Week in Review
Math 152

Week 09

Comparison Test

Alternating Series

Review

$\lim_{n\to\infty} a_n = 0$?

No → Divergent

Yes ↓

Any special type series ?
- Geometric series
- P-series
- Telescoping series
- Alternating series

Yes → Convergent Condition
- $|r| < 1$
- $p > 1$
- Converges
- $\lim_{n\to\infty} (-1)^n a_n = 0$

No ↓

Integral test
Known antiderivative ?

→ Convergent Condition
$\int_1^\infty f(x)dx < \infty$

No ↓

Long term behavior

Comparison test
$\Sigma a_k < \Sigma b_k < L$
(convergent)
$\infty = \Sigma b_k < \Sigma a_k$
(divergent)

Limit Comparison test
$\lim_{n\to\infty} \frac{a_n}{b_n} = L$
$\Sigma b_k < \infty$: convergent Σa_k
$\Sigma b_k = \infty$: divergent Σa_k

Ratio/root test
$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \rho < 1$
$\lim_{n\to\infty} (a_n)^{1/n} = \rho < 1$
convergent

For each of the following series, use the sequence of partial sums to determine whether the series converges or diverges.

a. $\sum_{n=1}^{\infty} \frac{n}{n+1}$

b. $\sum_{n=1}^{\infty} (-1)^n$

c. $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$

Review

$\lim_{n\to\infty} a_n = 0$?

No → Divergent

Yes ↓

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- P-series
- Telescoping series
- Alternating series

Yes → Convergent Condition

- $|r| < 1$
- $p > 1$
- Converges
- $\lim_{n\to\infty} (-1)^n a_n = 0$

No ↓

Integral test
Known antiderivative ?

→ Convergent Condition
$\int_1^{\infty} f(x)dx < \infty$

No ↓

Long term behavior

Comparison test
$\Sigma a_k < \Sigma b_k < L$
(convergent)
$\infty = \Sigma b_k < \Sigma a_k$
(divergent)

Limit Comparison test
$\lim_{n\to\infty} \frac{a_n}{b_n} = L$
$\Sigma b_k < \infty$: convergent Σa_k
$\Sigma b_k = \infty$: divergent Σa_k

Ratio/root test
$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \rho < 1$
$\lim_{n\to\infty} (a_n)^{1/n} = \rho < 1$
convergent

For each of the following series, apply the divergence test. If the divergence test proves that the series diverges, state so. Otherwise, indicate that the divergence test is inconclusive.

a. $\sum_{n=1}^{\infty} \frac{n}{3n-1}$

b. $\sum_{n=1}^{\infty} \frac{1}{n^3}$

c. $\sum_{n=1}^{\infty} e^{1/n^2}$

Review

$\lim_{n\to\infty} a_n = 0$?

- No → Divergent
- Yes → Any special type series ?
  - Geometric series
  - P-series
  - Telescoping series
  - Alternating series

Yes → Convergent Condition

- $|r| < 1$
- $p > 1$
- Converges
- $\lim_{n\to\infty} (-1)^n a_n = 0$

No → Integral test
Known antiderivative ? → Convergent Condition
$\int_1^\infty f(x)dx < \infty$

No → Long term behavior

Comparison test
$\Sigma a_k < \Sigma b_k < L$
(convergent)
$\infty = \Sigma b_k < \Sigma a_k$
(divergent)

Limit Comparison test
$\lim_{n\to\infty} \frac{a_n}{b_n} = L$
$\Sigma b_k < \infty$: convergent Σa_k
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Ratio/root test
$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \rho < 1$
$\lim_{n\to\infty} (a_n)^{1/n} = \rho < 1$
convergent

Determine whether the series

$\sum_{n=1}^{\infty} (n+1)/n$ converges or diverges.

Review

$\lim_{n\to\infty} a_n = 0$?

No → Divergent

Yes ↓

Any special type series ?
- Geometric series
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- Alternating series

Yes → Convergent Condition
- $|r| < 1$
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- $\lim_{n\to\infty} (-1)^n a_n = 0$

No ↓

Integral test
Known antiderivative ?

→ Convergent Condition
$\int_1^\infty f(x)dx < \infty$

No ↓

Long term behavior

Comparison test
$\Sigma a_k < \Sigma b_k < L$
(convergent)
$\infty = \Sigma b_k < \Sigma a_k$
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Limit Comparison test
$\lim_{n\to\infty} \frac{a_n}{b_n} = L$
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Ratio/root test
$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \rho < 1$
$\lim_{n\to\infty} (a_n)^{1/n} = \rho < 1$
convergent

Determine whether the telescoping series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{1}{2n+1}$$

Review

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Yes ↓

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No ↓

Integral test
Known antiderivative ?

→ Convergent Condition
$\int_1^\infty f(x)dx < \infty$

No ↓

Long term behavior

Comparison test
$\Sigma a_k < \Sigma b_k < L$
(convergent)
$\infty = \Sigma b_k < \Sigma a_k$
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Limit Comparison test
$\lim_{n\to\infty} \frac{a_n}{b_n} = L$
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Ratio/root test
$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \rho < 1$
$\lim_{n\to\infty} (a_n)^{1/n} = \rho < 1$
convergent

Evaluate

$$\sum_{n=1}^{\infty} \frac{3}{n\,(n+1)}$$

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No ↓

Integral test
Known antiderivative ?

→ Convergent Condition
$\int_1^\infty f(x)dx < \infty$

No ↓

Long term behavior

Comparison test
$\Sigma a_k < \Sigma b_k < L$
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Limit Comparison test
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Ratio/root test
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$\lim_{n\to\infty} (a_n)^{1/n} = \rho < 1$
convergent

Evaluate

$$\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{n-2}$$

Evaluate $\sum_{n=1}^{\infty} \frac{5}{2^{n-1}}$.

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No ↓

Integral test
Known antiderivative ?

→ Convergent Condition
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No ↓

Long term behavior

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$\lim_{n\to\infty} (a_n)^{1/n} = \rho < 1$
convergent

Determine whether each of the following geometric series converges or diverges, and if it converges, find its sum.

a. $\sum_{n=1}^{\infty} \frac{(-3)^{n+1}}{4^{n-1}}$

b. $\sum_{n=1}^{\infty} e^{2n}$

Review

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Yes → Convergent Condition
- $|r| < 1$
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No ↓

Integral test
Known antiderivative ?

→ Convergent Condition
$\int_1^\infty f(x)dx < \infty$

No ↓

Long term behavior

Comparison test
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Limit Comparison test
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Ratio/root test
$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \rho < 1$
$\lim_{n\to\infty} (a_n)^{1/n} = \rho < 1$
convergent

Determine whether the telescoping series

$$\sum_{n=1}^{\infty} \left[e^{1/n} - e^{1/(n+1)}\right]$$

converges or diverges. If it converges, find its sum.

Review

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- $|r| < 1$
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- $\lim_{n\to\infty} (-1)^n a_n = 0$

No ↓

Integral test
Known antiderivative ?

→ Convergent Condition
$\int_1^\infty f(x)dx < \infty$

No ↓

Long term behavior

Comparison test
$\Sigma a_k < \Sigma b_k < L$
(convergent)
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Limit Comparison test
$\lim_{n\to\infty} \frac{a_n}{b_n} = L$
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Ratio/root test
$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \rho < 1$
$\lim_{n\to\infty} (a_n)^{1/n} = \rho < 1$
convergent

Determine whether the telescoping series

$$\sum_{n=1}^{\infty} \left[\cos\left(\frac{1}{n}\right) - \cos\left(\frac{1}{n+1}\right)\right]$$

converges or diverges. If it converges, find its sum.

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- $|r| < 1$
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No ↓

Integral test
Known antiderivative ?

→ Convergent Condition
$\int_1^\infty f(x)dx < \infty$

No ↓

Long term behavior

Comparison test
$\Sigma a_k < \Sigma b_k < L$
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$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \rho < 1$
$\lim_{n\to\infty} (a_n)^{1/n} = \rho < 1$
convergent

For each of the following alternating series, determine whether the series converges or diverges.

a. $\sum_{n=1}^{\infty} (-1)^{n+1}/n^2$

b. $\sum_{n=1}^{\infty} (-1)^{n+1} n/(n+1)$

c. $\sum_{n=1}^{\infty} (-1)^{n+1} n/2^n$

Review

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Any special type series ?
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- Telescoping series
- Alternating series

Yes → Convergent Condition
- $|r| < 1$
- $p > 1$
- Converges
- $\lim_{n\to\infty} (-1)^n a_n = 0$

No ↓

Integral test
Known antiderivative ?

→ Convergent Condition
$\int_1^\infty f(x)dx < \infty$

No ↓

Long term behavior

Comparison test
$\Sigma a_k < \Sigma b_k < L$
(convergent)
$\infty = \Sigma b_k < \Sigma a_k$
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Limit Comparison test
$\lim_{n\to\infty} \frac{a_n}{b_n} = L$
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Ratio/root test
$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \rho < 1$
$\lim_{n\to\infty} (a_n)^{1/n} = \rho < 1$
convergent

For each of the following series, determine whether the series converges or diverges.

a. $\sum_{n=1}^{\infty} 1/n^3$

b. $\sum_{n=1}^{\infty} 1/\sqrt{2n-1}$

Review

$\lim_{n\to\infty} a_n = 0$?

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  - Alternating series

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- $|r| < 1$
- $p > 1$
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- $\lim_{n\to\infty} (-1)^n a_n = 0$

No → Integral test
Known antiderivative ?

→ Convergent Condition
$\int_1^\infty f(x)dx < \infty$

No → Long term behavior

Comparison test
$\Sigma a_k < \Sigma b_k < L$
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Ratio/root test
$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \rho < 1$
$\lim_{n\to\infty} (a_n)^{1/n} = \rho < 1$
convergent

Use the integral test to determine whether the series $\sum_{n=1}^{\infty} \frac{n}{3n^2+1}$ converges or diverges.

Review

$\lim_{n\to\infty} a_n = 0$?

No → Divergent

Yes ↓

Any special type series ?

- Geometric series
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- Alternating series

Yes → Convergent Condition

- $|r| < 1$
- $p > 1$
- Converges
- $\lim_{n\to\infty}(-1)^n a_n = 0$

No ↓

Integral test
Known antiderivative ?

→ Convergent Condition
$\int_1^\infty f(x)dx < \infty$

No ↓

Long term behavior

Comparison test
$\Sigma a_k < \Sigma b_k < L$
(convergent)
$\infty = \Sigma b_k < \Sigma a_k$
(divergent)

Limit Comparison test
$\lim_{n\to\infty}\frac{a_n}{b_n} = L$
$\Sigma b_k < \infty$: convergent Σa_k
$\Sigma b_k = \infty$: divergent Σa_k

Ratio/root test
$\lim_{n\to\infty}\frac{a_{n+1}}{a_n} = \rho < 1$
$\lim_{n\to\infty}(a_n)^{1/n} = \rho < 1$
convergent

For each of the following series, determine whether it converges or diverges.

a. $\sum_{n=1}^{\infty} \frac{1}{n^4}$

b. $\sum_{n=1}^{\infty} \frac{1}{n^{2/3}}$

Review

$\lim_{n\to\infty} a_n = 0$?

No → Divergent

Yes ↓

Any special type series ?
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Yes → Convergent Condition
- $|r| < 1$
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No ↓

Integral test
Known antiderivative ?

→ Convergent Condition
$\int_1^\infty f(x)dx < \infty$

No ↓

Long term behavior

Comparison test
$\Sigma a_k < \Sigma b_k < L$
(convergent)
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Limit Comparison test
$\lim_{n\to\infty} \frac{a_n}{b_n} = L$
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Ratio/root test
$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \rho < 1$
$\lim_{n\to\infty} (a_n)^{1/n} = \rho < 1$
convergent

For each of the following series, use the comparison test to determine whether the series converges or diverges.

a. $\sum_{n=1}^{\infty} \frac{1}{n^3+3n+1}$

b. $\sum_{n=1}^{\infty} \frac{1}{2^n+1}$

c. $\sum_{n=2}^{\infty} \frac{1}{\ln(n)}$

Review

$\lim\limits_{n\to\infty} a_n = 0$?

No → Divergent

Yes ↓

Any special type series ?
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- Alternating series

Yes → Convergent Condition
- $|r| < 1$
- $p > 1$
- Converges
- $\lim\limits_{n\to\infty} (-1)^n a_n = 0$

No ↓

Integral test
Known antiderivative ?

→ Convergent Condition
$\int_1^\infty f(x)dx < \infty$

No ↓

Long term behavior

Comparison test
$\Sigma a_k < \Sigma b_k < L$
(convergent)
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Limit Comparison test
$\lim\limits_{n\to\infty} \frac{a_n}{b_n} = L$
$\Sigma b_k < \infty$: convergent Σa_k
$\Sigma b_k = \infty$: divergent Σa_k

Ratio/root test
$\lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n} = \rho < 1$
$\lim\limits_{n\to\infty} (a_n)^{1/n} = \rho < 1$
convergent

For each of the following series, use the limit comparison test to determine whether the series converges or diverges.

a. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}+1}$

b. $\sum_{n=1}^{\infty} \frac{2^n+1}{3^n}$

c. $\sum_{n=1}^{\infty} \frac{\ln(n)}{n^2}$

Review

$\lim_{n\to\infty} a_n = 0$?

No → Divergent

Yes ↓

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Yes → Convergent Condition
- $|r| < 1$
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- $\lim_{n\to\infty} (-1)^n a_n = 0$

No ↓

Integral test
Known antiderivative ?

→ Convergent Condition
$\int_1^\infty f(x)dx < \infty$

No ↓

Long term behavior

Comparison test
$\Sigma a_k < \Sigma b_k < L$
(convergent)
$\infty = \Sigma b_k < \Sigma a_k$
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Limit Comparison test
$\lim_{n\to\infty} \frac{a_n}{b_n} = L$
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$\Sigma b_k = \infty$: divergent Σa_k

Ratio/root test
$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \rho < 1$
$\lim_{n\to\infty} (a_n)^{1/n} = \rho < 1$
convergent

Determine whether the series $\sum_{n=1}^{\infty} \frac{n^3}{3^n}$ converges or diverges.

For each of the following series, determine whether the series converges or diverges.

a. $\sum_{n=1}^{\infty} \frac{2^n}{n!}$

b. $\sum_{n=1}^{\infty} \frac{n^n}{n!}$

c. $\sum_{n=1}^{\infty} \frac{(-1)^n (n!)^2}{(2n)!}$

Review

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No ↓

Integral test
Known antiderivative ?

→ Convergent Condition
$\int_1^\infty f(x)dx < \infty$

No ↓

Long term behavior

Comparison test
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$\lim_{n\to\infty} (a_n)^{1/n} = \rho < 1$
convergent

Determine whether the series $\sum_{n=1}^{\infty} 1/n^n$ converges or diverges.

For each of the following series, determine whether the series converges or diverges.

a. $\sum_{n=1}^{\infty} \frac{(n^2+3n)^n}{(4n^2+5)^n}$

b. $\sum_{n=2}^{\infty} \frac{n^n}{(\ln(n))^n}$

Review

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No → Divergent

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- $|r| < 1$
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- $\lim_{n\to\infty}(-1)^n a_n = 0$

No ↓

Integral test
Known antiderivative ?

→ Convergent Condition
$\int_1^\infty f(x)dx < \infty$

No ↓

Long term behavior

Comparison test
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Ratio/root test
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$\lim_{n\to\infty} (a_n)^{1/n} = \rho < 1$
convergent

Determine whether the series converges or diverges.

a. $\sum_{n=1}^{\infty} \frac{n^2+2n}{n^3+3n^2+1}$

b. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(3n+1)}{n!}$

c. $\sum_{n=1}^{\infty} \frac{e^n}{n^3}$

d. $\sum_{n=1}^{\infty} \frac{3^n}{(n+1)^n}$

e. $\sum_{n=1}^{\infty} \frac{2^n}{3^n+n}$