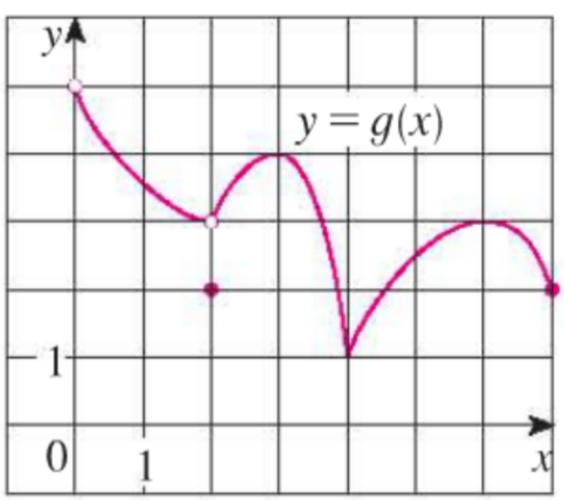
Problem 1. Find two positive numbers such that the sum of the first and thrice the second number is 60 and the product of the numbers is a maxima.

Problem 2. The sum of 2 positive numbers is 16. What is the smallest possible value of the sum of their squares? Show that this value is a minimum by using second derivative test.

Problem 3. Find the dimensions of a rectangle which has an area of 225 square centimeters and the smallest possible perimeter. What is the perimeter of this rectangle?

Problem 4. What is the absolute maxima and the absolute minima of the function given below? Are there any local extremas?



Problem 5. Find the absolute maxima and the absolute minima for $f(x) = x^4 - 18x^2 + 32$ on the interval $[-4, 4]$.

Problem 6. You would like to make an open rectangular box with a square base from 48 m^2 of material. Find the dimensions of the box that will result in it's largest possible volume.

Problem 7. A container, in the shape of a right circular cylinder with no top, has surface area of 3 m^2. Find the height h and base radius r that will maximize the volume of this cylinder. The volume of a cylinder is $\pi r^2 h$.

Problem 8. Find the absolute maxima and the absolute minima of the function

$f(x) = \frac{1}{3}x^3 + 2x^2 - 21x + 7$ on the interval $[0, 6]$.

Problem 9. A rectangular garden needs to be fenced. There is \$320 available for this project. Three sides of the fence will be constructed with wire fencing at the cost of \$2 per foot. The fourth side will be constructed with wood fencing at a cost of \$6 per foot. Find the length of the sides as well as the area of the largest garden that can be fenced in this way.

Problem 10. If a function $f(x)$ is continuous on an interval $[a, b]$, discuss whether the following statements are true of false.

(1) $f(x)$ must have a local maxima and a local minima on $[a, b]$.

(2) $f(x)$ must have an absolute maxima and an absolute minima on $[a, b]$.

(3) If $x = c$ is the only critical point on the interval $[a, b]$, and $f(c)$ is a local minima, then $f(x)$ has an absolute minima at $x = c$.

(4) The point $x = b$ can be a local maxima as well as an absolute maxima of the function $f(x)$.

Problem 11. Does the function $f(x) = \dfrac{1}{x}$ have an absolute maxima or an absolute minima on the interval $(-\infty, \infty)$? What about any local extema?

Problem 12. Find the absolute extremas for $f(x) = -\dfrac{x+4}{x-4}$ on the interval $[0, 3]$.

Problem 13. Find the absolute extremas for $f(x) = \dfrac{\ln x}{x}$ on the interval $[1, 4]$.