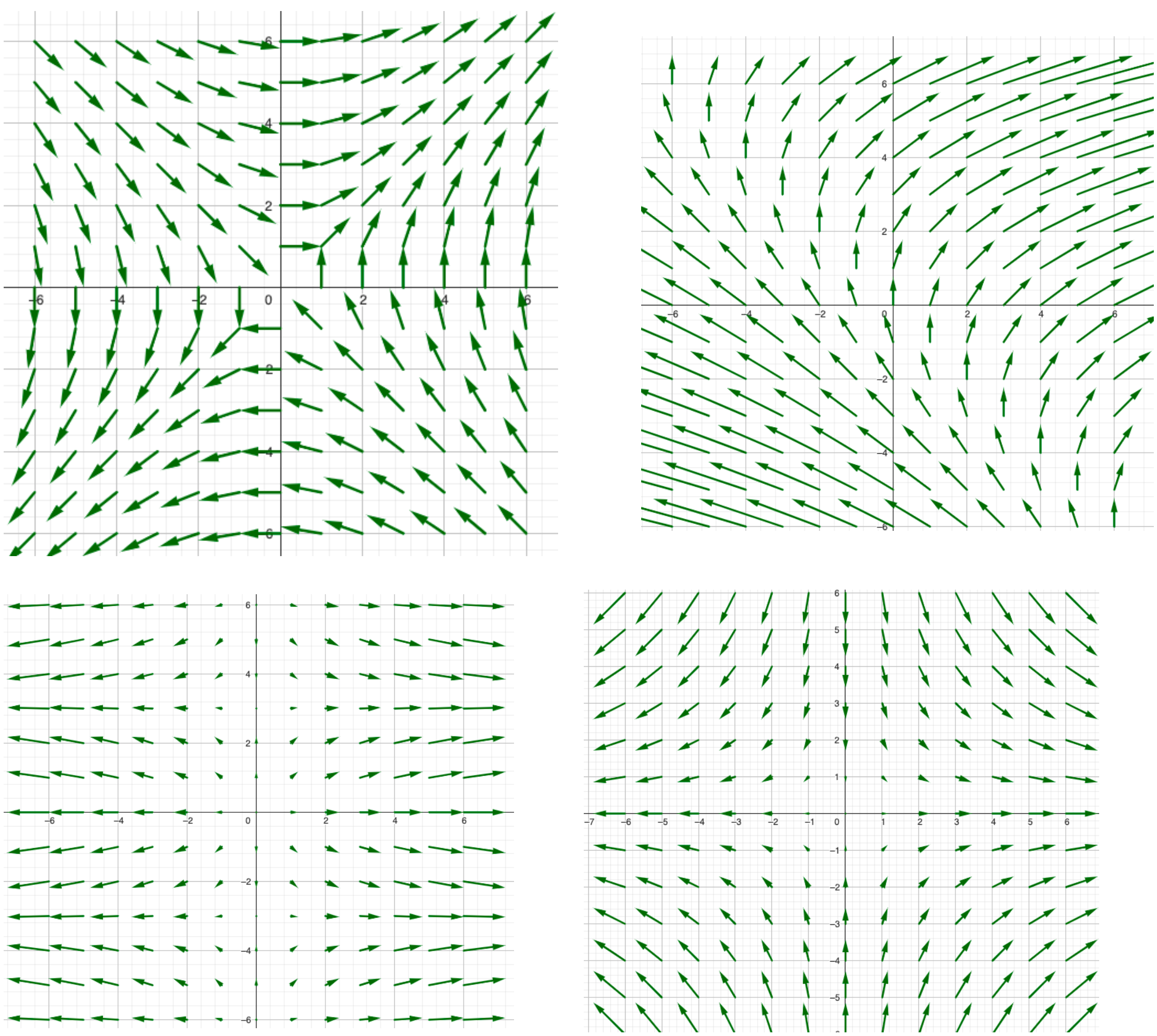
Example 1 (16.1). *Identify the vector field of the vector function* $\mathbf{F}$.

(a) $\mathbf{F}(x, y) = x\,\mathbf{i} - y\,\mathbf{j}$

(b) $\mathbf{F}(x, y) = \dfrac{y}{\sqrt{x^2+y^2}}\,\mathbf{i} + \dfrac{x}{\sqrt{x^2+y^2}}\,\mathbf{j}$

(c) $\mathbf{F}(x, y) = x\,\mathbf{i} + \sin y\,\mathbf{j}$

(d) $\mathbf{F}(x, y) = \langle x + y, 3\rangle$



Example 2 (16.1)**.** *Sketch the gradient vector field of f.*

(a) $f(x, y) = 2x + 3y$

(b) $f(x, y) = \dfrac{x^2}{2} + \dfrac{y^2}{2} - 3y.$

Example 3 (16.2)**.** *Evaluate the line integral*

$$\int_C x^2\,dx + y^2\,dy,$$

where C is the closed curve oriented counterclockwise in the upper half-plane formed by the circle $x^2 + y^2 = 9$ and line $y = 0$.

Example 4 (16.2)**.** *Let C be the curve* $\mathbf{r}(t) = \left\langle 2t,\, t^2,\, \dfrac{t^3}{3} \right\rangle;\ 0 \le t \le 2.$

(a) Evaluate the line integral $\displaystyle\int_C (xy + 3z)\, ds.$

(b) Evaluate the line integral $\displaystyle\int_C x^2\, dx + y\, dy + 12z\, dz.$

Example 5 (16.2). *Evaluate the line integral*

$$\int_C (x+y)\,dx + y^2\,dy + z\,dz$$

where C *consists of the line segments from* $(0,0,0)$ *to* $(1,1,0)$ *and from* $(1,1,0)$ *to* $(1,0,2)$.

Example 6 (16.2/16.3). *Evaluate the line integral*

$$\int_C \mathbf{F} \cdot d\mathbf{r},$$

where $\mathbf{F}(x, y) = \dfrac{x}{\sqrt{x^2 + y^2}}\,\mathbf{i} + \dfrac{y}{\sqrt{x^2 + y^2}}\,\mathbf{j}$ *and* C *is the part of the parabola* $y = x^2 + 1$ *from* $(-1, 2)$ *to* $(2, 5)$.

Example 7 (16.2)**.** *Find the work done by the force field* $\mathbf{F}(x, y, z) = x\,\mathbf{i} - y\,\mathbf{j} + (x + z)\,\mathbf{k}$ *acting along the circular helix* $\mathbf{r}(t) = \cos t\,\mathbf{i} + \sin t\,\mathbf{j} + 2t\,\mathbf{k}$ *from* $(0, 1, \pi)$ *to* $(-1, 0, 2\pi)$.

Theorem 2: *Let C be a (piecewise) smooth curve given by the vector function $\mathbf{r}(t)$, $a \le t \le b$. Let f be a differentiable function of two or three variables whose gradient vector ∇f is continuous on C. Then*

$$\int_C \nabla f \cdot d\mathbf{r} = f(\mathbf{r}(b)) - f(\mathbf{r}(a))$$

If C is a smooth curve in $\mathbb{R}^2$ with initial point $A(x_0, y_0)$ and terminal point $B(x_1, y_1)$, then

$$\int_C \nabla f \cdot d\mathbf{r} = f(x_1, y_1) - f(x_0, y_0)$$

If C is a smooth curve in $\mathbb{R}^3$ with initial point $A(x_0, y_0, z_0)$ and terminal point $B(x_1, y_1, z_1)$, then

$$\int_C \nabla f \cdot d\mathbf{r} = f(x_1, y_1, z_1) - f(x_0, y_0, z_0)$$

Theorem 6: *Let $\mathbf{F} = P\mathbf{i} + Q\mathbf{j}$ be a vector field on an open simply-connected region D. Suppose that P and Q have continuous first-order partial derivatives and*

$$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x} \qquad \text{throughout } D.$$

Then $\mathbf{F}$ is conservative.

Example 8 (16.3). *Determine whether or not* $\mathbf{F}(x, y) = (\sqrt{y}+2xy^2-3)\,\mathbf{i}+\left(\dfrac{x}{2\sqrt{y}} + 2x^2y + 1\right)\,\mathbf{j}$ *is conservative. If it is, find a potential function* f.

Example 9 (16.3)**.** *Find a potential function of the vector field* $\mathbf{F}(x, y, z) = \langle e^{yz},\, xze^{yz},\, xye^{yz}\rangle$, *and use it to find the line integral*

$$\int_C \mathbf{F}\cdot d\mathbf{r},\ \textit{where}\ C:\ x = 2t+1,\ \ y = t^2,\ z = \sqrt{t},\quad 0 \le t \le 4.$$

Example 10 (16.3). *Find the work done by the force field* $\mathbf{F}(x, y) = \langle 2x + y, x + 1 \rangle$ *acting along the circle* $x^2 + y^2 = 9$ *from* $(0, 3)$ *to* $(-3, 0)$.

Example 11 (16.3). *Find* $\displaystyle\int_C \mathbf{F} \cdot d\mathbf{r}$, *where* $\mathbf{F}$ *is a conservative vector field on* $\mathbb{R}^2$ *and* C *is a rectangle with vertices* $(1, 0)$, $(3, 0)$, $(3, 2)$, *and* $(1, 2)$.