Week in Review
Math 152

Week 08

Common Exam 2

Preparation

Part II

Common Exam II Prep. Part II

1. After long division, $\dfrac{x^4+5x^2+1}{x^2+1} = A + \dfrac{B}{x^2+1}$. What are A and B?

   (a) $A = x^2$ and $B = 4x^2 - 3$
   (b) $A = x^2 + 4$ and $B = -3$
   (c) $A = 1$ and $B = 5x + 1$
   (d) $A = 0$ and $B = 1$
   (e) $A = x^2 + 5x$ and $B = x + 1$

$$
\begin{array}{r|l}
 & \quad x^2 + 4 \\
\hline
x^2+1 & x^4 + 5x^2 + 1 \\
 & x^4 + x^2 \\
\hline
 & \qquad 4x^2 + 1 \\
 & \qquad 4x^2 + 4 \\
\hline
 & \qquad\quad -3
\end{array}
$$

$A = Quotient\ ;$ $\quad B = Remainder$

$A = x^2 + 4,$ $\quad B = -3$

Common Exam II Prep. Part II

2. What would be an appropriate substitution in order to evaluate $\int \frac{1}{\sqrt{x^2+10x}}\,dx$?

(a) $x = 5\sec\theta - 5$

(b) $x = 5\sin\theta + 5$

(c) $x = 5\tan\theta$

(d) $x = 25\sec\theta$

(e) $x = 25\sin\theta - 5$

$$\frac{1}{\sqrt{x^2+10x}} = \frac{1}{\sqrt{x^2+10x+5^2-5^2}}$$
$$= \frac{1}{\sqrt{(x+5)^2-5^2}}$$

- Trig sub for $\sqrt{x^2+a^2} \Rightarrow x = a\tan\theta$
- Trig sub for $\sqrt{x^2-a^2} \Rightarrow x = a\sec\theta$
- Trig sub for $\sqrt{a^2-x^2} \Rightarrow x = a\sin\theta$

$x + 5 = 5\sec\theta$

$x = 5\sec\theta - 5$

Common Exam II Prep. Part II

3. Evaluate the improper integral $\int_5^{\infty} \frac{1}{x(\ln x)^4}\, dx$.

(a) $-\frac{3}{(\ln 5)^3}$

(b) The integral diverges.

(c) $\frac{1}{3(\ln 5)^3}$

(d) $\frac{1}{375}$

(e) $\frac{3}{125}$

Improper integral w/ a u-subable integrand

$$\int_5^{\infty} \frac{1}{x(\ln x)^4}\, dx = \lim_{N\to\infty} \int_5^{N} \frac{1}{x(\ln x)^4}\, dx$$

U-sub: $u = \ln x \Rightarrow du = \frac{1}{x} dx$

$$\int_{x=5}^{x=N} \Rightarrow \int_{u=\ln 5}^{u=\ln N}$$

$$= \lim_{N\to\infty} \int_{\ln 5}^{\ln N} \frac{1}{u^4}\, du$$

$$= \lim_{N\to\infty} \left[-\frac{1}{3u^3}\right]_{\ln 5}^{\ln N}$$

$$= \lim_{N\to\infty} \left[-\frac{1}{3(\ln N)^3} + \frac{1}{3(\ln 5)^3}\right]$$

$$= \frac{1}{3(\ln 5)^3}$$

Common Exam II Prep. Part II

4. The following recursive sequence is bounded and increasing. Find the limit of the sequence, if it exists.

$a_1 = 4, \quad a_{n+1} = 8 - \dfrac{15}{a_n}.$

(a) 8

(b) 4

(c) 3

(d) 5

(e) The sequence diverges.

Bounded and increasing ⇒ Converges to the least upper bound

Since $\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} a_{n+1} = a$,

$$\lim_{n\to\infty} a_{n+1} = \lim_{n\to\infty}\left(8 - \frac{15}{a_n}\right)$$

$$a = 8 - \frac{15}{a}$$

$$a^2 - 8a + 15 = 0$$

$$(a-3)(a-5) = 0$$

$$a = 3{,}5$$

Since a_n is increasing, $a = 5$

Common Exam II Prep. Part II

5. When applying the comparison test for improper integrals, which of the following statements is true regarding

$$\int_2^\infty \frac{\cos(x)+6}{x^3}\,dx?$$

(a) $\int_2^\infty \frac{\cos(x)+6}{x^3}\,dx$ diverges because $\frac{\cos(x)+6}{x^3} < \frac{5}{x^3}$ and $\int_2^\infty \frac{5}{x^3}\,dx$ converges.

(b) $\int_2^\infty \frac{\cos(x)+6}{x^3}\,dx$ diverges because $\frac{\cos(x)+6}{x^3} > \frac{5}{x^3}$ and $\int_2^\infty \frac{5}{x^3}\,dx$ converges.

(c) $\int_2^\infty \frac{\cos(x)+6}{x^3}\,dx$ converges because $\frac{\cos(x)+6}{x^3} < \frac{5}{x^3}$ and $\int_2^\infty \frac{5}{x^3}\,dx$ diverges.

(d) $\int_2^\infty \frac{\cos(x)+6}{x^3}\,dx$ converges because $\frac{\cos(x)+6}{x^3} < \frac{7}{x^3}$ and $\int_2^\infty \frac{7}{x^3}\,dx$ converges.

(e) $\int_2^\infty \frac{\cos(x)+6}{x^3}\,dx$ converges because $\frac{\cos(x)+6}{x^3} > \frac{7}{x^3}$ and $\int_2^\infty \frac{7}{x^3}\,dx$ converges.

Note that $\frac{5}{x^3} \le \frac{\cos x+6}{x^3} \le \frac{7}{x^3}$ where $\int_2^\infty \frac{k}{x^3}\,dx < \infty$ for any $k \ge 0$ (p test)

$\int_2^\infty \frac{5}{x^3}\,dx \le \int_2^\infty \frac{\cos x+6}{x^3}\,dx \le \int_2^\infty \frac{7}{x^3}\,dx < \infty$

In particular, $\int_2^\infty \frac{\cos x+6}{x^3}\,dx \le \int_2^\infty \frac{7}{x^3}\,dx < \infty$

Common Exam II Prep. Part II

6. Find the sum of the geometric series $\sum_{n=1}^{\infty} \frac{3^{n+1}}{4^n}$.

(a) 3

(b) The series diverges.

(c) 9

(d) 4

(e) 1

$\sum_{i=1}^{\infty} \frac{3 \cdot 3^n}{4^n} = \sum_{i=1}^{\infty} 3\left(\frac{3}{4}\right)^n$ where $a_n = 3\left(\frac{3}{4}\right)^n$

$a_1 = \frac{9}{4}$ and $r = \frac{3}{4} (< 1)$

$S = \frac{a}{1-r} = \frac{\frac{9}{4}}{1-\frac{3}{4}} = \frac{\frac{9}{4}}{\frac{1}{4}} = 9$

Common Exam II Prep. Part II

7. Which of the following series fails the test for divergence?

(a) $\sum_{n=1}^{\infty} \frac{3n^2}{5n^2+1}$

(b) $\sum_{n=1}^{\infty} \frac{\ln n}{n}$

(c) $\sum_{n=1}^{\infty} \cos\left(\frac{1}{n}\right)$

(d) $\sum_{n=1}^{\infty} \frac{1}{\arctan n}$

(e) $\sum_{n=1}^{\infty} \frac{1}{1-e^{-n}}$

Divergence Test

If $\lim_{n\to\infty} S_n < \infty$, then $\lim_{n\to\infty} a_n = 0$

Contrapositive:

$\lim_{n\to\infty} a_n \neq 0$ then $\lim_{n\to\infty} S_n = \pm\infty$

(a) $\lim_{n\to\infty} \frac{3n^2}{5n^2+1} = \frac{3}{5} \quad \Rightarrow \lim_{n\to\infty} S_n = \infty$

(b) $\lim_{n\to\infty} \frac{\ln n}{n} = 0 \quad \Rightarrow \lim_{n\to\infty} S_n = ??$

(c) $\lim_{n\to\infty} \cos\left(\frac{1}{n}\right) = 1 \quad \Rightarrow \lim_{n\to\infty} S_n = \infty$

(d) $\lim_{n\to\infty} \frac{1}{\arctan n} = \frac{1}{\pi/2} \quad \Rightarrow \lim_{n\to\infty} S_n = \infty$

(e) $\lim_{n\to\infty} \frac{1}{1-e^{-n}} = 1 \quad \Rightarrow \lim_{n\to\infty} S_n = \infty$

Common Exam II Prep. Part II

8. Given the partial fraction decomposition $\dfrac{2x^2 - x + 4}{x(x^2+4)} = \dfrac{A}{x} + \dfrac{Bx + C}{x^2 + 4}$, what are the values of A, B and C?

(a) A= 1, B= 1, C= −1.

(b) A= 4, B= −2, C= −1.

(c) A= 1, B= 1, C= 1.

(d) A= 4, B= 2, C= 1.

(e) A= 1, B= 2, C= 4.

Cover up : $A = \left[\frac{2x^2-x+4}{x^2+4}\right]_{x=0} = 1$

Multiplying $(x-2i)$

$$\left[\frac{2x^2-x+4}{x(x+2i)}\right] = \frac{A}{x}(x-2i) = \frac{Bx+C}{x+2i}$$

$x = 2i$:

$$\frac{-8-2i+4}{(2i)(4i)} = \frac{2Bi+C}{4i} \quad \Rightarrow \frac{-4-2i}{2i} = 2Bi + C$$

$$-\frac{2}{i} - 1 = 2Bi + C$$

- $C = -1$
- $-\frac{2}{i} = 2Bi \Rightarrow -2 = -2B \quad \Rightarrow B = 1$

Common Exam II Prep. Part II

9. Which of the following statements is true about a series $\sum_{n=1}^{\infty} a_n$ whose nth partial sum is given by $s_n = \frac{n}{2n+1}$?

(a) The first term of the series is $a_1 = 1/3$ and the series diverges.

(b) The first term of the series is $a_1 = 0$ and the series converges to 1.

(c) The first term of the series is $a_1 = 1/3$ and the series converges to $1/2$.

(d) The first term of the series is $a_1 = 1/3$ and the series converges to 1.

(e) The first term of the series is $a_1 = 1$ and the series diverges.

- $S_1 = a_1$
- $S_{n+1} - S_n = a_{n+1}$

$$a_1 = S_1 = \frac{1}{3}$$

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{n}{2n+1} = \frac{1}{2}$$

Common Exam II Prep. Part II

10. Which of the following integrals are improper?

(I) $\int_1^e \ln(x-1)\, dx$ (II) $\int_{-5}^0 \frac{1}{5+2x}\, dx$ (III) $\int_1^\infty \frac{2}{x^3}\, dx$

(a) Only (III) is an improper integral.
(b) Only (I) and (III) are improper integrals.
(c) Only (II) and (III) are improper integrals.
(d) Only (I) is an improper integral.
(e) (I), (II) and (III) are all improper integrals.

$\ln(x-1)$ @ $x = 1$

$\frac{1}{5+2x}$ @ $x = -\frac{5}{2}$

$\int_1^\infty$

Common Exam II Prep. Part II

11. Let $s = \sum_{n=1}^{\infty} \frac{1}{n^4}$. Using The Remainder Estimate for the Integral Test, find the smallest value of n such that $R_n = s - s_n \leq \frac{1}{81}$.

(a) $n = 2$
(b) $n = 3$
(c) $n = 4$
(d) $n = 5$
(e) $n = 6$

$$R_n \leq \int_n^\infty f(x)dx$$

$$\int_n^\infty \frac{1}{x^4}dx \leq \frac{1}{81}$$

$$\left[-\frac{1}{3x^3}\right]_n^\infty = \frac{1}{3n^3} \leq \frac{1}{3^4}$$

$$3n^3 \geq 3^4$$

$$n^3 \geq 3^3$$

$$n \geq 3$$

Common Exam II Prep. Part II

12. Evaluate the integral $\int_{1/2}^{\sqrt{3}/2} \frac{1}{\sqrt{1-x^2}}\,dx$.

(a) $\pi/3 - \pi/6$

(b) $\frac{\sqrt{3}-1}{2}$

(c) $\pi/6 - \pi/3$

(d) $\frac{\sqrt{3}+1}{2}$

(e) $\pi/3 - \pi/4$

Not an improper integral

$\sqrt{1-x^2}$: Trig substitution

Trig sub: $x = \sin\theta \;\Rightarrow dx = \cos\theta \; d\theta$

$$\int_{x=\frac{1}{2}}^{x=\frac{\sqrt{3}}{2}} \Rightarrow \int_{u=\frac{\pi}{6}}^{u=\frac{\pi}{3}}$$

$$\int_{u=\frac{\pi}{6}}^{u=\frac{\pi}{3}} \frac{1}{\cos\theta}(\cos\theta\, d\theta)$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} d\theta$$

$$= \frac{\pi}{3} - \frac{\pi}{6}$$

13. Which of the following sequences will converge?

(I) $a_n = \dfrac{\sin n}{n}$ (II) $a_n = \ln(5n+1) - \ln(3n+2)$ (III) $a_n = \dfrac{(-1)^n 5n}{n+5}$

(a) (I) and (III) only.

(b) (II) only.

(c) (I) and (II) only.

(d) (I), (II) and (III).

(e) (II) and (III) only.

(a) $\lim\limits_{n\to\infty} \dfrac{\sin n}{n} = 0$

(b) $\lim\limits_{n\to\infty} \ln\left(\dfrac{5n+1}{3n+2}\right) = \ln\dfrac{5}{3}$

(c) $\lim\limits_{n\to\infty} \dfrac{(-1)^n(5n)}{n+5} \to \pm 5$ (Diverges)

Common Exam II Prep. Part II

14. Which of the following is true for the sequence $a_n = 3 + \frac{(-1)^n n}{3n^2+1}$ for $n \geq 1$?

(a) The sequence is decreasing and the sequence converges to 3.

(b) The sequence is increasing and the sequence diverges.

(c) The sequence is neither increasing nor decreasing and the sequence diverges.

(d) The sequence is neither increasing nor decreasing and the sequence converges to 3.

(e) The sequence is increasing and the sequence converges to 3.

$\frac{(-1)^n n}{3n^2+1}$ oscillate with decreasing amplitudes

$3 + \frac{(-1)^n n}{3n^2+1}$ oscillate with decreasing amplitudes and converges to 3

Common Exam II Prep. Part II

15. What is a general formula for the sequence $a_n = \left\{\frac{1}{5}, \frac{-4}{8}, \frac{9}{11}, \frac{-16}{14}, \frac{25}{17}, \dots\right\}$, for $n \geq 1$?

(a) $a_n = \dfrac{(-1)^{n-1}n^2}{2n+3}$

(b) $a_n = \dfrac{(-1)^{n-1}2n}{n^2+4}$

(c) $a_n = \dfrac{(-1)^{n+1}n^2}{4n+1}$

(d) $a_n = \dfrac{(-1)^{n}n^2}{n+3}$

(e) $a_n = \dfrac{(-1)^{n+1}n^2}{3n+2}$

Sign: $(-1)^{n+1}$

Numerator : 1,4,9,16, 25 $\Rightarrow n^2$

Denominator: 5,8,14,17 $\Rightarrow$ arithmetic sequence w/ $a = 5$ and $d = 3$

$$5 + (n-1) \cdot 3 = 3n + 2$$

$$a_n = (-1)^{n+1} \frac{n^2}{3n+2}$$

Common Exam II Prep. Part II

16. (10 points) Does the telescoping series $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty}(e^{1/n} - e^{1/(n+1)})$ converge or diverge? If the series converges, find the sum of the series.

Telescoping series

$$a_n = e^{\frac{1}{n}} - e^{\frac{1}{n+1}}$$

$$S_n = \left(e^{\frac{1}{1}} - e^{\frac{1}{2}}\right) + \left(e^{\frac{1}{2}} - e^{\frac{1}{3}}\right) + \left(e^{\frac{1}{3}} - e^{\frac{1}{4}}\right) + \cdots + \left(e^{\frac{1}{n}} - e^{\frac{1}{n+1}}\right)$$

$$= e^{\frac{1}{1}} - e^{\frac{1}{n+1}}$$

$$\lim_{n\to\infty} S_n = e - 1$$

Common Exam II Prep. Part II

17. (10 points) Compute $\int \frac{\sqrt{9+x^2}}{x^4}\,dx$.

Your final answer MUST be presented in x, without any inverse trigonometric function(s).

$\sqrt{3^2+x^2} \Rightarrow$ Trig-sub $x = 3\tan\theta$; $dx = 3\sec^2\theta\, d\theta$

$$\sqrt{3^2+3^2\tan^2\theta} = \sqrt{3^2\sec^2\theta} = 3\sec\theta$$

$$\int \frac{\sqrt{9+x^2}}{x^4}\,dx = \int \frac{3\sec\theta}{3^4\tan^4\theta}(3\sec^2\theta\, d\theta)$$

$$= \frac{1}{3^2}\int \frac{1}{\tan^4\theta}\cdot \sec^3\theta\, d\theta$$

$$= \frac{1}{3^2}\int \frac{\cos^4\theta}{\sin^4\theta}\cdot \frac{1}{\cos^3\theta}\, d\theta$$

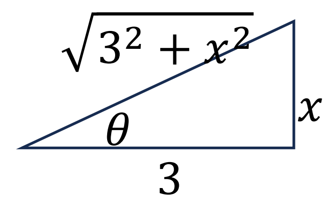
$$= \frac{1}{3^2}\int \frac{\cos\theta}{\sin^4}\, d\theta$$

u-sub: $u = \sin\theta \Rightarrow du = \cos\theta d\theta$

$$= \frac{1}{3^2}\int \frac{du}{u^4} = \frac{1}{3^2}\left[-\frac{1}{3u^3}\right] + C = -\frac{1}{3^3u^3} + C$$

$$= -\frac{1}{(3\sin\theta)^3} + C$$

$$= -\frac{1}{\left(\frac{3x}{\sqrt{3^2+x^2}}\right)^3} + C$$



Common Exam II Prep. Part II

19. (10 points) Use the Integral Test to show that the series $\sum_{n=1}^{\infty} \frac{3e^{1/n}}{n^2}$ converges or prove that it diverges.

Your answer MUST be presented as a complete coherent sentence with justification.

(is positive, continuous, and decreasing)

$$\int_1^\infty \frac{3e^{1/x}}{x^2}\,dx = \lim_{N\to\infty} \int_1^N \frac{3e^{1/x}}{x^2}\,dx = \lim_{N\to\infty} \int_1^N 3e^{\frac{1}{x}} \frac{dx}{x^2}$$

u −sub: $u = \frac{1}{x} \Rightarrow du = -\frac{dx}{x^2}$

$$\int_{x=1}^{x=N} \Rightarrow \int_{u=1}^{1/N}$$

$$= \lim_{N\to\infty} \int_1^{1/N} 3e^u(-du)$$

$$= \lim_{N\to\infty} [-3e^u]_1^{1/N}$$

$$= \lim_{N\to\infty} \left(-3e^{1/N} + 3e^1\right)$$

$$= 3e - 3$$

Common Exam II Prep. Part II

18. (10 points) Evaluate the integral $\int \frac{4x^2-5x+11}{(x-1)(x^2+4)}\,dx$.

Partial fraction (Approach 1 : Standard way)

$$\frac{4x^2-5x+11}{(x-1)(x^2+4)} = \frac{A}{x-1} + \frac{Bx+C}{(x^2+4)}$$

- Multiply $(x-1)(x^2+4)$ on both sides : $4x^2-5x+11 = A(x^2+4) + (Bx+C)(x-1)$
- Let $x=1$: $4 = 2A \Rightarrow A = 2$
  - $4x^2-5x+11 = (2+B)x^2 + (C-B)x + 8 - C$
  - $\begin{cases} 4 = 2+B \\ -5 = C-B \\ 11 = 8-C \end{cases} \Rightarrow B = 2, C = -3$

$$\int \left(\frac{2}{x-1} + \frac{2x-3}{(x^2+4)}\right) dx$$

$$= \int \left(\frac{2}{x-1} + \frac{2x}{(x^2+4)} - \frac{3}{(x^2+4)}\right) dx$$

- $\int \frac{2}{x-1}\,dx = 2\ln|x-1| + C$
- $\int \frac{2x}{(x^2+4)}\,dx = \ln(x^2+4) + C$
- $-3\int \frac{1}{x^2+2^2}\,dx = -\frac{3}{2}\operatorname{atan}\left(\frac{x}{2}\right) + C$

$$= 2\ln|x-1| + \ln(x^2+4) - \frac{3}{2}\operatorname{atan}\left(\frac{x}{2}\right) + C$$

Common Exam II Prep. Part II

18. (10 points) Evaluate the integral $\int \frac{4x^2-5x+11}{(x-1)(x^2+4)}\,dx$.

Partial fraction (Approach 2 : Using derivatives)

$$\frac{4x^2-5x+11}{(x-1)(x^2+4)} = \frac{A}{x-1} + \frac{Bx+C}{(x^2+4)}$$

- Multiply $(x-1)(x^2+4)$ on both sides : $4x^2-5x+11 = A(x^2+4) + (Bx+C)(x-1)$
- Let $x=1: 4 = 2A \Rightarrow A = 2$
- By differentiating : $8x-5 = 4x + Bx - B + Bx + C$
- $\begin{cases} 8 = 4 + 2B \\ -5 = -B + C \end{cases} \Rightarrow B = 2,\ C = -3$

$$\int \left(\frac{2}{x-1} + \frac{2x-3}{(x^2+4)}\right) dx$$

$$= \int \left(\frac{2}{x-1} + \frac{2x}{(x^2+4)} - \frac{3}{(x^2+4)}\right) dx$$

- $\int \frac{2}{x-1}\,dx = 2\ln|x-1| + C$
- $\int \frac{2x}{(x^2+4)}\,dx = \ln(x^2+4) + C$
- $-3\int \frac{1}{x^2+2^2}\,dx = -\frac{3}{2}\operatorname{atan}\left(\frac{x}{2}\right) + C$

$$= 2\ln|x-1| + \ln(x^2+4) - \frac{3}{2}\operatorname{atan}\left(\frac{x}{2}\right) + C$$

Common Exam II Prep. Part II

18. (10 points) Evaluate the integral $\int \frac{4x^2-5x+11}{(x-1)(x^2+4)}\,dx$.

Partial fraction (Approach 3 : Using complex roots)

$$\frac{4x^2-5x+11}{(x-1)(x^2+4)} = \frac{A}{x-1} + \frac{Bx+C}{(x^2+4)}$$

- Multiply $(x-1)$ on both sides : $\frac{4x^2-5x+11}{(x^2+4)} = A + \frac{Bx+C}{(x^2+4)}(x-1)$
  - Let $x = 1$: $\frac{10}{5} = A \Rightarrow A = 2$
- Multiply (x^2+4) on both sides : $\frac{4x^2-5x+11}{(x-1)} = \frac{A}{x-1}(x^2+4) + Bx + C$
  - Let $x = 2i$: $\frac{-16-10i+11}{2i-1} = 2Bi + C$
  - Multiply $(2i-1)$ on both sides: $-5-10i = -4B + (2C-2B)i - C$
  - $\begin{cases} 4B + C = 5 \\ B - C = 5 \end{cases} \Rightarrow B = 2,\ C = -3$

$$\int \left(\frac{2}{x-1} + \frac{2x-3}{(x^2+4)}\right) dx$$

$$= \int \left(\frac{2}{x-1} + \frac{2x}{(x^2+4)} - \frac{3}{(x^2+4)}\right) dx$$

$$= 2\ln|x-1| + \ln(x^2+4) - \frac{3}{2}\operatorname{atan}\left(\frac{x}{2}\right) + C$$

- $\int \frac{2}{x-1}dx = 2\ln|x-1| + C$
- $\int \frac{2x}{(x^2+4)}dx = \ln(x^2+4) + C$
- $-3\int \frac{1}{x^2+2^2}dx = -\frac{3}{2}\operatorname{atan}\left(\frac{x}{2}\right) + C$

Common Exam II Prep. Part II

18. (10 points) Evaluate the integral $\int \frac{4x^2-5x+11}{(x-1)(x^2+4)}\,dx$.

Partial fraction (Approach 4 : Fractional manipulation – most reliable)

$$\frac{4x^2-5x+11}{(x-1)(x^2+4)} = \frac{A}{x-1} + \frac{Bx+C}{(x^2+4)}$$

- $A = \left[\frac{4x^2-5x+11}{(x^2+4)}\right]_{x=1} = \frac{10}{5} = 2$
- $\frac{4x^2-5x+11}{(x-1)(x^2+4)} = \frac{2}{x-1} + \frac{Bx+C}{(x^2+4)}$

$$\frac{4x^2-5x+11}{(x-1)(x^2+4)} - \frac{2}{x-1} = \frac{Bx+C}{(x^2+4)}$$

$$\frac{Bx+C}{(x^2+4)} = \frac{4x^2-5x+11}{(x-1)(x^2+4)} - \frac{2(x^2+4)}{(x-1)(x^2+4)}$$

$$= \frac{2x^2-5x+3}{(x-1)(x^2+4)}$$

$$= \frac{(x-1)(2x-3)}{(x-1)(x^2+4)}$$

$$= \frac{(2x-3)}{(x^2+4)}$$

$B = 2, C = -3$

$$\int \left(\frac{2}{x-1} + \frac{2x-3}{(x^2+4)}\right) dx$$

$$= \int \left(\frac{2}{x-1} + \frac{2x}{(x^2+4)} - \frac{3}{(x^2+4)}\right) dx$$

- $\int \frac{2}{x-1}\,dx = 2\ln|x-1| + C$
- $\int \frac{2x}{(x^2+4)}\,dx = \ln(x^2+4) + C$
- $-3\int \frac{1}{x^2+2^2}\,dx = -\frac{3}{2}\operatorname{atan}\left(\frac{x}{2}\right) + C$

$$= 2\ln|x-1| + \ln(x^2+4) - \frac{3}{2}\operatorname{atan}\left(\frac{x}{2}\right) + C$$

Common Exam II Prep. Part II

19. (10 points) Use the Integral Test to show that the series $\sum_{n=1}^{\infty} \frac{3e^{1/n}}{n^2}$ converges or prove that it diverges.

Your answer MUST be presented as a complete coherent sentence with justification.

$\frac{3e^{1/x}}{x^2} \geq 0$, continuous, and decreasing $\Rightarrow$ May use the integral test

$$\int_1^{\infty} \frac{3e^{1/x}}{x^2} dx = \lim_{N\to\infty} \int_1^N \frac{3e^{1/x}}{x^2} dx = \lim_{N\to\infty} \int_1^N 3e^{\frac{1}{x}} \frac{dx}{x^2}$$

u −sub: $u = \frac{1}{x} \Rightarrow du = -\frac{dx}{x^2}$

$$\int_{x=1}^{x=N} \Rightarrow \int_{u=1}^{1/N}$$

$$= \lim_{N\to\infty} \int_1^{1/N} 3e^u(-du)$$

$$= \lim_{N\to\infty} [-3e^u]_1^{1/N}$$

$$= \lim_{N\to\infty} \left(-3e^{1/N} + 3e^1\right)$$

$$= 3e - 3 < \infty$$

∴ The series converges by the Integral Test.