\ \\ \end{bmatrix} \begin{array}{l} \text{animated} \\ \text{live action} \\ \text{docum} \end{array}$$

$$c_{11} = \underbrace{3000 \times 1.99}_{\text{an. ser.}} + \underbrace{6500 \times .99}_{\text{an. films}}$$

b. How much income does the online streaming service bring in, in January, from each studio?

$$C = \begin{bmatrix} 3000 \times 1.99 + 6500 \times .99 \\ 6200 \times 1.99 + 5000 \times .99 \\ 1200 \times 1.99 + 6800 \times .99 \end{bmatrix} = \begin{bmatrix} \$12{,}405 \\ \$17{,}288 \\ \$9{,}120 \end{bmatrix}$$

c. How much income does the online streaming service bring in, for January and February combined, from each studio?

$$(J + F) \cdot C = J \cdot C + F \cdot C$$

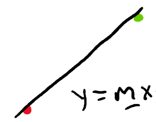
using calculator

$$= \begin{bmatrix} \$22{,}838 \\ \$27{,}511 \\ \$31{,}460 \end{bmatrix}$$

Section 2.1: Review of Lines

(x_1, y_1) (x_2, y_2)

- Slope of a line between two points, $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ $\Leftarrow = \dfrac{\text{"rise"}}{\text{"run"}}$
- Equations of a Line,
  - Point-Slope Form: $y - y_1 = m(x - x_1)$ ☆
  - Slope-Intercept Form: $y = mx + b$ ☆
  - Standard Form: $Ax + BY = C$
  - Vertical Line: $x = a$
  - Horizontal Line: $y = b$
- Intercepts of a Line
  - x-intercept: $(x, 0)$
  - y-intercept: $(0, y)$
- Interpreting Change, $m = \dfrac{\Delta y}{\Delta x} =$



$y = mx + b$

$\Delta y = y_2 - y_1 = \text{"rise"}$

$\Delta x = x_2 - x_1 = \text{"run"}$

Pr 1. Determine the slope between each of the given pair of points.

(a) $(2, -5)$ and $(-9, 11)$

$$\frac{\text{rise}}{\text{run}} = \frac{11 - (-5)}{-9 - 2} = \frac{11 + 5}{-11} = \frac{16}{-11} = -\frac{16}{11}$$

$$\frac{a}{-b} = \frac{-a}{b} \leftarrow \frac{-5 - 11}{+2 - (-9)} = \frac{-16}{11} = -\frac{16}{11}$$

$2 + 9$

$$\boxed{m = \frac{-16}{11}}$$

(b) $(2.5, 1.3)$ and $(2.5, -2.8)$

$$= \frac{-2.8 - 1.3}{2.5 - 2.5} = \frac{-4.1}{0}$$

$\frac{0}{*}$ kay $\quad \frac{0}{k} = 0$

$\frac{*}{0} \quad \frac{n}{0} =$ undefined

$\boxed{m \text{ is undefined / does not exist}}$

(c) $\left(\dfrac{2}{2}, \dfrac{2}{5}\right)$ and $\left(-\dfrac{7}{11}, \dfrac{2}{5}\right)$

$$\frac{\frac{2}{5} - \frac{2}{5}}{-\frac{7}{11} - \frac{2}{2}} = \frac{0}{k} = 0$$

$\rightarrow$ not zero

$$= \frac{0}{-\frac{7}{11}\cdot\frac{2}{2} - \frac{2}{2}\cdot\frac{11}{11}} = \ldots = 0$$

$$\boxed{m = 0}$$

Pr 2. Write the equation of the line given the slope which passes throught the given point in the stated form.

(a) $m = \frac{2}{7}$ and $(-9, 11)$, in point-slope form

$y - y_1 = m(x - x_1)$

or $y = m(x - x_1) + y_1$

$y - y_1 = \frac{2}{7}(x - x_1)$

$y - 11 = \frac{2}{7}(x - (-9))$

$\boxed{y - 11 = \frac{2}{7}(x + 9)}$

$y = mx + b$

(b) $m = -\frac{5}{2}$ and $(4, -7)$, in slope-intercept form

Approach I: $y - (-7) = -\frac{5}{2}(x - 4)$

$y + 7 = -\frac{5}{2}x - \frac{5}{2}(-4)$

$y + 7 = -\frac{5}{2}x + 10$

$-7 \qquad -7$

$\boxed{y = -\frac{5}{2}x + 3}$

Approach II: $-7 = -\frac{5}{2}(4) + b$ and solve for b.

(c) $m = \frac{6}{7}$ and $\left(\frac{7}{2}, 0\right)$, in standard form

$Ax + By = C$, A, B, and C are whole integers.

$Ax + By = C$

$B = \Delta x$

$A = -\Delta y$

$y - 0 = \frac{6}{7}\left(x - \frac{7}{2}\right)$

$y = \frac{6}{7}x - \frac{6}{7}\cdot\frac{7}{2} = \frac{6}{7}x - 3$

$y = \frac{6}{7}x - 3$

$-\frac{6}{7}x \qquad -\frac{6}{7}x$

$7 \cdot \left(y - \frac{6}{7}x\right) = -3 \cdot 7$

$7y - 6x = -21$

$\boxed{-6x + 7y = -21}$

(d) $m = 0$ and $(17, 20)$, in standard form

horizontal

$y = a$

$\boxed{y = 20}$

$0x + 1y = 20$

Pr 3. Write the equation of the line which passes through the given pair of points.

which form?
our choice ◡̈
point-slope is quickest

(a) $(2,-5)$ and $(-9,11)$

$$m = \frac{11-(-5)}{-9-2} = \frac{16}{-11} = -\frac{16}{11}$$

Answer 1:

$$y-(-5) = -\frac{16}{11}(x-2)$$

$$\boxed{y+5 = -\frac{16}{11}(x-2)}$$

$$y-11 = -\frac{16}{11}(x-(-9))$$

$$\boxed{y-11 = -\frac{16}{11}(x+9)}$$

same line

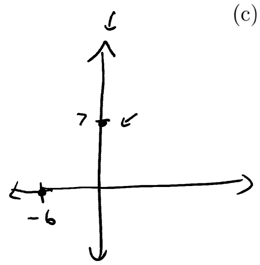
(b) $(7,10)$ and $(7,-12)$

$$m = \frac{-12-10}{7-7} = \frac{-22}{0}$$

m is undefined
vertical

$$x = 7$$

y-axis is $x=0$



(c) intersects the y-axis at $y=7$ and the x-axis at $x=-6$

$(0,7)$ $(-6,0)$

$$m = \frac{0-7}{-6-0} = \frac{-7}{-6} = \frac{7}{6}$$

$$y = \frac{7}{6}x + 7$$

slope-intercept

$$y-7 = \frac{7}{6}(x-0)$$

Pr 4. Determine the x- and y-intercept without graphing. Write the coordinates of each intercept. Then use the points to graph each line.

(a) $5x - 6y = 30$

x-intercept $(c,0)$

$5x - 6(0) = 30$

$\frac{5x}{5} = \frac{30}{5}$

$x = 6$

$\boxed{\text{x-intercept: } (6,0)}$

y-intercept: $(0,c)$

$5(0) - 6y = 30$

$\frac{-6y}{-6} = \frac{30}{-6}$

$y = -5$

$\boxed{\text{y-int. } (0,-5)}$

(b) $\frac{2}{3}y = -\frac{2}{3}x + 12$

x-intercept set $y=0$

$\frac{2}{3}\cdot 0 = -\frac{2}{3}x + 12$

$+\frac{2}{3}x \quad +\frac{2}{3}x$

$\frac{3}{2}\cdot\frac{2}{3}x = 12\cdot\frac{3}{2}$

$x = 18$

$\boxed{\text{x-int. } (18,0)}$

y-int. set $x=0$

$\frac{2}{3}y = -\frac{2}{3}(0) + 12$

$\frac{3}{2}\cdot\frac{2}{3}y = 12\cdot\frac{3}{2}$

$y = 18$

y-int. $(0,18)$

off the graph

(c) $x = -4$

Vertical

x-intercept: $(-4,0)$

y-intercept DNE

because $-4 \neq 0$

(d) $y = 7$

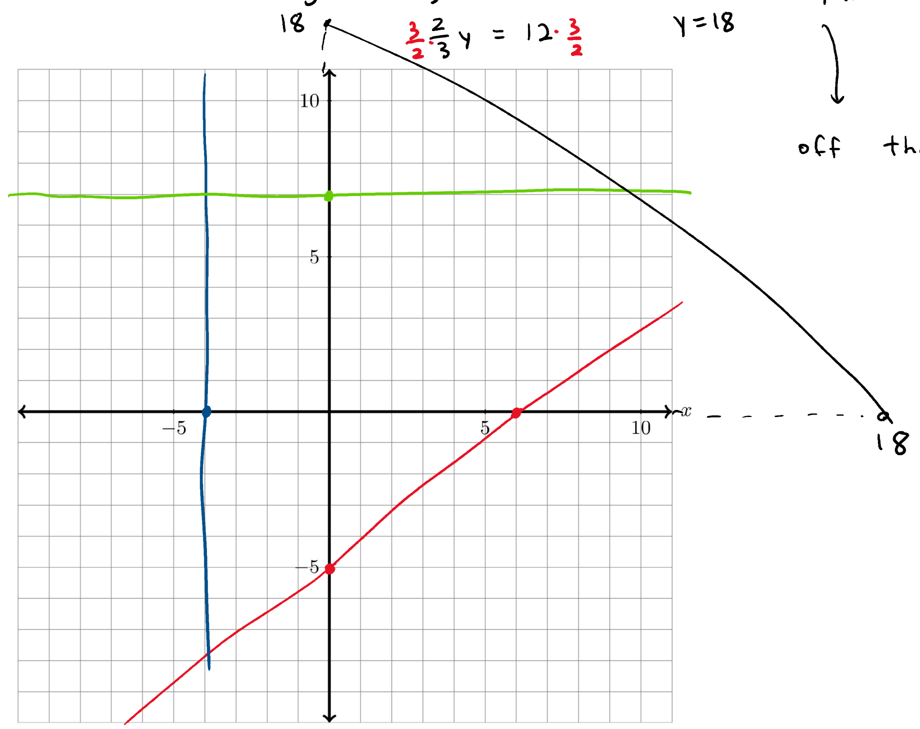
horizontal

x-intercept: DNE

$7 \neq 0$

y-intercept

$(0,7)$



Pr 5. Given the line $x = \frac{2}{3}y - \frac{11}{4}$,

Approach: use $m = \frac{\Delta y}{\Delta x}$

$\frac{\Delta x}{\Delta y} = \frac{2}{3}$

(a) If x increases by 2 units, what is the corresponding change in y?

$\Delta x = 2$ $\quad\Delta y$

$$\frac{3}{2}x = \frac{3}{2}\left(\frac{2}{3}y - \frac{11}{4}\right) = y - \frac{3}{2}\cdot\frac{11}{4}$$

$+\frac{3}{2}\cdot\frac{11}{4} \qquad + \frac{3}{2}\cdot\frac{11}{4}$

$$y = \frac{3}{2}x + \frac{3}{2}\cdot\frac{11}{4} = \frac{3}{2}x + \frac{33}{8}$$

$\frac{3}{2}x + \underset{\text{number}}{a} = y$

$\frac{\Delta y}{\Delta x} = \frac{3}{2}$, $\quad\Delta x = 2$

$2\cdot\frac{\Delta y}{2} = \frac{3}{2}\cdot 2$

$\Delta y = 3$

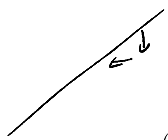
y increases by 3 units

(b) If y decreases by 9 units, what is the corresponding change in x?

$\Delta y = -9$ $\quad$ want Δx

increases $\Delta y > 0$

decreases $\Delta y < 0$



$\frac{\Delta y}{\Delta x} = \frac{3}{2}$, $\quad \Delta y = -9$

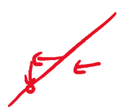
$\Delta x \cdot \frac{-9}{\Delta x} = \frac{3}{2}\Delta x \qquad \frac{2}{3}(-9) = \left(\frac{3}{2}\Delta x\right)\frac{2}{3}$

$-6 = \Delta x$

x decreases by 6 units

(c) If x decreases by 7 units, what is the corresponding change in y?

$\Delta x = -7$



$\frac{\Delta y}{\Delta x} = \frac{3}{2}$

$\frac{\Delta y}{-7} = \frac{3}{2}$

$\Delta y = \frac{3}{2}\cdot(-7)$

$\Delta y = -\frac{21}{2}$

y decreases by $\frac{21}{2}$ units

y decreases by 10.5 units

Pr 6. Suppose that when x increases by 2.1 units, y decreases by 4.5 units, what is the slope of the line containing any point (x, y)?

$\Delta y = -4.5$

$\Delta x = 2.1$

$$m = \frac{\Delta y}{\Delta x} = \frac{-4.5}{2.1} = \frac{-1.5}{.7} = \boxed{\frac{-15}{7}}$$

- Linear Depreciation, $V(t) = mt + b$
- Cost, variable cost + fixed costs $C(x) = mx + F$
- Revenue, price per item times quantity sold $R(x) = px$
- Profit, revenue minus cost $P(x) = R(x) - C(x)$

Pr 1. A piece of machinery is purchased new for \$225,000 and has a value of \$165,000 after 5 years.

(a) Assuming the value of the machinery depreciates at a constant rate each year, determine the rate of depreciation.

rate of depreciation = slope

t=0 time it was purchased

$(0, 225000)$ to $(5, 165000)$

$$\frac{165000 - 225000}{5 - 0} = \frac{-60000}{5} = -12000$$

$$\boxed{\text{rate of depreciation} = \$12000 \text{ per year}}$$

(b) Write the linear depreciation model for the value of the machinery, V, after t years.

$$V(t) = -12000t + b$$

Approach II:

$$225000 = V(0) = -12000(0) + b$$

$$225000 = b$$

$$\boxed{V(t) = -12000t + 225000}$$

(c) What is the value of the machinery after 47 months?

wrong answer: $V(47)$

right answer: $V\left(\frac{47}{12}\right) = -12000\left(\frac{47}{12}\right) + 225000$

$$= -47000 + 225000$$

$$= \$178000$$

(d) If the machinery reaches scrap value in 15 years, what is the scrap value of the machinery?

t

0?

scrap value $= V(15)$

$$= -12000(15) + 225000$$

$$\$45{,}000$$

Alternative question: given a scrap value S, find the first time when $V(t) = S$.

Solve $mt + b = S$

Pr 2. An item purchased 6 years ago has a current value of \$2000. After a little research you find the item reaches its scrap value of \$800 after 107 months. Assuming the item is depreciating linearly, what was the purchase price of the item?

Skip for now

What was the purchase price? $V(0)$

$V(t) = mt + b$

Two coordinates: $(6, 2000)$ and $\left(\frac{107}{12}, 800\right)$

Find $V(t) = mt + b$ ...

$$m = \frac{800 - 2000}{\frac{107}{12} - 6} = \frac{-1200}{\frac{107-72}{12}} = \frac{-1200}{\frac{35}{12}} = \frac{-14400}{35} = -\frac{2880}{7}$$

$$2000 = V(6) = \frac{-2880}{7} \cdot 6 + b$$

⋮

Pr 3. Ted runs a food truck that sells gyros. The cost of maintaining the food truck is \$255 per week. The stand makes a profit of \$124 when 50 gyros are sold in a week. If only 20 gyros are sold, Ted knows the total cost for that week is \$234.

(a) Write the cost function for producing x gyros at ~~Munchkin~~ Ted's stand.

cost function $C(x) = mx + F$ fixed costs

$F = 255$

$$234 = C(20) = m \cdot 20 + 255$$

$$20m + 255 = 234$$

$$-255 \quad -255$$

$$20m = -21 \rightarrow m = -\frac{21}{20}$$

$$\boxed{C(x) = -\frac{21}{20}x + 255}$$

should be pos. 1) we made a mistake very rare 2) Dr. White made mistake ✓

(b) Write the profit function for producing and selling x gyros.

$P(x) = mx + b$

$b = -F$

$= R(x) - C(x)$

$$124 = P(50) = m(50) - 255$$

$$124 = 50m - 255$$

$$+255 \quad +255$$

$$\rightarrow 50m = 379$$

$$m = \frac{379}{50}$$

$$\boxed{P(x) = \frac{379}{50}x - 255}$$

(c) Write the revenue function for the sale of x gyros at Ted's food truck.

$$P(x) = R(x) - C(x)$$

$$P(x) + C(x) = R(x)$$

$$R(x) = P(x) + C(x)$$

$$= \frac{379}{50}x - 255 + \left(-\frac{21}{20}x + 255\right)$$

$$= \frac{379}{50}x - \frac{21}{20}x - \cancel{255} + \cancel{255}$$

$$= \left(\frac{379}{50} - \frac{21}{20}\right)x = \left(\frac{758}{100} - \frac{105}{100}\right)x = \frac{653}{100}x$$

$$\boxed{R(x) = 6.53x}$$