Exam 1 review (1.1 - 1.4, 2.1 - 2.2)

Problem 1. Given the function $f(x) = x^2 + 2x - 15$,

(1) What is the average rate of change of $f(x)$ on the interval $[5, 7]$?

(2) What is the instantaneous rate of change of $f(x)$ at $x = 2$?

Problem 2. Given the function $f(x) = \sqrt{3x+1}$,

(1) Use the limit definition of the derivative to find $f'(x)$.

(2) Find the equation of the tangent line to the graph of $f(x)$ at $x = 5$.

Problem 3. List the different ways

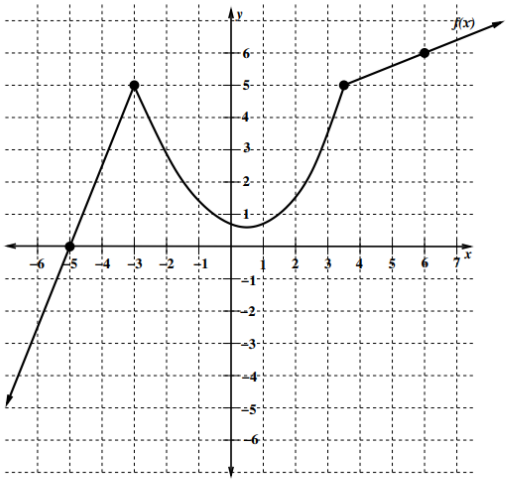
(1) to describe the slope of a secant line

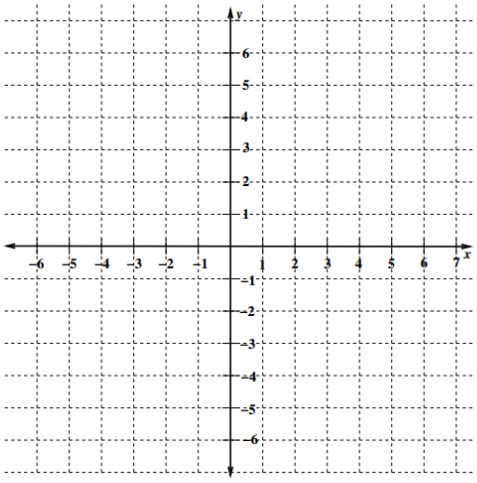
(2) to describe the slope of a tangent line

(3) that a function can be non-differntiable.

Problem 4. .

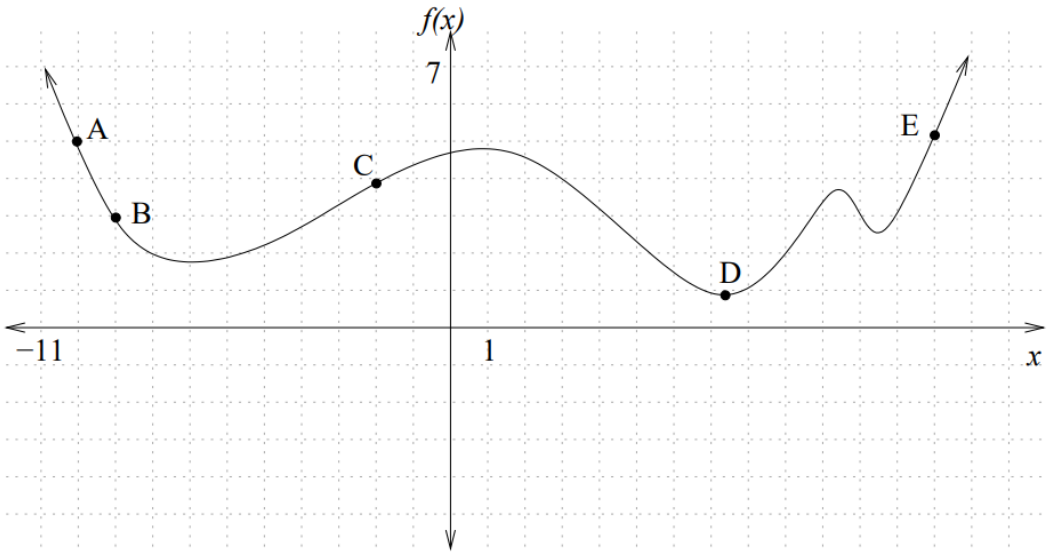
Given the graph of $f(x)$, sketch a graph of $f'(x)$





Problem 5. Find all the vertical asymptotes and holes for $f(x) = \dfrac{(x+2)(x-3)(x-8)^2}{(x-8)^3(x-5)(x-3)}$

Problem 6. Given the graph of $f(x)$ below



(1) At which labeled point(s) is the derivative positive?

(2) At which labeled point(s) is the derivative negative?

(3) At which labeled point(s) is the derivative zero?

(4) At which labeled point(s) is the derivative largest?

(5) At which labeled point(s) is the derivative smallest?

(6) Between which two labeled points is the average rate of change largest?

Problem 7. Find the following limits if they exist

(1) $\lim_{x \to -5^-} \dfrac{|x+5|}{x^2-25}$

(2) $\lim_{x \to -\infty} \dfrac{2e^x - 11}{3e^{-x} + 5e^x + 2}$

Problem 8. The monthly revenue of a local candy shop is given by $R(x) = -x^2 + 12x$ dollars when x gift baskets are sold each month. The shop's monthly cost function, $C(x) = 40x + 1500$ dollars when x gift baskets are made each month.

(1) Find the average rate of change of revenue when the number of gift baskets sold each month changes from 35 to 40 baskets. Interpret your answer.

(2) Find the rate of change of profit when 35 gift baskets are made and sold each month. Interpret your answer.

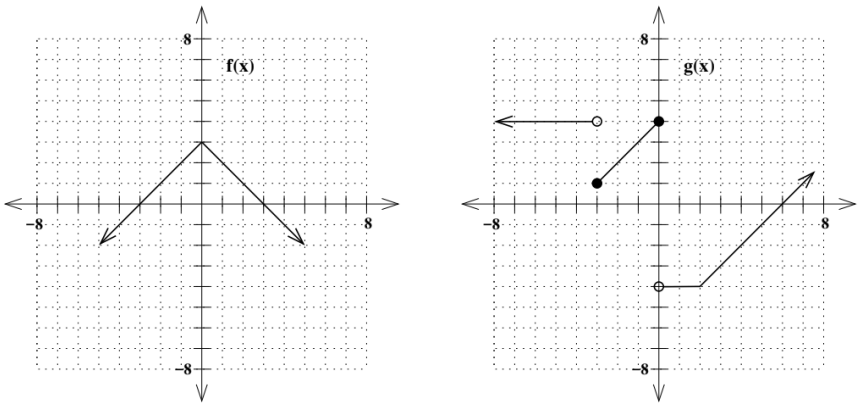
Problem 9. Given the function $f(x) = \dfrac{5x}{3x-4}$

(1) find the equation of the secant line from the point $(-2, 1)$ to the point $(1, -5)$.

(2) find the equation of the tangent line at the point $(1, -5)$.

Problem 10. Where are the following functions continuous? Where are they non-differentiable?

(1)



(2) $f(x) = \begin{cases} \dfrac{x+1}{2x^2-3x-9} & \text{if } x < 1 \\ x^2+1 & \text{if } x \geq 1 \end{cases}$

(3) $f(x) = \dfrac{x-5}{\sqrt{x+1}} + \ln(3x-7)$