Week in Review
Math 152

Week 03

Volumes by Slicing: Disks and Washers

Volume by Cylindrical Shells

General slicing method for <u>any</u> volume

The solid whose base is the region bounded by semicircle $y = \sqrt{1-x^2}$ and the x −axis. And whose cross section through the solid perpendicular to the x axis are squares. Find the volume of the solid.

Step 1: Plot the graph

Step2. Find the size of a slice at x or y

- At x: Thickness $dx \Rightarrow$ Cross section $A(x)$
- At y: Thickness $dy \Rightarrow$ Cross section $A(y)$

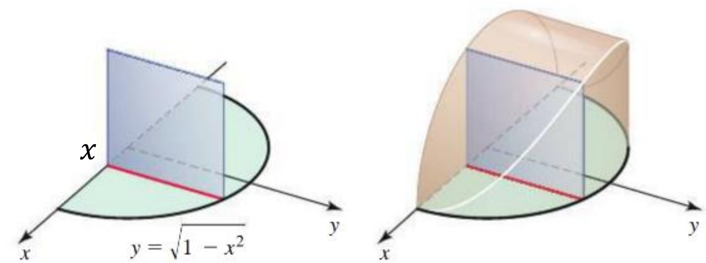
Step3. Find the volumnof a slice at x or y

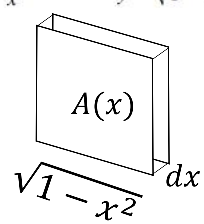
- At x: Thickness $dx \Rightarrow$ Cross section $A(x)$
  - $dV = A(x)dx$
- At y: Thickness $dy \Rightarrow$ Cross section $A(y)$
  - $dV = A(y)dy$

Step4. Find the upper/lower limits for x or y

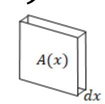
Step5. Set up integral and evaluate

$V = \int_a^b A(x)dx$ or $V = \int_c^d A(y)dy$





$A(x) = \left[\sqrt{1-x^2}\right]^2$

$V($  $) = A(x)dx$

$= (1-x^2)dx$

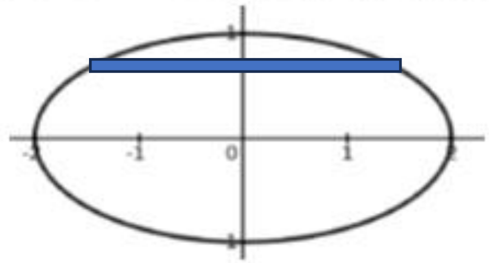
- Limit for x : $[-1,1]$
- $V = \int_a^b A(x)dx$

$= \int_{-1}^{1}(1-x^2)dx = 2\int_0^1(1-x^2)dx$

$= 2\left[x - \frac{x^3}{3}\right]_0^1 = \frac{4}{6}$

Example

Find the volume of the solid whose base is the ellipse $x^2 + 4y^2 = 4$ and whose cross-sections perpendicular to the $y-$axis are squares. Evaluate your integral.

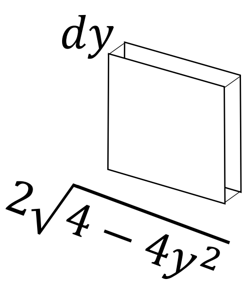


$$V = \int_{-1}^{1} 4(4 - 4y^2)dy$$
$$= 2\int_{0}^{1} 4(4 - 4y^2)dy$$
$$= 32\int_{0}^{1} (1 - y^2)dy$$
$$= 32\left[y - \frac{1}{3}y^3\right]_0^1$$
$$= 32\left[1 - \frac{1}{3}\right]$$
$$= 32 \cdot \frac{2}{3} = \frac{64}{3}$$

$$x = \pm\sqrt{4 - 4y^2}$$

dy

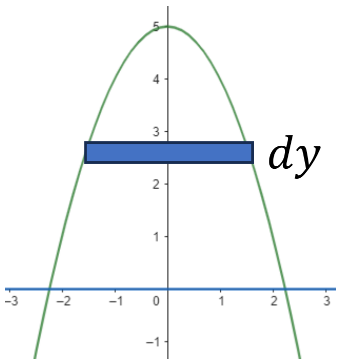
$2\sqrt{4 - 4y^2}$



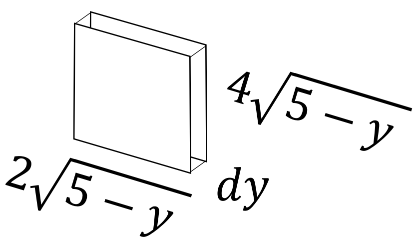
$$dV = \left[2\sqrt{4 - 4y^2}\right]^2 dy$$
$$= 4(4 - y^2)dy$$

Example

The base of a solid is the region bounded by the curve $y = 5 - x^2$ and the x-axis. Cross-Sections perpendicular to the y-axis are rectangles with height equal to twice the base. Find the volume of this solid.



$$x = \pm\sqrt{5-y}$$

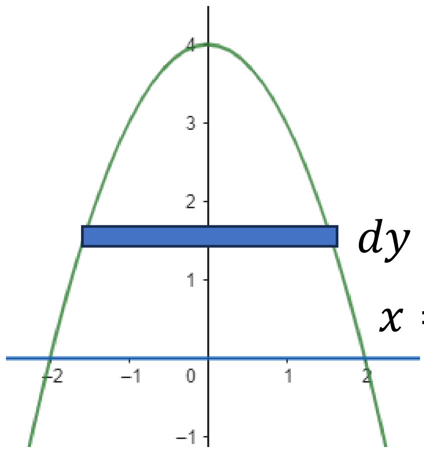


$$dV = \left(2\sqrt{5-y}\right)^2 dy$$

$$\begin{aligned} V &= \int_0^5 2\left(2\sqrt{5-y}\right)^2 dy \\ &= 8\int_0^5 (5-y)\,dy \\ &= 8\left[5y - \tfrac{1}{2}y^2\right]_0^5 \\ &= 8\left[25 - \tfrac{25}{2}\right] \\ &= 100 \end{aligned}$$

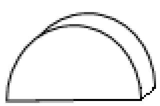
Example

Consider the solid S whose base is the region bounded by $y = 4 - x^2$ and $y = 0$. Cross sections perpendicular to the y - axis are semicircles. Find the volume of S.



dy

$x = \pm\sqrt{4-y}$



dy

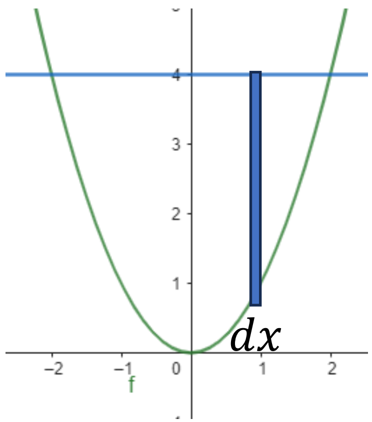
$2\sqrt{4-y}$

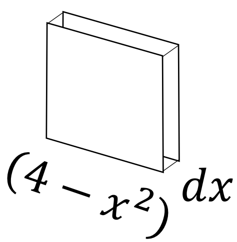
$$dV = \frac{\pi}{2}\left(\sqrt{4-y}\right)^2 dy$$

$$\begin{aligned} V &= \frac{1}{2}\int_0^4 \pi\left(\sqrt{4-y}\right)^2 dy \\ &= \frac{\pi}{2}\int_0^4 (4-y)\,dy \\ &= \frac{\pi}{2}\left[4y - \frac{1}{2}y^2\right]_0^4 \\ &= \frac{\pi}{2}(16-8) = 4\pi \end{aligned}$$

Example

Consider the solid S described here. The base of S is the region bounded by $y = x^2$ and $y = 4$. Cross sections perpendicular to the x - axis are squares. Find the volume of S.





$$dV = (4 - x^2)^2 dx$$

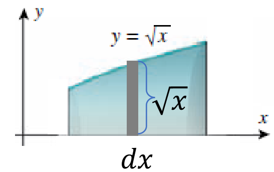
$$
\begin{aligned}
V &= \int_{-2}^{2} (4 - x^2)^2 dx \\
&= 2\int_0^2 (4 - x^2)^2 dx \\
&= 2\int_0^2 (16 - 8x^2 + x^4) dx \\
&= 2\left[16x - \frac{8}{3}x^3 + \frac{1}{5}x^5\right]_0^2 \\
&= 2\left[2^5 - \frac{2^6}{3} + \frac{2^5}{5}\right] \\
&= 2^6\left[1 - \frac{2}{3} + \frac{1}{5}\right] \\
&= 2^6\left[\frac{8}{15}\right] = \frac{2^9}{15} = \frac{512}{15}
\end{aligned}
$$

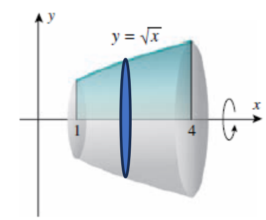
Volume by disks perpendicular to the x axis

Find the volume of the solid that is obtained when the region under the curve $y = \sqrt{x}$ over the interval [1, 4] is revolved about the *x*-axis

Volume of solid of revolution around x axis

Step 1: Plot the graph

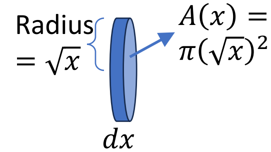




Step2. Find the size of a <u>perpendicular</u> slice at x

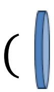
At x: Thickness $dx \Rightarrow$ Cross section $A(x)$

$$\pi[f(x)]^2$$



Step3. Find the volume of a slice at x

- At x: Thickness $dx \Rightarrow$ Cross section $A(x)$
  - $dV = \pi[f(x)]^2 dx$

$V($  $) = \pi[f(x)]^2 dx$

Step4. Find the upper/lower limits for x

$1 \le x \le 4$ (limits)

Step5. Set up integral and evaluate

$$V = \int_a^b A(x)dx = \int_a^b \pi[f(x)]^2 dx$$

$$\begin{aligned} V &= \int_a^b \pi[f(x)]^2 dx \\ &= \int_1^4 \pi x dx \\ &= \frac{\pi}{2}[x^2]_1^4 \\ &= \frac{\pi}{2}[16-1] = \frac{15}{2}\pi \end{aligned}$$

Volume by Washer perpendicular to the x axis

Volume of solid of revolution around x axis

Step 1: Plot the graph of $f(x)$, $g(x)$ w/ $f > g$

Step2. Find the size of a perpendicular slice at x

- At x: Thickness $dx \Rightarrow$ Cross section $A(x)$
- Washer = Large disc – small disc
  $= \pi[f(x)]^2 - \pi[g(x)]^2$

Step3. Find the volume of a slice at x

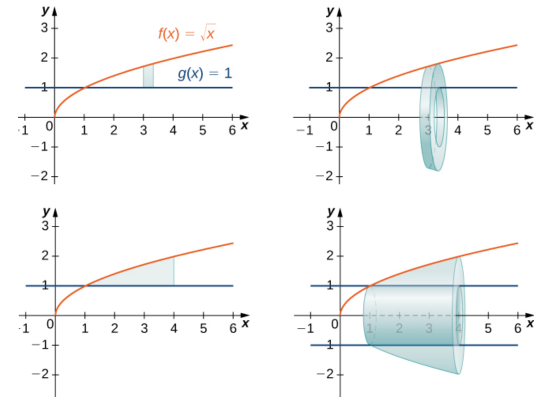
- At x: Thickness $dx \Rightarrow$ Cross section $A(x)$
  - $dV = \pi([f(x)]^2 - [g(x)]^2)dx$

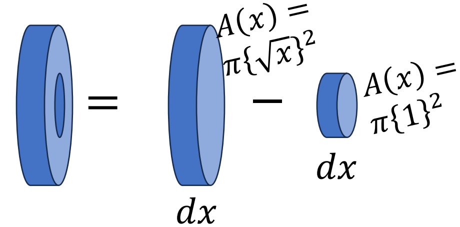
Step4. Find the upper/lower limits for x

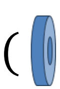
Step5. Set up integral and evaluate

$$V = \int_a^b A(x)dx$$
$$= \int_a^b \pi([f(x)]^2 - [g(x)]^2)dx$$

Find the volume of the solid that is obtained when the region between the curve $y = \sqrt{x}$ *and* $y = 1$ over the interval [1, 4] is revolved about the *x*-axis





$V($  $) = \pi\{\sqrt{x}\}^2 dx - \pi\{1\}^1 dx = \pi[x - 1]dx$

Limit = [1,4]

$$V = \int_1^4 \pi(x - 1)dx$$
$$= \pi\left[\frac{1}{2}x^2 - x\right]_1^4 = \frac{9}{2}\pi$$

Example

The region bounded by $y = \cos x$ and the x-axis on the interval $\left[0, \frac{\pi}{2}\right]$ is rotated about the x-axis. Find the volume of the resulting solid.

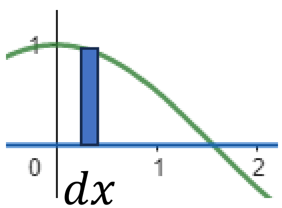
(a) 1

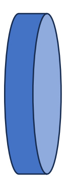
(b) $\frac{\pi^2}{2}$

(c) $\frac{\pi}{2}$

(d) $\frac{\pi}{4}$

(e) $\frac{\pi^2}{4}$ ← correct





$r = \cos x$

dx

$dV = \pi(\cos x)^2 dx$

$$\int_0^{\frac{\pi}{2}} \pi \cos^2 x \, dx$$

$$= \int_0^{\frac{\pi}{2}} \pi \left[\frac{\cos 2x + 1}{2}\right] dx$$

$$= \frac{\pi}{2}\left[\frac{1}{2}\sin 2x + x\right]_0^{\pi/2}$$

$$= \frac{\pi}{2}\left[\frac{1}{2}(0 - 0) + \left(\frac{\pi}{2} - 0\right)\right]$$

$$= \left(\frac{\pi}{2}\right)^2$$

Example

The region bounded by $y = e^x$ and the x-axis on the interval $[0, 2]$ is rotated about the x-axis. Find the volume of the resulting solid.

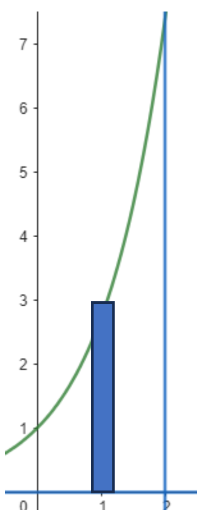
(a) $\frac{\pi e^4}{2}$

(b) $\frac{\pi e^2}{2}$

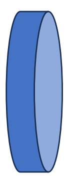
(c) $\frac{\pi}{2}(e^4 - 1)$ ← correct

(d) $\frac{\pi}{2}(e^2 - 1)$

(e) $2\pi(e^4 - 1)$



dx



dx

$r = e^x$

$dV = \pi e^{2x} dx$

$$\int_0^2 \pi e^{2x} dx$$
$$= \frac{\pi}{2}[e^{2x}]_0^2$$
$$= \frac{\pi}{2}(e^4 - 1)$$

Example

The region bounded by the curves $y = x^2$ and $y = 1$ is rotated about the line $y = 1$. Find the volume of the resulting solid.

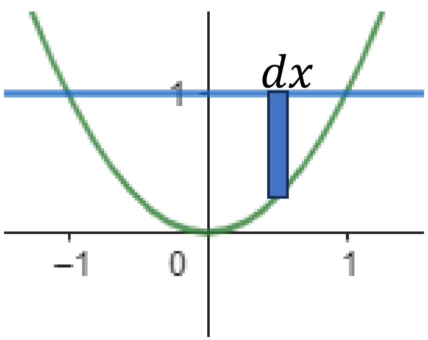
(a) $\frac{8\pi}{15}$

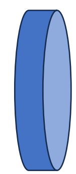
(b) $\frac{8\pi}{5}$

(c) $\frac{4\pi}{3}$

(d) $\frac{12\pi}{5}$

(e) $\frac{16\pi}{15}$





$r = (1 - x^2)$

dx

$dV = \pi(1 - x)^2 dx$

Limits: $x^2 = 1 \Rightarrow x = \pm 1$

$$V = \int_{-1}^{1} \pi(1 - x^2)^2 dx$$
$$= 2\pi \int_0^1 (1 - 2x^2 + x^4)\, dx$$
$$= 2\pi \left[x - \frac{2}{3}x^3 + \frac{1}{5}x^5\right]_0^1$$
$$= 2\pi \left[1 - \frac{2}{3} + \frac{1}{5}\right] = 2\pi \left[\frac{1}{3} + \frac{1}{5}\right]$$
$$= \frac{16\pi}{15}$$

Example

If we revolve the region bounded by $y = 1 - x^2$ and $x - y = 1$ about the line $y = 3$, which of the following integrals gives the resulting volume?

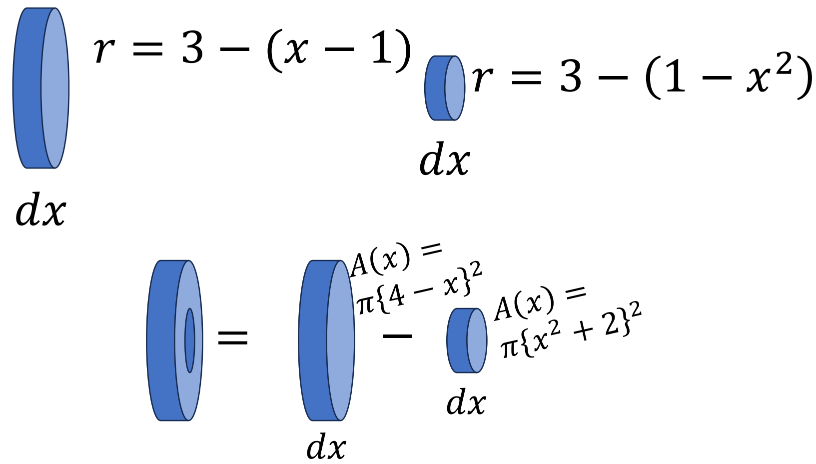
(a) $\int_{-1}^{2} 2\pi(3-x)(x^2-x+2)\,dx$

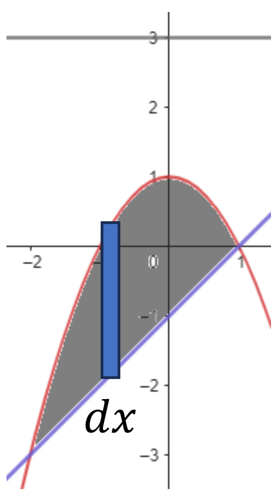
(b) $\int_{-2}^{1} \pi\left((2+x^2)^2-(4-x)^2\right)\,dx$

(c) $\int_{-1}^{2} 2\pi(x-3)(x^2-x+2)\,dx$

(d) $\int_{-2}^{1} \pi\left((4-x)^2-(2+x^2)^2\right)\,dx$

(e) $\int_{-1}^{2} \pi\left((2+x^2)^2-(4-x)^2\right)\,dx$





$$dV = \pi[(4-x)^2-(x^2+2)^2]dx$$

Limits: $1 - x^2 = x - 1$

$x^2 + x - 2 = 0;\ (x-1)(x+2) = 0$

$x = -2, 1$

$$V = \int_{-2}^{1} \pi[(4-x)^2-(x^2+2)^2]dx$$

Example

Consider the region R bounded by $y = \sqrt{x}$, $y = 1$, $x = 0$. Find the volume obtained by rotating the region R about the line $y = 1$.

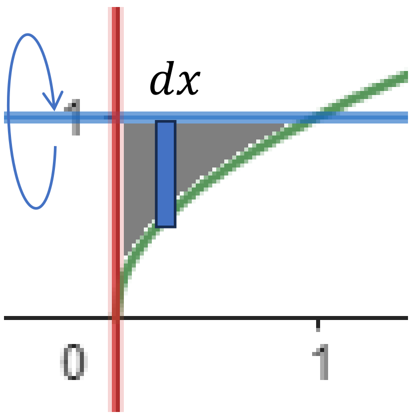
(a) $\frac{\pi}{6}$

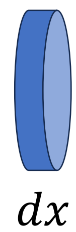
(b) $\frac{\pi}{2}$

(c) $\frac{7\pi}{6}$

(d) $\frac{\pi}{3}$

(e) $\frac{5\pi}{6}$





$$r = (1 - \sqrt{x})$$

$$dV = \pi(1 - \sqrt{x})^2 dx$$

Limits: $\sqrt{x} = 1 \Rightarrow x = 1$

$$V = \int_0^1 \pi(1 - 2\sqrt{x} + x)dx$$

$$= \pi\left[x - \frac{4}{3}x^{\frac{3}{2}} + \frac{x^2}{2}\right]_0^1$$

$$= \pi \cdot \left[1 - \frac{4}{3} + \frac{1}{2}\right]$$

$$= \frac{\pi}{6}$$

Example

Which of the following integrals gives the volume of the solid obtained by rotating the region bounded by $y = 5 - x^2$ and $y = 1$ about the x-axis.

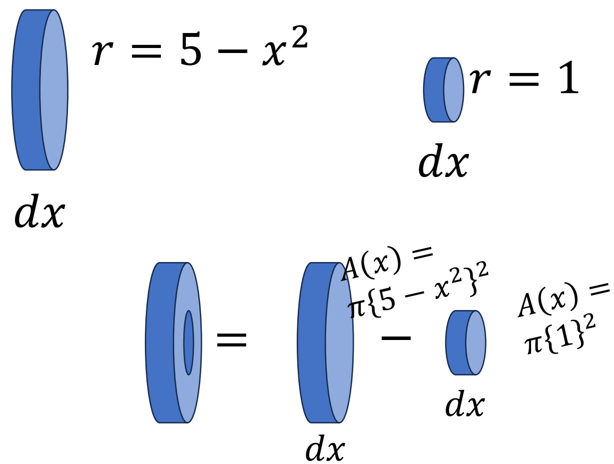
(a) $\pi \int_{-2}^{2} \left(1 - (5 - x^2)^2\right) dx$

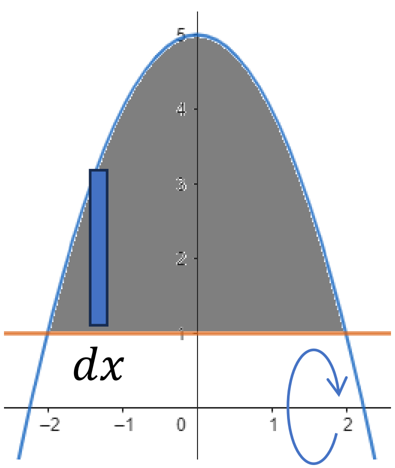
(b) $\pi \int_{-2}^{2} (4 - x^2)^2 \, dx$

(c) $2\pi \int_{-2}^{2} x(4 - x^2) \, dx$

(d) $\pi \int_{-2}^{2} \left((5 - x^2)^2 - 1\right) dx$ ← correct

(e) $2\pi \int_{-2}^{2} x(x^2 - 4) \, dx$





$$dV = \pi[(5 - x^2)^2 - 1]dx$$

Limits: $5 - x^2 = 1$

$x^2 = 4; \quad x = \pm 2$

$$V = \int_{-2}^{2} \pi[(5 - x^2)^2 - 1]dx$$

Volume by disks perpendicular to the y axis

Find the volume of the solid that is obtained when the region between the curve $y = x^3$ and $x = 0$ between the interval $0 \le y \le 8$ is revolved about the y-axis

Volume of solid of revolution around y axis

Step 1: Plot the graph

Step2. Find the size of a perpendicular slice at y

- At y: Thickness $dy \Rightarrow$ Cross section $A(y)$

  $\pi[g(y)]^2$

- For $y = f(x)$, solve for $x = f^{-1}(y)$

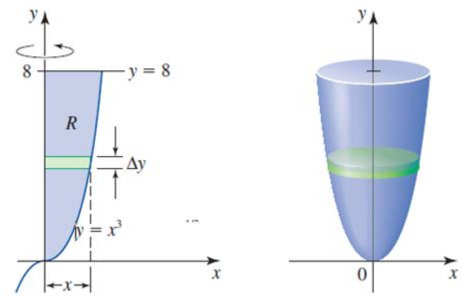
Step3. Find the volume of a slice at y

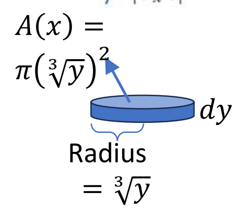
- At y: Thickness $dy \Rightarrow$ Cross section $A(y)$
  - $dV = \pi[g(y)]^2 dy$

Step4. Find the upper/lower limits for y

Step5. Set up integral and evaluate

$$V = \int_c^d A(y)dy = \int_c^d \pi[g(y)]^2 dy$$





$V(\text{disk}) = \pi[g(y)]^2 dy$

$0 \le y \le 8$ (limits)

$$V = \int_a^b \pi[g(y)]^2 dy$$

$$= \int_0^8 \pi y^{\frac{2}{3}} dy$$

$$= \pi\left[\frac{3}{5}x^{\frac{5}{3}}\right]_0^8$$

Volume by Washer perpendicular to the y axis

Volume of solid of revolution around y axis
Step 1: Plot the graph of $f(y)$, $g(y)$ w/ $f > g$

Step2. Find the size of a perpendicular slice at y
- At y: Thickness $dy \Rightarrow$ Cross section $A(y)$
- Washer = Large disc – small disc
  $= \pi[f(y)]^2 - \pi[g(y)]^2$

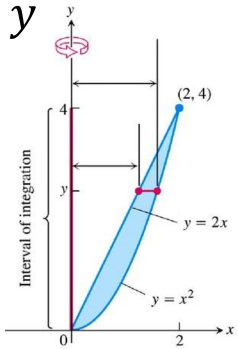
Step3. Find the volume of a slice at y
- At x: Thickness $dx \Rightarrow$ Cross section $A(y)$
  - $dV = \pi([f(y)]^2 - [g(y)]^2)dy$

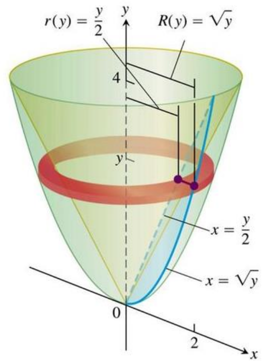
Step4. Find the upper/lower limits for y

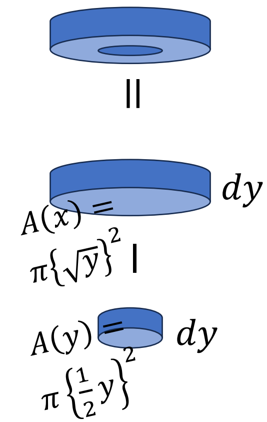
Step5. Set up integral and evaluate

$$V = \int_a^b A(y)dy$$
$$= \int_a^b \pi([f(y)]^2 - [g(y)]^2)dy$$

Find the volume of the solid that is obtained when the region between the curve $y = x^2$ *and* $y = 2x$ is revolved about the y-axis







$V($$) = \pi\{\sqrt{x}\}^2 - \pi\{1\}^1 = \pi[x - 1]$

Limit = [1,4]

$$V = \int_1^4 \pi(x - 1)dx$$
$$= \pi\left[\frac{1}{2}x^2 - x\right]_1^4 = \frac{9}{2}\pi$$

Example

Find the volume of the solid obtained by rotating the region bounded by $x = y^2$ and $x = y^3$ around the y-axis.

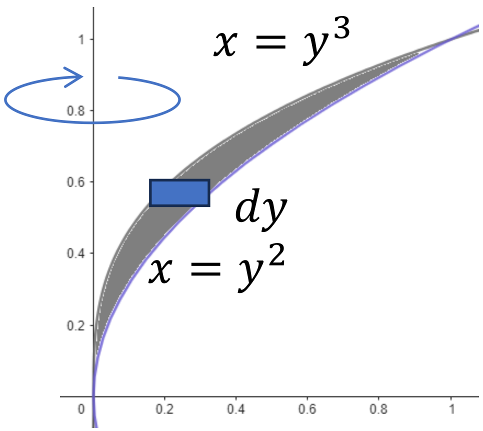
(a) $\frac{\pi}{35}$

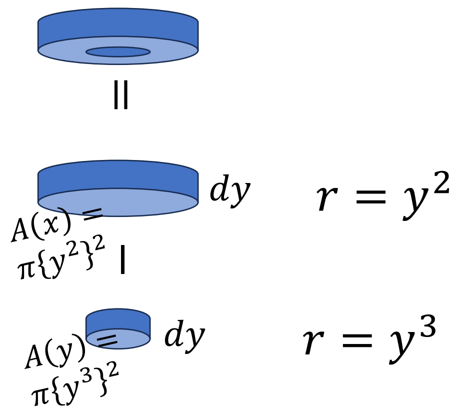
(b) $\frac{\pi}{10}$

(c) $\frac{\pi}{12}$

(d) $\frac{2\pi}{35}$ ← correct

(e) $\frac{\pi}{105}$





$$dV = \pi[y^4 - y^6]dy$$

Limit: $y^3 = y^2 \Rightarrow y = 0,1$

$$V = \int_0^1 \pi[y^4 - y^6]dy$$

$$= \pi\left[\frac{y^5}{5} - \frac{y^7}{7}\right]_0^1$$

$$= \pi\left[\frac{1}{5} - \frac{1}{7}\right] = \frac{2\pi}{35}$$

Example

Consider the region R bounded by $y = 2x^2$ and $y = 1$, first quadrant only.
Find the volume obtained by rotating R about the y-axis.

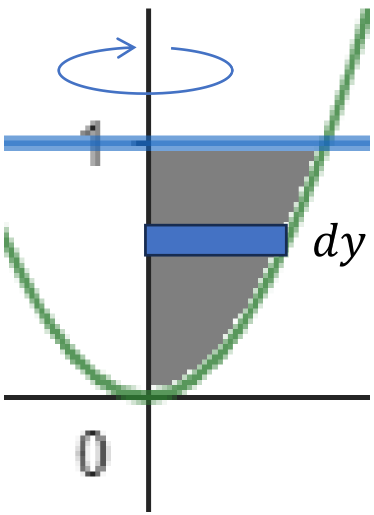
(a) $\frac{\pi}{4}$

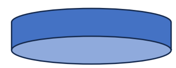
(b) $\frac{\pi}{2}$

(c) π

(d) $\frac{4\pi}{5}$

(e) None of the above





$$r = \sqrt{\frac{y}{2}}$$

$$dV = \pi\left[\sqrt{\frac{y}{2}}\right]^2 dy$$
$$= \frac{\pi}{2} y\,dy$$

Limits: $y = 0,1$

$$V = \int_0^1 \frac{\pi}{2} y\,dy$$
$$= \frac{\pi}{2}\left[\frac{y^2}{2}\right]_0^1 = \frac{\pi}{4}$$

Volume by cylindrical shells about the x-axis

Volume of solid of revolution around x axis

Step 1: Plot the graph

Step2. Find the size of a <u>parallel</u> slice at y

At y: Thickness $dy \Rightarrow$ Cross section $A(y)$

$$2\pi f(y)dy$$

Step3. Find the volume of a slice at y

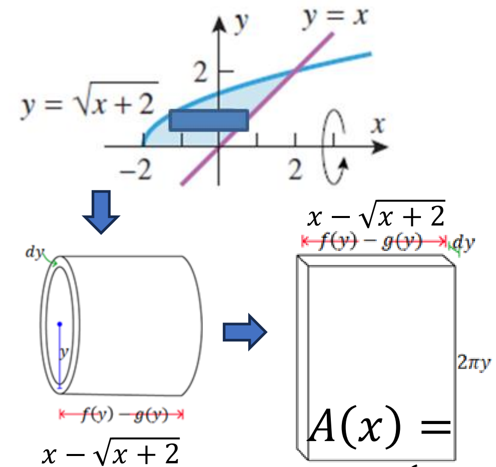
- At y: Thickness $dx \Rightarrow$ Cross section $A(y)$
    - $dV = 2\pi f(y)dy$

Step4. Find the upper/lower limits for y

Step5. Set up integral and evaluate

$V = \int_a^b A(y)dy = \int_a^b 2\pi y f(y)dy$

Use cylindrical shells to find the volume of the solid generated when the shaded region is revolved about the indicated axis.



$A(x) =$

$2\pi y\left(x - \sqrt{x-2}\right)dy$

$$V = \int_0^2 2\pi y[y - y^2 + 2]dy$$
$$= 2\pi \int_0^2 [y^2 - x^3 + 2y]dy$$
$$= 2\pi \left[\frac{1}{3}y^3 - \frac{1}{4}y^4 + y^2\right]_0^2$$
$$= 2\pi \left[\frac{8}{3} - \frac{16}{4} + 4\right]$$
$$= \frac{16\pi}{3}$$

Example

Consider the region bounded by the two curves $y = \cos x$, $y = \sin x$ and the two lines $x = 0$ and $x = \frac{\pi}{4}$.
Which of the following represents the volume of this region being rotated about the line $x = -1$?

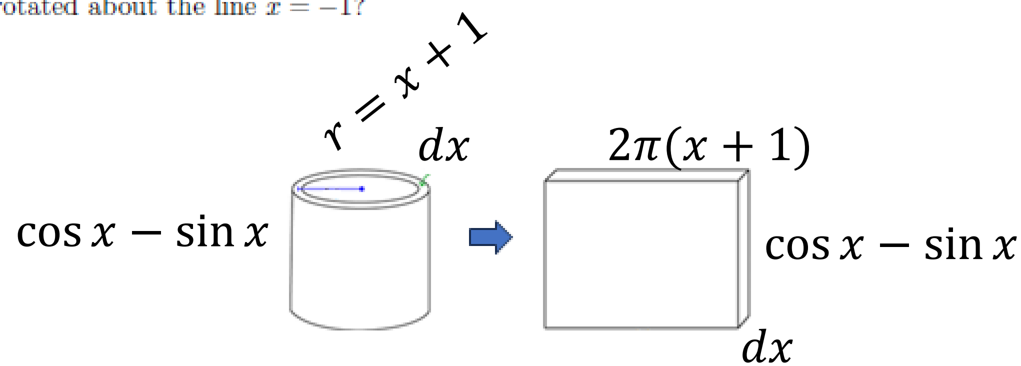
(a) $\int_0^{\frac{\pi}{4}} 2\pi(x+1)(\cos x - \sin x)\, dx$ ← correct

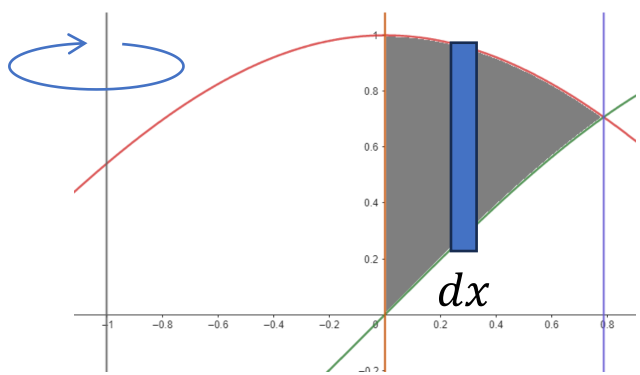
(b) $\int_0^{\frac{\pi}{4}} 2\pi(x+1)(\sin x - \cos x)\, dx$

(c) $\int_{-1}^{\frac{\pi}{4}} 2\pi(x+1)(\cos x - \sin x)\, dx$

(d) $\int_0^{\frac{\pi}{4}} 2\pi(x+1)(\cos^2 x - \sin^2 x)\, dx$

(e) $\int_0^{\frac{\pi}{4}} \pi(\cos^2 x - \sin^2 x)\, dx$





$r = x + 1$

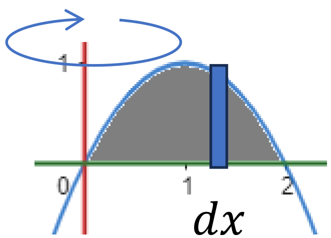
$dV = 2\pi(x+1)[\cos x - \sin x]dx$
Limits: $x = 0, \pi/4$

$V = \int_0^{\pi/4} 2\pi(x+1)[\cos x - \sin x]dx$

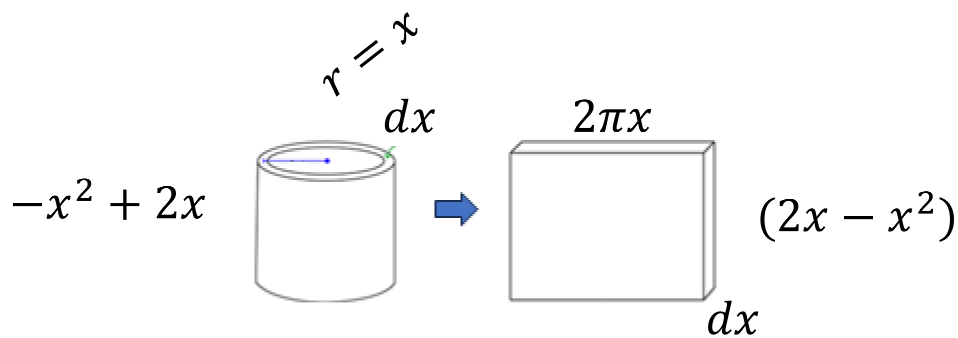
Example

Find the volume of the solid found by rotating the region bounded by the curves $y = -x^2 + 2x$ and $y = 0$ about the y-axis.

(a) $\frac{16}{3}\pi$
(b) $\frac{8}{3}\pi$
(c) $\frac{4}{3}\pi$
(d) $\frac{2}{3}\pi$
(e) $\frac{1}{3}\pi$



$r = x$



$dV = 2\pi x(2x - x^2)dx$

Limits: $-x^2 + 2x = 0 \quad \Rightarrow x = 0, 2$

$$V = \int_0^2 2\pi x(2x - x^2)dx$$
$$= 2\pi \int_0^2 (2x^2 - x^3)dx$$
$$= 2\pi \left[\frac{2}{3}x^3 - \frac{1}{4}x^4\right]_0^2 = 2\pi\left[\frac{2^4}{3} - \frac{2^4}{4}\right]$$
$$= 2^5\pi\left[\frac{1}{12}\right] = \frac{8\pi}{3}$$

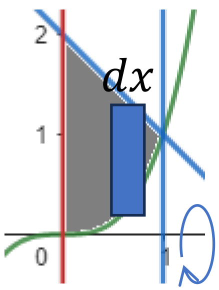
Example

Consider the region R bounded by $y = x^3$, $y = -x + 2$, $x = 0$, and $x = 1$.

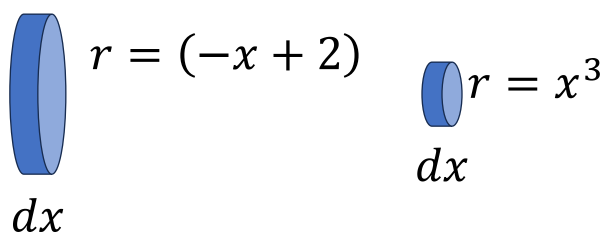
(a) Sketch the region R.

(b) Set up the integral that gives the volume obtained by revolving the region R about the x-axis using the method of washers. **DO NOT EVALUATE THE INTEGRAL.**

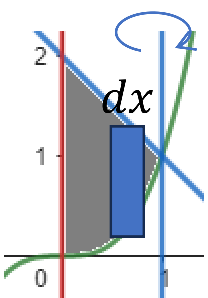
(c) Set up the integral that gives the volume obtained by revoling the region R about the line $x = 1$ using the method of cylindrical shells. **DO NOT EVALUATE THE INTEGRAL.**



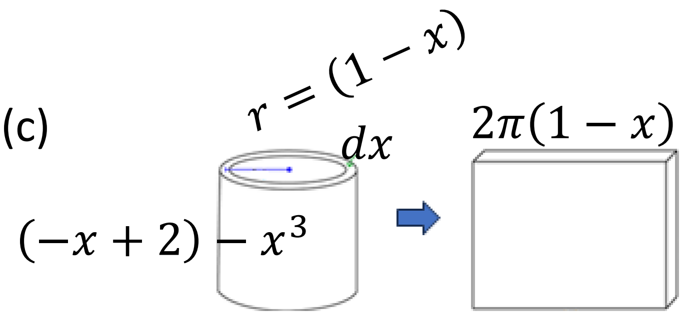
(b)



$$dV = \pi[(-x+2)^2 - (x^3)^2]dx$$
$$= \pi[x^2 - 4x + 4 - x^6]dx$$
$$V = \int_0^1 \pi[x^2 - 4x + 4 - x^6]dx$$
$$= \pi\left[\frac{1}{3}x^3 - 2x^2 + 4x - \frac{1}{7}x^7\right]_0^1$$
$$= \pi\left[\frac{1}{3} - 2 + 4 - \frac{1}{7}\right]$$
$$= \frac{46\pi}{21}$$



(c)



$$V = \int_0^1 2\pi(2 - x^3 + x^2 - 3x + x^4)dx$$
$$= 2\pi\left[2x - \frac{x^4}{4} + \frac{x^3}{3} - \frac{3x^2}{2} + \frac{x^5}{5}\right]_0^1$$
$$= 2\pi\left[2 - \frac{1}{4} + \frac{1}{3} - \frac{3}{2} + \frac{1}{5}\right]$$
$$= \frac{47\pi}{30}$$

$$dV = 2\pi(1-x)[-x + 2 - x^3]dx$$
$$= 2\pi(-x + 2 - x^3 + x^2 - 2x + x^4)dx$$
$$= 2\pi(2 - x^3 + x^2 - 3x + x^4)dx$$

Example

Consider the region R bounded by $y = 4x - x^2$ and $y = 0$. Which of the following integrals gives the volume of the solid obtained by revolving R about the line $x = -2$?

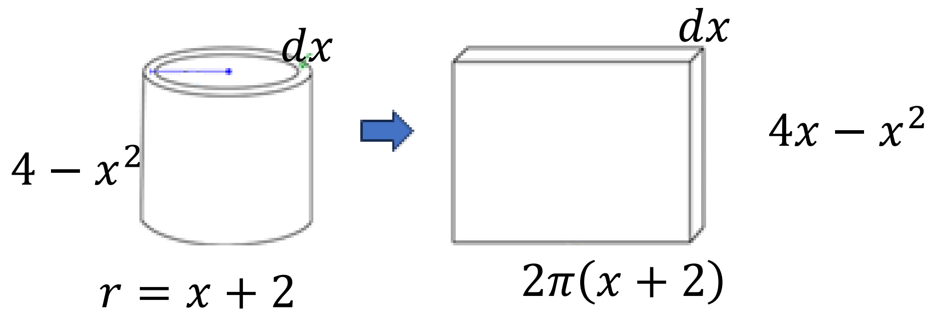
(a) $\int_0^4 2\pi(2-x)(4x-x^2)\,dx$

(b) $\int_0^4 2\pi x(4x-x^2)\,dx$

(c) $\int_0^4 2\pi(x+2)(4x-x^2)\,dx$

(d) $\int_0^4 2\pi(x-2)(4x-x^2)\,dx$

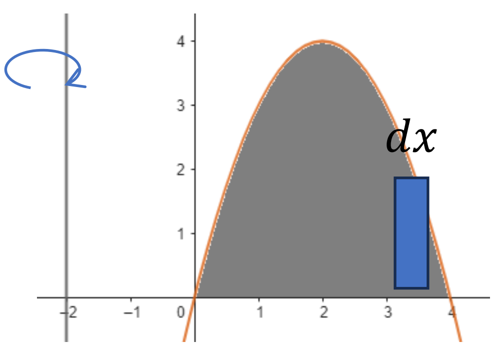
(e) None of the above



$$dV = 2\pi(x+2)(4x-x^2)dx$$

Limits : $4x - x^2 = 0 \Rightarrow x = 0,4$

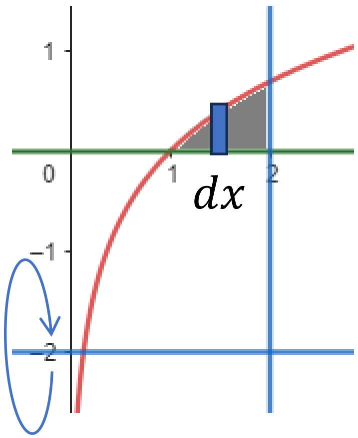
$$V = \int_0^4 2\pi(x+2)(4x-x^2)dx$$



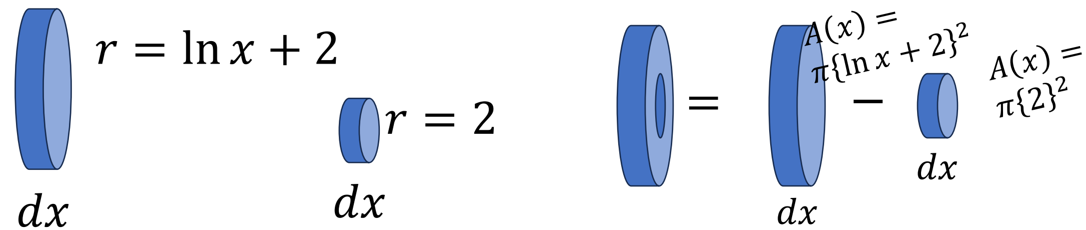
Example

Consider the region R bounded by $y = \ln x$, $y = 0$, and $x = 2$. If this region is revolved about the line $y = -2$:

(a) Set up but **do not evaluate** the integral that gives the volume using the method of shells.
(b) Set up but **do not evaluate** the integral that gives the volume using the method of washers.



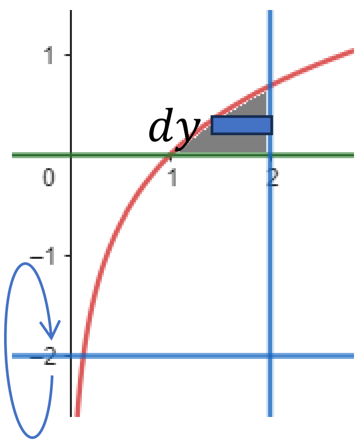
(a)



$dV = \pi[(\ln x + 2)^2 - 4]dx$

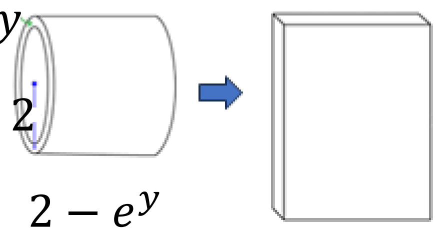
Limits $x = 0,2$

$V = \int_1^2 \pi[(\ln x + 2)^2 - 4]dx$



(b) dy

$r = y + 2$



$2 - e^y$

$dV = 2\pi(y + 2)(2 - e^y)dy$

Limits $e^y = 2, y = \ln 2$

$V = \int_0^{\ln 2} 2\pi(y + 2)(2 - e^y)dy$

Example

Consider the region bounded by the curves $x = y^2 - 2y$ and the y-axis. Which of the following represents the volume of solid formed when the region is rotated about $y = 4$?

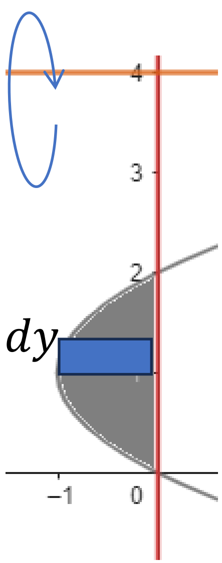
(a) $\int_0^2 2\pi y(y^2 - 2y)\, dy$

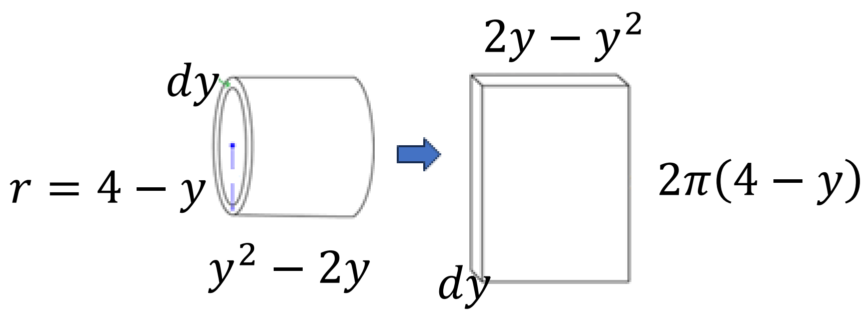
(b) $\int_0^2 2\pi y(2y - y^2)\, dy$

(c) $\int_0^2 2\pi(4 - y)(y^2 - 2y)\, dy$

(d) $\int_0^2 \pi(y - 4)(4y^2 - y^4)\, dy$

(e) $\int_0^2 2\pi(4 - y)(2y - y^2)\, dy$ ← correct





Limits: $y^2 - 2y = 0 \Rightarrow y = 0,2$

$dV = 2\pi(4 - y)(2y - y^2)dy$

$$V = \int_0^2 2\pi(4 - y)(2y - y^2)dy$$