TEXAS A&M UNIVERSITY
Math Learning Center

Math 251 - Fall 2024
"HANDS ON GRADES UP"
EXAM 4 REVIEW
TUESDAY, DEC 3, 7-9 PM
ILCB 229

Exam 4 Review: Covering sections 16.1-16.9, 14.7

PLEASE SCAN THE QR CODE BELOW

We will begin at 7:00 PM. A problem will be displayed on the wall monitors. Collaborate with your table on how to solve each problem. If you have a question, raise your hand. At the end of a predetermined number of minutes, the solutions will be displayed on the table monitors. Feel free to take a picture of the solution, as the solutions are not posted.

Problem 1. Evaluate $\int_C (x^2 + y^2 + z^2)\, ds$, where C is parameterized by $\mathbf{r}(t) = \langle\ t,\ \cos 2t, \sin 2t\rangle$, $0 \leq t \leq 2\pi$.

Problem 2. Evaluate $\int_C y^2\,dx + xy\,dy$, where C is the positively oriented rectangle with vertices $(0,0)$, $(3,0)$, $(3,2)$, and $(0,2)$.

Problem 3. Given $\mathbf{F} = \langle 4xe^z, \cos(y), 2x^2e^z \rangle$ and $\mathbf{r}(t) = \langle \sin(t), t, \cos(t) \rangle$, compute $\int_C \mathbf{F} \cdot d\mathbf{r}$ for $0 \leq t \leq \frac{\pi}{2}$.

Problem 4. Evaluate $\int_C (x - z + y)\, ds$, where C is the line segment from $(2, 1, 1)$ to $(3, -1, 0)$.

Problem 5. Set up but do not evaluate $\int_C xy\,dx + 2y\,dy$, where C is the arc of the curve $y = \sqrt{x}$ from $(0, 0)$ to the point $(9, 3)$, then the line segment from the point $(9, 3)$ to the point $(6, 0)$.

Problem 6. Find the work done by the force field $\mathbf{F} = \langle x \cos y,\ y \rangle$ in moving a particle along the parabola $y = 2x^2$ from the point $(1, 2)$ to the point $(2, 8)$.

Problem 7. A particle is moving along a triangular path. The particle starts at the point $(1, 1)$, then to the point $(2, 2)$, then from $(2, 2)$ to the point $(3, 1)$, then back to the point $(1, 1)$. Find the work done on this particle by the force field $\mathbf{F} = \langle x + 1, y - 2x \rangle$.

Problem 8. Evaluate $\int_C x\,dz + y\,dx + (xz)\,dy$, where C is parameterized by $\mathbf{r}(t) = \langle t^2, t^3, 2t \rangle$, $0 \leq t \leq 1$.

Problem 9. Consider the part of the plane $6x + 2y + 8z = 24$ that lies in the first octant. Set up but **do not evaluate** a double integral that gives the surface area of this plane in the order $dA = dzdx$.

Problem 10. Find the surface area of the part of the paraboloid $x = 4y^2 + 4z^2$ that lies inside the cylinder $y^2 + z^2 = 64$.

Problem 11. Consider the surface S that is the part of the the cylinder $y^2 + z^2 = 9$, $0 \leq x \leq 2$, including the disk $x = 2$. Find a parameterization of S.

Problem 12. Evaluate $\iint_S z\,dS$ where S is the part of the sphere $x^2 + y^2 + z^2 = 16$ that lies above the plane $z = 2$.

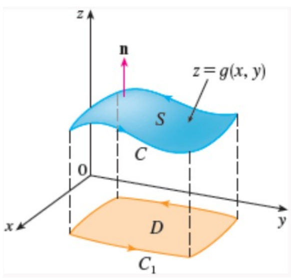
Recall: A sphere of radius ρ is parameterized by $\mathbf{r}(\phi, \theta) = \langle \rho\sin(\phi)\cos(\theta),\ \rho\sin(\phi)\sin(\theta),\ \rho\cos(\phi)\rangle$, where $0 \le \theta \le 2\pi$ and $0 \le \phi \le \pi$. Furthermore, $|\mathbf{r}_\phi \times \mathbf{r}_\theta| = \rho^2\sin(\phi)$. NOTE: When parameterizing a sphere, ρ is the radius of the sphere, which is a constant!!

Problem 13. Evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F} = \langle xy, 4x^2,\ yz \rangle$ and S is the surface $z = xe^y$, $0 \leq x \leq 1$, $0 \leq y \leq 1$, with upward orientation.

Problem 14. Set up but do not evaluate a double integral that gives the flux of $\mathbf{F} = \langle\, z - 3,\ x,\ y\rangle$ across S, where S is the part of the paraboloid $z = x^2 + y^2 + 3$ that is below the plane $z = 9$. Use the positive (outward) orientation.

Stokes' Theorem: Let S be an oriented piecewise-smooth surface parameterized by $\mathbf{r}(u, v)$, $u, v \in D$, that is bounded by a simple, closed, piecewise-smooth boundary curve C with positive (counterclockwise) orientation. Let $\mathbf{F}$ be a vector field whose components have continuous partial derivatives on an open region in $\Re^3$ that contains the surface S. Then

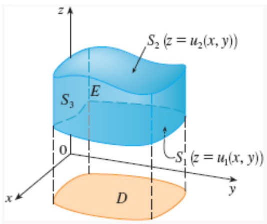
$$\int_C \mathbf{F} \cdot d\mathbf{r} = \iint_S \text{curl } \mathbf{F} \cdot d\mathbf{S}$$



Problem 15. Use Stokes' Theorem to set up but **not evaluate** $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F} = \langle yz, 2xy, 4xz \rangle$, and where C is the boundary curve of the part of the plane $3x + y + z = 3$ in the first octant, oriented counterclockwise when looking from above.

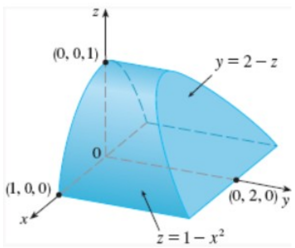
Problem 16. Use Stokes' Theorem to evaluate $\iint_S \text{curl } \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F} = \langle x^2 \sin(z-3), y^2, xy \rangle$, and S is the part of the paraboliod $z = 9 - x^2 - y^2$ that lies above the plane $z = 3$, oriented upward.

The Divergence Theorem: Let E be a simple solid region whose boundary surface S has positive (outward) orientation. Let $\mathbf{F}$ be a vector field whose component functions have continuous partial derivatives on an open region that contains E. Then $\iint\limits_S \mathbf{F} \cdot d\mathbf{S} = \iiint\limits_E \operatorname{div} \mathbf{F} \, dV$.



Problem 17. Using the The Divergence Theorem, set up but do not evaluate a triple integral used to find the flux of $\mathbf{F} = \left\langle ye^{z^2},\ ze^x,\ 2z + 8 \right\rangle$ across S, where S is the surface of the solid bounded by the plane $x + z = 7$, the cylinder $x^2 + y^2 = 9$, and the plane $z = 1$.

Problem 18. Using The Divergence Theorem, set up but do not evaluate a triple integral used to find $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F} = \left\langle xy, y^2 + e^{xz^2}, \sin(xy) \right\rangle$ and S is the surface of the region E bounded by the parabolic cylinder $z = 1 - x^2$ and the planes $z = 0$, $y = 0$, and $y + z = 2$. Assume S is positively orientated (see picture below).



Problem 19. Don't forget to review section 14.7, local and absolute extrema. Find all local extrema and/or saddle points if $f(x, y) = y^3 - 6y^2 - 2x^3 - 6x^2 + 48x + 20$.