Math 308: Week-in-Review 12

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Review for the Final Exam - Part 1

1. (Chapter 2) Find the solution to the initial value problem

   (a) $2\sqrt{x}\dfrac{dy}{dx} = \cos^2 y, \qquad y(4) = \dfrac{\pi}{4}$

(b) $\dfrac{dy}{dt} + \dfrac{2y}{t} = \dfrac{\cos t}{t^2}, \qquad y(1) = \dfrac{1}{2}, \qquad t > 0.$

(c) $y' + 2y = e^{-2t}, \qquad y(0) = 1.$

2. (Chapter 1, 2) A tank contains 100 gal of brine in which 50 lbs of salt are dissolved. Brine containing 2 lbs of salt per galon flows into the tank at a rate of 6 gal per min. The mixture, which is kept uniform by stirring, flows out of the tank at the rate of 4 gal per min.

   (a) Find the amount of salt in the tank at the end of t minutes.

   (b) After 50 mins, how much salt will be in the tank, and what will be the volume of brine?

3. (Chapter 2) Given the differential equation

$$\frac{dy}{dt} = y^3 - 2y^2 + y$$

   (a) Find the equilibrium solutions

   (b) Graph the phase line. Classify each equilibrium solution as either stable, unstable, or semi-stable.

   (c) Sketch the graph of some solutions.

   (d) If $y(t)$ is the solution of the equation satisfying the initial condition $y(0) = y_0$, where $-\infty < y_0 < \infty$, find the limit of $y(t)$ when t increases.

4. (Chapter 2) Find an integrating factor for the equation

$$(3xy + y^2)dx + (x^2 + xy)dy = 0$$

and then solve the equation.

5. (Chapter 3) Find the general solution of the equation $y'' + 2y' + y = 4e^{-t}$.

6. (Chapter 3) Find the form of a particular solution for each of the following nonhomogeneous equations. Do **not** solve the equation.

   (a) $y'' + 2y' + 2y = e^{-t}\sin t + e^{-t}\cos 2t$

   (b) $y'' - 2y' + y = te^{t} + t^{2}e^{-t} + e^{t}\cos t + t^{2}$

7. (Chapter 3) Find the general solution of the equation

$$y'' + 6y' + 9y = \frac{e^{-3x}}{1+2x}$$