Week in Review
Math 152

Week 07

Common Exam 2

Preparation

After an appropriate substitution, the integral $\int \sqrt{x^2 + x}\,dx$ is equivalent to which of the following?

(a) $\int \tan^2\theta \sec\theta \, d\theta$

(b) $\frac{1}{4}\int \sec^3\theta \, d\theta$

(c) $-\frac{1}{4}\int \sin^2\theta \, d\theta$

(d) $\frac{1}{4}\int \tan^2\theta \sec\theta \, d\theta$ ← correct

(e) $\int \cos^2\theta \, d\theta$

The region bounded by $y = \cos x$ and the x-axis on the interval $\left[0, \frac{\pi}{2}\right]$ is rotated about the x-axis. Find the volume of the resulting solid.

(a) 1

(b) $\frac{\pi^2}{2}$

(c) $\frac{\pi}{2}$

(d) $\frac{\pi}{4}$

(e) $\frac{\pi^2}{4}$ $\leftarrow$ correct

Common Exam II Prep

. Compute $\int_0^{\pi/2} \sin^2(\theta)\cos^3(\theta)\,d\theta$.

(a) $\frac{2}{5}$

(b) $\frac{2}{15}$ $\leftarrow$ correct

(c) $\frac{4}{5}$

(d) $\frac{8}{5}$

(e) None of the above

Common Exam II Prep

Evaluate $\int \tan^3(x)\sec^5(x)\,dx$.

(a) $\frac{1}{7}\tan^7 x - \frac{1}{5}\sec^5 x + C$

(b) $\frac{1}{7}\sec^7 x - \frac{1}{5}\tan^5 x + C$

(c) $\frac{1}{4}\sec^6 x - \frac{1}{6}\tan^{10} x + C$

(d) $\frac{1}{4}\sec^4 x - \frac{1}{6}\tan^6 x + C$

(e) $\frac{1}{7}\sec^7 x - \frac{1}{5}\sec^5 x + C$

Common Exam II Prep

After an appropriate substitution, the integral $\int \sqrt{x^2 + x}\, dx$ is equivalent to which of the following?

(a) $\frac{1}{4}\int \tan^2\theta \sec\theta \, d\theta$ ← correct

(b) $\int \tan^2\theta \sec\theta \, d\theta$

(c) $\frac{1}{4}\int \sec^3\theta \, d\theta$

(d) $-\frac{1}{4}\int \sin^2\theta \, d\theta$

(e) $\int \cos^2\theta \, d\theta$

Which of the following is the form of the partial-fraction decomposition for the rational function?

$$\frac{1}{(x+1)(x^2-2x-3)(x^2-2x+2)}$$

(a) $\frac{A}{x+1} + \frac{Bx+C}{x^2-2x-3} + \frac{Dx+E}{x^2-2x+2}$

(b) $\frac{A}{x+1} + \frac{Bx+C}{(x+1)^2} + \frac{D}{x-3} + \frac{Ex+F}{x^2-2x+2}$

(c) $\frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x-3} + \frac{D}{x^2-2x+2}$

(d) $\frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x-3} + \frac{Dx+E}{x^2-2x+2}$ ← correct

(e) $\frac{A}{x+1} + \frac{B}{x^2-2x-3} + \frac{C}{x^2-2x+2}$

Common Exam II Prep

$$\int_1^\infty xe^{-x^2}\,dx =$$

(a) 1

(b) $2e$

(c) $\dfrac{1}{2e}$ $\leftarrow$ correct

(d) $\dfrac{1}{2}$

(e) ∞

$$\int_0^1 \frac{2}{x^2 - 1}\, dx =$$

(a) $-\infty$ $\leftarrow$ correct

(b) ∞

(c) $\ln 2$

(d) $-\ln 2$

(e) 0

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$\lim_{n\to\infty} a_n = 0$?

- No → Divergent
- Yes → Any special type series ?
  - Geometric series
  - P-series
  - Telescoping series
  - Alternating series

Any special type series? Yes → Convergent Condition
- $|r| < 1$
- $p > 1$
- Converges
- $\lim_{n\to\infty} (-1)^n a_n = 0$

Any special type series? No → Integral test
Known antiderivative ?

Integral test → Convergent Condition
$\int_1^\infty f(x)dx < \infty$

Integral test: No → Long term behavior

Long term behavior →

Comparison test
$\Sigma a_k < \Sigma b_k < L$
(convergent)
$\infty = \Sigma b_k < \Sigma a_k$
(divergent)

Limit Comparison test
$\lim_{n\to\infty} \frac{a_n}{b_n} = L$
$\Sigma b_k < \infty$: convergent Σa_k
$\Sigma b_k = \infty$: divergent Σa_k

Ratio/root test
$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \rho < 1$
$\lim_{n\to\infty} (a_n)^{1/n} = \rho < 1$
convergent

Common Exam II Prep

Which of the following statements is true for the series $\sum_{n=1}^{\infty} \frac{2+\cos n}{n}$?

(a) The series converges because $\sum_{n=1}^{\infty} \frac{\cos n + 2}{n} \leq \sum_{n=1}^{\infty} \frac{3}{n}$, which converges.

(b) The series converges because $\sum_{n=1}^{\infty} \frac{\cos n + 2}{n} \leq \sum_{n=1}^{\infty} \frac{1}{n}$, which converges.

(c) The series diverges because $\sum_{n=1}^{\infty} \frac{\cos n + 2}{n} \geq \sum_{n=1}^{\infty} \frac{3}{n}$, which diverges.

(d) The series diverges because $\sum_{n=1}^{\infty} \frac{\cos n + 2}{n} \geq \sum_{n=1}^{\infty} \frac{1}{n}$, which diverges. $\leftarrow$ correct

(e) None of the above

Common Exam II Prep

Which of the following statements is true regarding the improper integral $\int_1^\infty \frac{\sin x + 2}{x(x+1)}\,dx$?

(a) The integral converges because $\int_1^\infty \frac{\sin x + 2}{x(x+1)}\,dx \le \int_1^\infty \frac{1}{x^2}\,dx$, which converges.

(b) The integral diverges because $\int_1^\infty \frac{\sin x + 2}{x(x+1)}\,dx \ge \int_1^\infty \frac{3}{x}\,dx$, which diverges.

(c) The integral converges because $\int_1^\infty \frac{\sin x + 2}{x(x+1)}\,dx \le \int_1^\infty \frac{3}{x^2}\,dx$, which converges. ← correct

(d) The integral diverges because $\int_1^\infty \frac{\sin x + 2}{x(x+1)}\,dx \ge \int_1^\infty \frac{1}{x}\,dx$, which diverges.

(e) The integral diverges by oscillation.

Let $s = \sum_{n=1}^{\infty} \frac{3}{n^4}$. Using the Remainder Estimate for the Integral Test, find the smallest value of n that will ensure that $R_n = s - s_n \leq \frac{1}{100}$.

(a) $n = 2$

(b) $n = 3$

(c) $n = 4$

(d) $n = 5$ $\leftarrow$ correct

(e) $n = 6$

The series $\sum_{n=1}^{\infty}\left(3^{1/n} - 3^{1/(n+1)}\right)$

(a) converges to 3

(b) converges to 2 $\leftarrow$ correct

(c) converges to 1

(d) converges to $\frac{1}{3}$

(e) diverges

Which of the following is true for the three sequences below?

(I) $a_n = \ln(2n+3) - \ln n$ (II) $a_n = \dfrac{\ln(n^2)}{n}$ (III) $a_n = n \sin\left(\dfrac{1}{n}\right)$

(a) Only (I) and (II) converge.

(b) Only (II) converges.

(c) Only (II) and (III) converge.

(d) Only (I) and (III) converge.

(e) All three converge. ← correct

Find the 2023rd term, a_{2023}, of the sequence $\{a_n\}$, assuming that the pattern of the first few terms continues beginning with $n = 1$.

$$\left\{-\frac{4}{4}, \frac{8}{9}, -\frac{16}{16}, \frac{32}{25}, -\frac{64}{36}, \dots\right\}$$

(a) $a_{2023} = -\dfrac{2^{2023}}{(2023)^2}$

(b) $a_{2023} = \dfrac{2^{2024}}{(2023)^2}$

(c) $a_{2023} = -\dfrac{2^{2024}}{(2024)^2}$ ← correct

(d) $a_{2023} = \dfrac{2^{2024}}{(2024)^2}$

(e) $a_{2023} = \dfrac{2^{2023}}{(2024)^2}$

Find the sum of series $\sum_{n=1}^{\infty} a_n$ if

$$a_1 + a_2 + \cdots + a_n = \frac{2}{n(n+1)}$$

(a) 0 $\leftarrow$ correct

(b) $\frac{1}{2}$

(c) 1

(d) 2

(e) This series diverges

Common Exam II Prep

Suppose a sequence $\{a_n\}$ is increasing and bounded above by 5 for all positive integers n. Determine the convergence if the sequence satisfies

$$a_1 = 3 \quad \text{and} \quad a_{n+1} = 5 - \frac{4}{a_n}$$

(a) convergent to 4 $\leftarrow$ correct

(b) convergent to 1

(c) convergent to 5

(d) convergent to $\frac{2}{9}$

(e) divergent

Find TRUE statements in the following.

I. If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ converges.

II. If the nth partial sum, $\{s_n\}$, converges, then $\sum_{n=1}^{\infty} a_n$ converges.

III. The geometric series $\sum_{n=1}^{\infty} ar^{n-1}$ converges if $|r| < 1$.

IV. If $\{a_n\}$ is decreasing and $a_n \geq 0$ for all n, then $\lim_{n\to\infty} a_n = 0$.

(a) I and II only

(b) II and III only $\leftarrow$ correct

(c) III and IV only

(d) I, II, and III only

(e) II, III, and IV only

Which of the following series diverges by the Test for Divergence?

(I) $\sum_{n=1}^{\infty} \cos\left(\frac{n}{2n+3}\right)$ (II) $\sum_{n=1}^{\infty} \frac{2}{3+2^{4n}}$ (III) $\sum_{n=1}^{\infty} \frac{1}{\arctan n}$

(a) I only

(b) II only

(c) III only

(d) I and II only

(e) I and III only ← correct

(8 points) Determine whether the series converges or diverges. If it converges, find the sum. Simplify your final answer.

$$\sum_{n=1}^{\infty} \frac{(-3)^n + 2^{2n}}{5^n}$$

(11 points) Compute the following integral showing all necessary work clearly.

$$\int \frac{1}{(x^2+9)^{5/2}}\,dx$$

(11 points) Compute the following integral showing all necessary work clearly.

$$\int \frac{4x^2 - 5x + 11}{(x+1)(x-1)(x^2+4)}\, dx$$

Common Exam II Prep

(10 points) Compute the following integral or show it diverges. Correct mathematical notations must be shown throughout the work for full credit.

$$\int_0^1 x^2 \ln x \, dx$$