2024 Fall Math 140 Week-In-Review

Week 1: Sections 1.1-1.2

Section 1.1: Basic Matrix Operations

Some Key Words and Terms: Dimensions, Elements/Entries, Scalar, Transpose, Matrix Equality, Commutative, Associative

Dimensions of Matrices: $m \times n$: m rows then n columns

★ rows go first ★

i.e. 2×3: 2 row, 3 column

4×10: 4 row, 10 column

Elements, or Entries, of Matrices:

i.e. a_{12} "the first row, second column of matrix A

$$C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \quad c_{13}$$

Adding/Subtracting Matrices:

★ First: we must check if it is possible ⇒ they must be the exact same size

★ If Possible: we add/subtract "entry-wise", so corresponding entries add/subtract

$A = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ 2×1 $\quad B = \begin{bmatrix} 4 \\ x \end{bmatrix}$ 2×1 $\quad A+B = \begin{bmatrix} (1+4) \\ (2+x) \end{bmatrix} = \begin{bmatrix} 5 \\ (2+x) \end{bmatrix}$

$\boxed{\text{Scalar}}$ Multiplication:

↳ just is a single term, which for us will always be a #

We just "distribute" the scalar (#) to each entry of the matrix and multiply

$C = \begin{bmatrix} 1 & 2 \\ x & y \end{bmatrix}$ $\quad 3C = 3\begin{bmatrix} 1 & 2 \\ x & y \end{bmatrix} = \begin{bmatrix} 3\cdot1 & 3\cdot2 \\ 3\cdot x & 3\cdot y \end{bmatrix} = \begin{bmatrix} 3 & 6 \\ 3x & 3y \end{bmatrix}$

★ scalars don't change the size of a matrix ★

Transpose of a Matrix:

We do one of two things:

1) rows become the columns
OR
2) columns become the rows

$\Big\}$ Pick One

$$A = \begin{array}{c} \begin{bmatrix} 1 & 2 & 3 \\ x & y & z \end{bmatrix} \begin{array}{l} r1 \\ r2 \end{array} \end{array} \quad (2\times3) \text{ so } A^T \text{ is } (3\times2)$$

$$L = \begin{array}{c} \begin{bmatrix} a & b \\ 1 & 2 \\ x & y \\ 99 & 100 \end{bmatrix} \\ c_1 \quad c_2 \end{array} \quad (4\times2) \text{ so } L^T \text{ is } (2\times4)$$

$$A^T = \begin{array}{c} c1 \quad c2 \\ \begin{bmatrix} 1 & x \\ 2 & y \\ 3 & z \end{bmatrix} \end{array}$$

$$L^T = \begin{bmatrix} a & 1 & x & 99 \\ b & 2 & y & 100 \end{bmatrix} \begin{array}{l} r1 \\ r2 \end{array}$$

Matrix Equality: In general, one matrix = one matrix only if both:
① they are the exact same size
AND
② each corresponding entry equals

$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$

$M = \begin{bmatrix} a & b & c \\ x & y & z \end{bmatrix}$

$A = M$:
$1 = a$
$2 = b$
$3 = c$
$4 = x$
$5 = y$
$6 = z$

Other Properties: I) Add/Subtract: (i) order doesn't matter
$A+B+C = C+A+B$, $A-B = -B+A$
(ii) grouping doesn't matter
$(A+B)+C = A+(B+C)$

II) Tranpose: ★ transpose of transpose undo each other
$(A^T)^T = A$

Examples:

1. Complete the given matrix operations, if possible. If it is not possible, explain why.

$$7\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \boxed{- 5\begin{bmatrix} a & 0 & -b \\ 9 & 2c & -3 \end{bmatrix}} + \begin{bmatrix} 11 & -8 \\ -1 & 5 \\ 3 & -4 \end{bmatrix}^T$$

negative scalars:
① distribute the number but not the negative & subtract matrices
② distribute the negative w/ the number but then ADD matrices

$\text{scalar} \cdot (2\times3) - \text{scalar} \cdot (2\times 3) + (3\times2)^T$

$2\times3 - 2\times3 + 2\times3 \Rightarrow 2\times3$ ✓ doable

★ In general, follow PEMDAS transpose works like an exponent

$$\begin{bmatrix} 7 & 14 & 21 \\ 28 & 35 & 42 \end{bmatrix} - \begin{bmatrix} 5a & 0 & -5b \\ 45 & 10c & -15 \end{bmatrix} + \begin{bmatrix} 11 & -1 & 3 \\ -8 & 5 & -4 \end{bmatrix}$$

OR

$$\begin{bmatrix} 7 & 14 & 21 \\ 28 & 35 & 42 \end{bmatrix} + \begin{bmatrix} -5a & 0 & 5b \\ -45 & -10c & 15 \end{bmatrix} + \begin{bmatrix} 11 & -1 & 3 \\ -8 & 5 & -4 \end{bmatrix}$$

Then ...

$$\begin{bmatrix} (7-5a+11) & (14+0-1) & (21+5b+3) \\ (28-45-8) & (35-10c+5) & (42+15-4) \end{bmatrix} = \boxed{\begin{bmatrix} 18-5a & 13 & 24+5b \\ -25 & 40-10c & 53 \end{bmatrix}}$$

2. Determine the values for w, x, y, and z that make the following matrix equation true.

$$2\begin{bmatrix}1 & w\\ x & 2\end{bmatrix} - 3\begin{bmatrix}y & 3\\ 4 & z\end{bmatrix} = 5\begin{bmatrix}-1 & 2\\ 3 & -2\end{bmatrix}$$

matrix equation, first
one matrix = one matrix
so add/subtract/transpose/scalar

$$\begin{bmatrix}2 & 2w\\ 2x & 4\end{bmatrix} - \begin{bmatrix}3y & 9\\ 12 & 3z\end{bmatrix} = \begin{bmatrix}-5 & 10\\ 15 & -10\end{bmatrix}$$

$$\begin{bmatrix}(2-3y) & (2w-9)\\ (2x-12) & (4-3z)\end{bmatrix} = \begin{bmatrix}-5 & 10\\ 15 & -10\end{bmatrix}$$

second set corresponding entries equal & solve

$2-3y=-5$, $-3y=-7$, $y=\frac{-7}{-3}=\boxed{\frac{7}{3}}$

$2w-9=10$, $2w=19$, $w=\boxed{\frac{19}{2}}$

$2x-12=15$, $2x=27$, $x=\boxed{\frac{27}{2}}$

$4-3z=-10$, $-3z=-14$, $z=\frac{-14}{-3}=\boxed{\frac{14}{3}}$

Section 1.2: Matrix Multiplication

Some Key Words and Terms: Dimensions, Elements/Entries, Scalar vs. Matrix, Transpose, Matrix Equality, Associative, Distributive

Multiplying Matrices: ① Must check dimensions to see if possible

$(m \times n) \cdot (n \times p) \Rightarrow m \times p$

must much in number & category word-problems

② the actual multiplication is about the process not the entries

Scalar Multiplication vs. Matrix Multiplications:

this does not change the dimensions, just the entries ("distributing")

this likely changes the dimensions & changes the entries (an entire, specific process)

Other Properties:

Two properties:

(i) grouping doesn't matter: $(AB)C = A(BC)$
order cannot change!
in general $AB \neq BA$

(ii) distributes over +/− but is side specific: $A(B \pm C) = AB \pm AC$
on left on left

$(B \pm C)A = BA \pm CA$
on right on right

Examples:

1. Determine the dimensions of the resultant matrix, if possible. If it is not possible, explain why.

A is 2x3, B is 3x3, C is 2x2, D is 2x4

a. $3CD + 5D$

3C: scalar $\cdot$ (2x2) $\to$ 2x2

(3C)$\cdot$D: (2x2) $\cdot$ (2x4) $\to$ 2x4 ✓

} 3CD $\to$ 2x4

5D: scalar $\cdot$ (2x4) $\to$ 2x4

3CD + 5D: (2x4) + (2x4) $\to$ $\boxed{2\text{x}4}$

b. $2CA - 4B$

2C: scalar $\cdot$ (2x2) $\to$ 2x2

(2C)$\cdot$A: (2x2) $\cdot$ (2x3) $\to$ 2x3 ✓

4B: scalar $\cdot$ (3x3) $\to$ 3x3

2CA − 4B: (2x3) − (3x3)

not possible b/c cannot subtract matrices of different sizes

c. AA^TA

A $\cdot$ A^T $\cdot$ A

A^T: (2x3)^T $\to$ 3x2

A$\cdot$A^T: (2x3) $\cdot$ (3x2) $\to$ 2x2 ✓

(AA^T)$\cdot$A: (2x2) $\cdot$ (2x3) $\Rightarrow$ $\boxed{2\text{x}3}$ ✓

d. $ABCD$

AB: (2x3) $\cdot$ (3x3) $\Rightarrow$ 2x3 ✓

(AB)C: (2x3) $\cdot$ (2x2) ✗

Impossible because inner dimensions did not match

2. Complete the given matrix operations, if possible. If it is not possible, explain why.

$$\begin{bmatrix} 1 & -a \\ -2 & 5 \end{bmatrix} \cdot \begin{bmatrix} 7 & 0 \\ 3 & x \end{bmatrix}$$

① Check if possible

$(2\times 2)\cdot(2\times 2) \to 2\times 2$

$$\overset{A}{\begin{bmatrix} 1 & -a \\ -2 & 5 \end{bmatrix}} \cdot \overset{B}{\begin{bmatrix} 7 & 0 \\ 3 & x \end{bmatrix}} = \overset{C}{\begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}}$$

c_{11} = row 1 of A, column 1 of B

c_{12} = row 1 of A, column 2 of B

c_{21} = row 2 of A, column 1 of B

c_{22} = row 2 of A, column 2 of B

$$= \begin{bmatrix} (1)(7)+(-a)(3) & (1)(0)+(-a)(x) \\ (-2)(7)+(5)(3) & (-2)(0)+(5)(x) \end{bmatrix} = \boxed{\begin{bmatrix} 7-3a & -ax \\ 1 & 5x \end{bmatrix}}$$

3. Complete the given matrix operations, if possible. If it is not possible, explain why.

$$\begin{bmatrix} 1 & w & 2 \\ x & 3 & y \\ 4 & z & 5 \end{bmatrix} \cdot \begin{bmatrix} -1 & a \\ b & -2 \\ -3 & c \end{bmatrix}^{\mathrm{T}}$$

$(3\times 3)\cdot(3\times 2)^T$

$\star$ PEMDAS

transpose

$\underbrace{(3\times 3)\cdot(2\times 3)}_{\times}$

Impossible, inner dimensions don't match

4. Complete the given matrix operations, if possible. If it is not possible, explain why.

$$\begin{bmatrix} 1 & 2 \\ 2 & -1 \\ 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} a & 0 \\ 5 & b \\ c & 3 \end{bmatrix}^{\mathrm{T}}$$

$(3\times 2) \cdot (3 \times 2)^T$

so $\underbrace{(3\times 2) \cdot (2 \times 3)}$

so (3×3) result

$$A \cdot B = C$$

$$\begin{bmatrix} 1 & 2 \\ 2 & -1 \\ 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} a & 5 & c \\ 0 & b & 3 \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix}$$

$$= \begin{bmatrix} (1)(a) + \cancel{(2)(0)} & (1)(5) + (2)(b) & (1)(c) + (2)(3) \\ (2)(a) + \cancel{(-1)(0)} & (2)(5) + (-1)(b) & (2)(c) + (-1)(3) \\ (2)(a) + \cancel{(3)(0)} & (2)(5) + (3)(b) & (2)(c) + (3)(3) \end{bmatrix}$$

$$= \boxed{\begin{bmatrix} a & 5+2b & c+6 \\ 2a & 10-b & 2c-3 \\ 2a & 10+3b & 2c-9 \end{bmatrix}}$$