

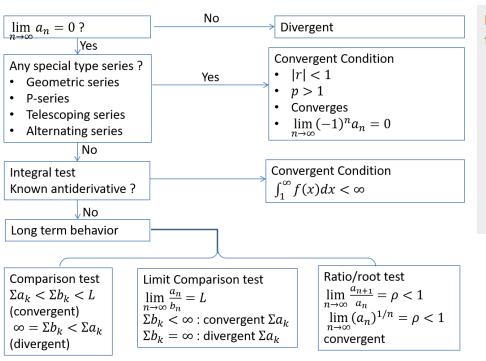
Week in Review Math 152

Week 09

Comparison Test Alternating Series

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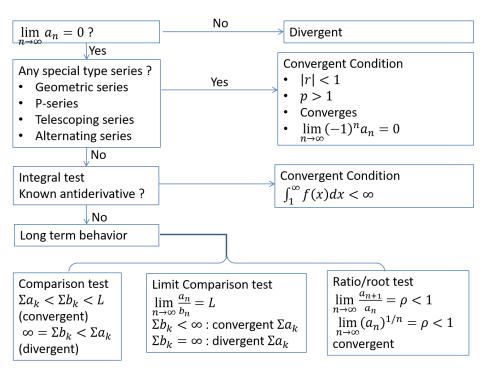
For each of the following series, use the sequence of partial sums to determine whether the series converges or diverges.

a.
$$\sum_{n=1}^{\infty} \frac{n}{n+1}$$

b.
$$\sum_{n=1}^{\infty} (-1)^n$$

c.
$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$$



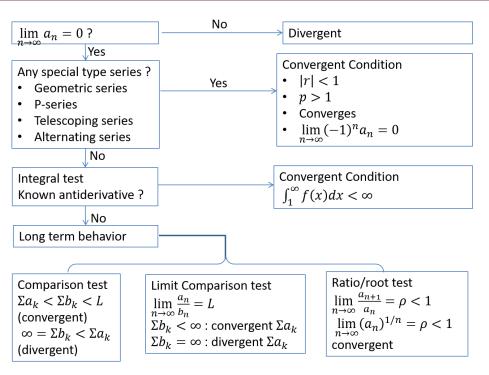


For each of the following series, apply the divergence test. If the divergence test proves that the series diverges, state so. Otherwise, indicate that the divergence test is inconclusive.

a.
$$\sum_{\substack{n=1 \ \infty}}^{\infty} \frac{n}{3n-1}$$

b. $\sum_{\substack{n=1 \ \infty}}^{\infty} \frac{1}{n^3}$
c. $\sum_{n=1}^{\infty} e^{1/n^2}$

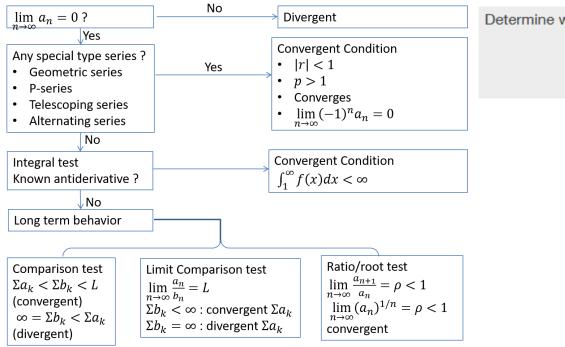




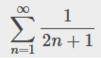
Determine whether the series

 $\sum\nolimits_{n=1}^{\infty} (n+1)/n$ converges or diverges.

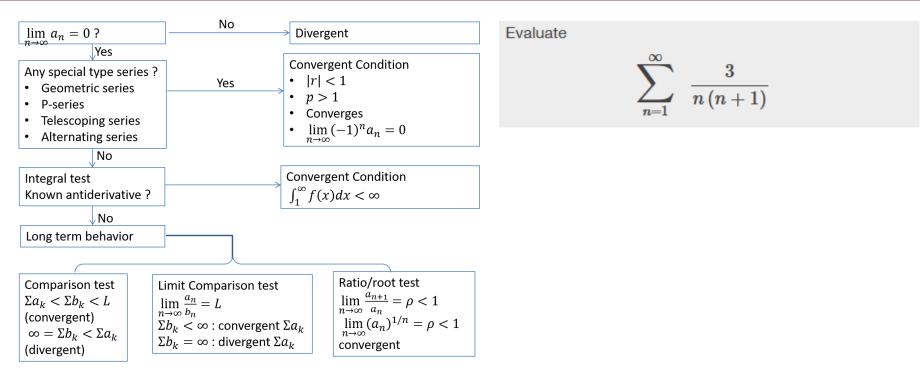




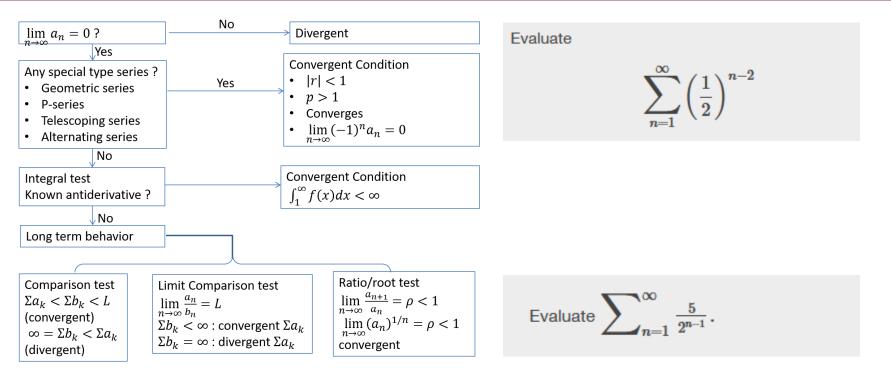
Determine whether the telescoping series converges or diverges.



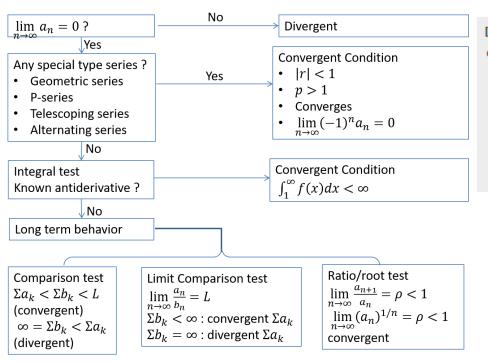




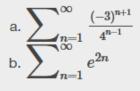








Determine whether each of the following geometric series converges or diverges, and if it converges, find its sum.



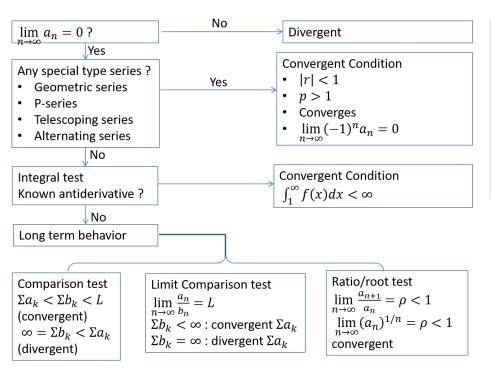


	No		
$\lim_{n\to\infty}a_n=0?$		Divergent	Determine whether the telescoping series
_n→∞Yes			
Any special type serie Geometric series P-series Telescoping series Alternating series	Yes	Convergent Condition r < 1 p > 1 Converges $\lim_{n \to \infty} (-1)^n a_n = 0$	$\sum\nolimits_{n=1}^{\infty} \big[e^{1/n} - e^{1/(n+1)} \big]$
No			converges or diverges. If it converges, find its sum.
Integral test Known antiderivative		Convergent Condition $\int_{1}^{\infty} f(x) dx < \infty$	
Long term behavior			
Comparison test $\Sigma a_k < \Sigma b_k < L$ (convergent) $\infty = \Sigma b_k < \Sigma a_k$ (divergent)	Limit Comparison test $\lim_{n \to \infty} \frac{a_n}{b_n} = L$ $\Sigma b_k < \infty : \text{ convergent } \Sigma a_k$ $\Sigma b_k = \infty : \text{ divergent } \Sigma a_k$	$n \rightarrow \infty$	



$\lim_{n \to \infty} a_n = 0 ?$ Yes		rgent	Determine whether the telescoping series
 Any special type series ? Geometric series P-series Telescoping series Alternating series 	Yes • r • p > • Co		$\sum_{n=1}^{\infty} \left[\cos\left(\frac{1}{n}\right) - \cos\left(\frac{1}{n+1}\right) \right]$
No			converges or diverges. If it converges, find its sum.
Integral test Known antiderivative ?		rgent Condition $(x)dx < \infty$	
√No Long term behavior	<u>]</u>		
$\begin{array}{c c} \Sigma a_k < \Sigma b_k < L \\ \text{(convergent)} \\ \infty = \Sigma b_k < \Sigma a_k \end{array}$	imit Comparison test $im_{\rightarrow\infty} \frac{a_n}{b_n} = L$ $ib_k < \infty : \text{ convergent } \Sigma a_k$ $ib_k = \infty : \text{ divergent } \Sigma a_k$	$\begin{array}{l} \text{Ratio/root test} \\ \lim_{n \to \infty} \frac{a_{n+1}}{a_n} = \rho < 1 \\ \lim_{n \to \infty} (a_n)^{1/n} = \rho < 1 \\ \text{convergent} \end{array}$	





For each of the following alternating series, determine whether the series converges or diverges.

a.
$$\sum_{n=1}^{\infty} (-1)^{n+1}/n^2$$

b. $\sum_{n=1}^{\infty} (-1)^{n+1}n/(n+1)$
c. $\sum_{n=1}^{\infty} (-1)^{n+1}n/2^n$



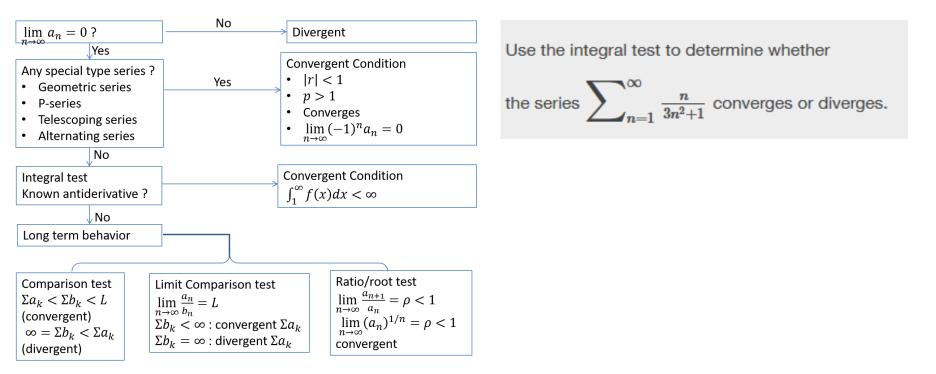
$\lim_{n \to \infty} a_n = 0 ?$	No	→ Divergent	For each				
 Any special type series Geometric series P-series Telescoping series Alternating series 	? Yes	Convergent Condition • $ r < 1$ • $p > 1$ • Converges • $\lim_{n \to \infty} (-1)^n a_n = 0$	a. $\sum_{n=1}^{\infty}$				
Integral test Known antiderivative ?	>	Convergent Condition $\int_{1}^{\infty} f(x) dx < \infty$					
No Long term behavior							
$\begin{array}{l} \text{Comparison test} \\ \Sigma a_k < \Sigma b_k < L \\ (\text{convergent}) \\ \infty = \Sigma b_k < \Sigma a_k \\ (\text{divergent}) \end{array} \qquad $		$\lim_{n \to \infty} \frac{a_{n+1}}{a_n} = \rho < 1$ $\lim_{n \to \infty} (a_n)^{1/n} = \rho < 1$					

For each of the following series, determine whether the series converges or diverges.

a.
$$\sum_{n=1}^{\infty} 1/n^3$$

b. $\sum_{n=1}^{\infty} 1/\sqrt{2n-1}$

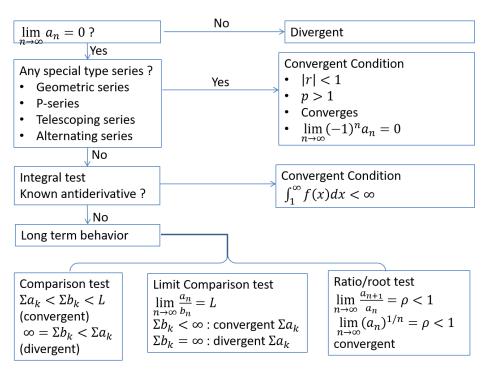






$\lim_{n\to\infty}a_n=0?$	No >	Divergent	For each of the following series, determine whether it converges or diverges.
 Yes Any special type seri Geometric series P-series Telescoping series 	S Yes	Convergent Condition • $ r < 1$ • $p > 1$ • Converges • $\lim_{x \to 0} (-1)^n a = 0$	a. $\sum_{\substack{n=1 \ \infty}}^{\infty} \frac{1}{n^4}$ b. $\sum_{n=1}^{\infty} \frac{1}{n^{2/3}}$
 Alternating series 		• $\lim_{n \to \infty} (-1)^n a_n = 0$	
No			
Integral test	(Convergent Condition	
Known antiderivative	e?	$\int_{1}^{\infty} f(x) dx < \infty$	
No		-	1
Long term behavior			
Comparison test	Limit Comparison test	Ratio/root test	
$\Sigma a_k < \Sigma b_k < L$ (convergent) $\infty = \Sigma b_k < \Sigma a_k$ (divergent)	$\begin{split} &\lim_{n\to\infty}\frac{a_n}{b_n}=L\\ &\Sigma b_k<\infty: \text{convergent }\Sigma\\ &\Sigma b_k=\infty: \text{divergent }\Sigma a_k \end{split}$	$\eta \rightarrow 0$	



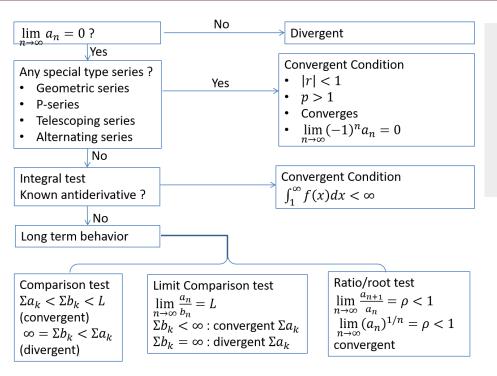


For each of the following series, use the comparison test to determine whether the series converges or diverges.

a.
$$\sum_{n=1}^{\infty} \frac{1}{n^3+3n+1}$$

b. $\sum_{n=1}^{\infty} \frac{1}{2^n+1}$
c. $\sum_{n=2}^{\infty} \frac{1}{\ln(n)}$





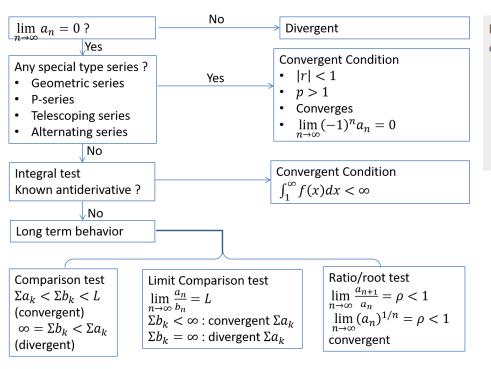
For each of the following series, use the limit comparison test to determine whether the series converges or diverges.

a.
$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+1}}$$

b.
$$\sum_{n=1}^{\infty} \frac{\frac{2^{n}+1}{3^{n}}}{\frac{1}{3^{n}}}$$

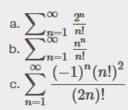
c.
$$\sum_{n=1}^{\infty} \frac{\ln(n)}{n^{2}}$$



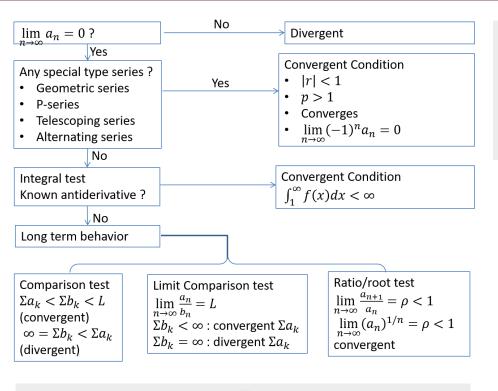


Determine whether the series $\sum_{n=1}^{\infty} \frac{n^3}{3^n}$ converges or diverges.

For each of the following series, determine whether the series converges or diverges.







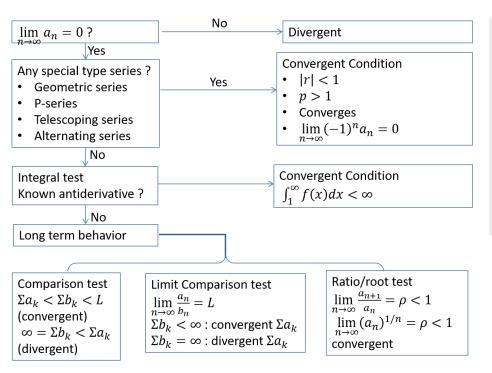
Determine whether the series $\sum_{n=1}^\infty 1/n^n$ converges or diverges.

For each of the following series, determine whether the series converges or diverges.

a.
$$\sum_{n=1}^{\infty} \frac{(n^2+3n)^n}{(4n^2+5)^n}$$

b.
$$\sum_{n=2}^{\infty} \frac{n^n}{(\ln(n))^n}$$





Determine whether the series converges or diverges.

a.
$$\sum_{n=1}^{\infty} \frac{\frac{n^2+2n}{n^3+3n^2+1}}{\frac{(-1)^{n+1}(3n+1)}{n!}}$$

b.
$$\sum_{n=1}^{\infty} \frac{\frac{(-1)^{n+1}(3n+1)}{n!}}{\frac{n!}{n!}}$$

c.
$$\sum_{n=1}^{n} \frac{\frac{e^n}{n^3}}{\frac{3^n}{(n+1)^n}}$$

e.
$$\sum_{n=1}^{\infty} \frac{2^n}{3^n+n}$$