Math 251/221. WEEK in REVIEW 2. Fall 2024

1. Let $\mathbf{v} = 5\mathbf{j}$ and let $\mathbf{u}$ be a vector with length 3 that starts at the origin and rotates in the xy-plane. Find the maximum and the minimum values of the length of the vector $\mathbf{u} \times \mathbf{v}$. In what direction does $\mathbf{u} \times \mathbf{v}$ point?

2. Find two **unit** vectors orthogonal to both $< 3, 2, 1 >$ and $< -1, 1, 0 >$.

3. Find the area of the parallelogram with the vertices $A(1, 0, 2)$, $B(3, 3, 3)$, $C(7, 5, 8)$, $D(5, 2, 7)$.

4. Use the scalar triple product to determine whether the points $P(1, 0, 1)$, $(Q(2, 4, 6))$, $R(3, -1, 2)$, and $S(6, 2, 8)$ lie in the same plane.

5. Find the volume of the parallelepiped determined by the vectors $\mathbf{a} = 2\mathbf{i}+3\mathbf{j}-2\mathbf{k}$, $\mathbf{b} = \mathbf{i}-\mathbf{j}$, and $\mathbf{c} = 2\mathbf{i}+3\mathbf{k}$.

6. Find a **vector** equation and **parametric** equations for the line passing through the point $P(-3, 2, 1)$ and parallel to the vector $\mathbf{v} = -2\mathbf{i} + \mathbf{j} - 3\mathbf{k}$.

7. Find **parametric** and **symmetric** equations for the line through the points $(-8, 1, 4)$ and $(3, -2, 4)$.

   Find the points in which the line intersects the coordinate planes.

8. Find symmetric equations for the line that passes through the point $(0, 2, -1)$ and is parallel to the line with parametric equations $x = -1 + 2t$, $y = 6 - 3t$, and $z = 5 + 9t$.

9. Determine whether the lines L_1 and L_2 are parallel, skew (do not intersect and are not parallel), or intersecting. If they intersect, find the point of intersection.

   (a) $L_1 : \ x = 2 - 6t, y = 9t, z = 1 - 3t,$ $\quad L_2 : \ x = 1 + 2s, y = 4 - 3s, z = s.$

   (b) $L_1 : \ x = 3 + 2t, y = 4 - t, z = 1 + 3t,$ $\quad L_2 : \ x = 1 + 4s, y = 3 - 2s, z = 4 + 5s.$

   (c) $L_1 : \ \dfrac{x-2}{1} = \dfrac{y-3}{-2} = \dfrac{z-1}{-3},$ $\quad L_2 : \ \dfrac{x-3}{1} = \dfrac{y+4}{3} = \dfrac{z-2}{-7}$

10. Find an equation of the plane through the point $(3, 5, 3)$ with the normal vector $\mathbf{n} =< 2, 1, -1 >$. Find the intercepts and sketch the plane.

11. Find an equation of the plane through the point $(2, 0, 1)$ and perpendicular to the line $x = 3t$, $y = 2 - t$, $z = 3 + 4t$.

12. Find an equation of the plane through the point $(1, -1, -1)$ and parallel to the plane $5x - y - z = 6$.

13. Find an equation of the plane that contains the line $x = 1 + t$, $y = 2 - t$, $z = 4 - 3t$ and is parallel to the plane $5x + 2y + z = 1$.

14. Find an equation of the plane passing through the points $(-1, 1, -1)$, $(1, -1, 0)$, $(1, 0, 1)$.

15. Find an equation of the plane that passes through the point $(3, 5, -1)$ and contains the line $x = 4 - t$, $y = 2t - 1$, $z = -3t$.

16. Find the point at which the line $x = t - 1$, $y = 1 + 2t$, $z = 3 - t$ intersects the plane $2x - y + 2z = 5$.

17. (a) Find the angle between the planes $x - 2y + z = 1$ and $2x + y + z = 1$.

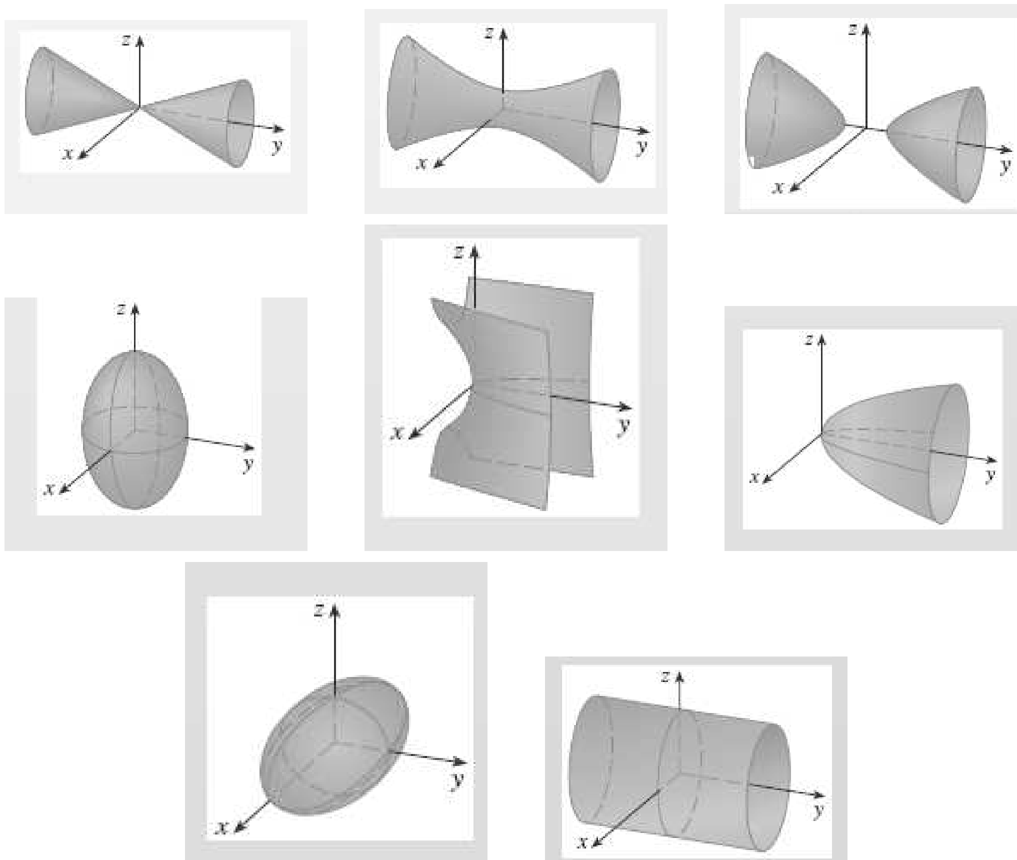
    (b) Find symmetric equation for the line of intersection of the planes.

18. Find the distance from the point $(1, -2, 4)$ to the plane $4x - 6y + 2z = 3$.

19. Find the distance between the parallel planes $2x - 3y + z = 4$ and $4x - 6y + 2z = 3$.

20. Match the equation with its graph. Give reasons for your choice.

a) $x^2 + 4y^2 + 9z^2 = 1$
b) $x^2 - y^2 + z^2 = 1$
c) $y = 2x^2 + z^2$
d) $x^2 + 2z^2 = 1$
e) $9x^2 + 4y^2 + z^2 = 1$
f) $-x^2 + y^2 - z^2 = 1$
g) $y^2 = x^2 + 2z^2$
h) $y = x^2 - z^2$



21. Use traces to sketch and identify the surface:

(a) $4x^2 + 9y^2 + 9z^2 = 36$
(b) $9y^2 + 4z^2 = x^2 + 36$
(c) $-4x^2 + y^2 - 4z^2 = 4$
(d) $3x^2 - y^2 + 3z^2 = 0$
(e) $y = z^2 + x^2$
(f) $z = x^2 - y^2$
(g) $x^2 + z^2 = 1$
(h) $z = y^2$
(i) $xy = 1$

22. Reduce the equation to the standard form, classify the surface, and sketch it.

(a) $z = (x - 1)^2 + (y + 5)^2 + 7$
(b) $4x^2 - y^2 + (z - 4)^2 = 20$
(c) $x^2 + y^2 + z + 6x - 2y + 10 = 0$