

## Week in Review Math 152

## Week 07 Common Exam 2 Preparation

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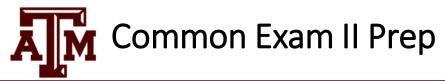
After an appropriate substitution, the integral  $\int \sqrt{x^2 + x} \, dx$  is equivalent to which of the following?

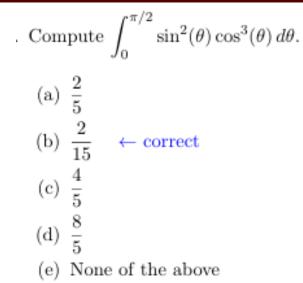
(a) 
$$\int \tan^2 \theta \sec \theta \, d\theta$$
  
(b)  $\frac{1}{4} \int \sec^3 \theta \, d\theta$   
(c)  $-\frac{1}{4} \int \sin^2 \theta \, d\theta$   
(d)  $\frac{1}{4} \int \tan^2 \theta \sec \theta \, d\theta \quad \leftarrow \text{ correct}$   
(e)  $\int \cos^2 \theta \, d\theta$ 



The region bounded by  $y = \cos x$  and the x-axis on the interval  $\left[0, \frac{\pi}{2}\right]$  is rotated about the x-axis. Find the volume of the resulting solid.

(a) 1 (b)  $\frac{\pi^2}{2}$ (c)  $\frac{\pi}{2}$ (d)  $\frac{\pi}{4}$ (e)  $\frac{\pi^2}{4} \leftarrow \text{correct}$ 





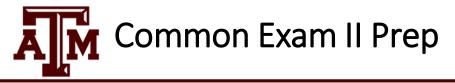


Evaluate  $\int \tan^3(x) \sec^5(x) dx$ .

- (a)  $\frac{1}{7} \tan^7 x \frac{1}{5} \sec^5 x + C$
- (b)  $\frac{1}{7}\sec^7 x \frac{1}{5}\tan^5 x + C$
- (c)  $\frac{1}{4}\sec^6 x \frac{1}{6}\tan^{10} x + C$
- (d)  $\frac{1}{4} \sec^4 x \frac{1}{6} \tan^6 x + C$
- (e)  $\frac{1}{7}\sec^7 x \frac{1}{5}\sec^5 x + C$

After an appropriate substitution, the integral  $\int \sqrt{x^2 + x} \, dx$  is equivalent to which of the following?

(a) 
$$\frac{1}{4} \int \tan^2 \theta \sec \theta \, d\theta \quad \leftarrow \text{ correct}$$
  
(b)  $\int \tan^2 \theta \sec \theta \, d\theta$   
(c)  $\frac{1}{4} \int \sec^3 \theta \, d\theta$   
(d)  $-\frac{1}{4} \int \sin^2 \theta \, d\theta$   
(e)  $\int \cos^2 \theta \, d\theta$ 



Which of the following is the form of the partial-fraction decomposition for the rational function?

$$\frac{1}{(x+1)(x^2-2x-3)(x^2-2x+2)}$$
(a)  $\frac{A}{x+1} + \frac{Bx+C}{x^2-2x-3} + \frac{Dx+E}{x^2-2x+2}$   
(b)  $\frac{A}{x+1} + \frac{Bx+C}{(x+1)^2} + \frac{D}{x-3} + \frac{Ex+F}{x^2-2x+2}$   
(c)  $\frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x-3} + \frac{D}{x^2-2x+2}$   
(d)  $\frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x-3} + \frac{Dx+E}{x^2-2x+2} \leftarrow \text{correct}$   
(e)  $\frac{A}{x+1} + \frac{B}{x^2-2x-3} + \frac{C}{x^2-2x+2}$ 



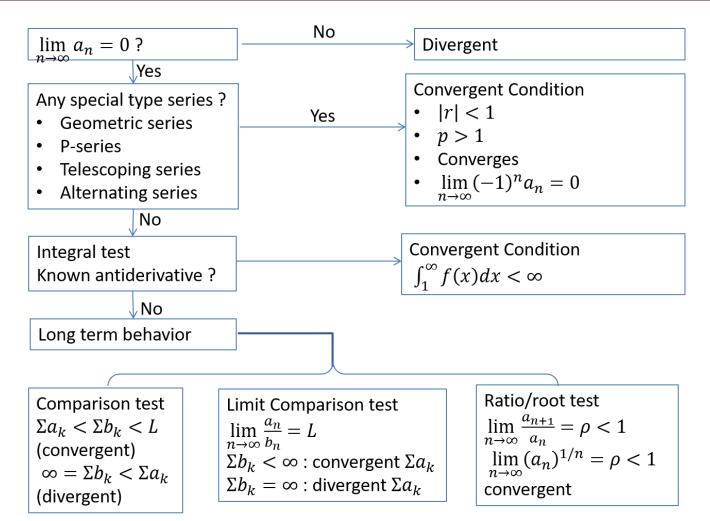
$$\int_{1}^{\infty} x e^{-x^{2}} dx =$$
(a) 1
(b) 2e
(c)  $\frac{1}{2e} \leftarrow \text{correct}$ 
(d)  $\frac{1}{2}$ 
(e)  $\infty$ 



$$\int_0^1 \frac{2}{x^2 - 1} \, dx =$$

- (a)  $-\infty \leftarrow \text{correct}$
- (b)  $\infty$
- (c)  $\ln 2$
- $(d)\ -\ln 2$
- (e) 0





Which of the following statements is true for the series  $\sum_{n=1}^{\infty} \frac{2 + \cos n}{n}$ ? (a) The series converges because  $\sum_{n=1}^{\infty} \frac{\cos n + 2}{n} \le \sum_{n=1}^{\infty} \frac{3}{n}$ , which converges. (b) The series converges because  $\sum_{n=1}^{\infty} \frac{\cos n + 2}{n} \le \sum_{n=1}^{\infty} \frac{1}{n}$ , which converges. (c) The series diverges because  $\sum_{n=1}^{\infty} \frac{\cos n + 2}{n} \ge \sum_{n=1}^{\infty} \frac{3}{n}$ , which diverges. (d) The series diverges because  $\sum_{n=1}^{\infty} \frac{\cos n + 2}{n} \ge \sum_{n=1}^{\infty} \frac{1}{n}$ , which diverges.  $\leftarrow$  correct

(e) None of the above

Which of the following statements is true regarding the improper integral  $\int_{1}^{\infty} \frac{\sin x + 2}{x(x+1)} dx$ ?

(a) The integral converges because 
$$\int_{1}^{\infty} \frac{\sin x + 2}{x(x+1)} dx \leq \int_{1}^{\infty} \frac{1}{x^2} dx$$
, which converges.  
(b) The integral diverges because  $\int_{1}^{\infty} \frac{\sin x + 2}{x(x+1)} dx \geq \int_{1}^{\infty} \frac{3}{x} dx$ , which diverges.  
(c) The integral converges because  $\int_{1}^{\infty} \frac{\sin x + 2}{x(x+1)} dx \leq \int_{1}^{\infty} \frac{3}{x^2} dx$ , which converges.  $\leftarrow$  correct  
(d) The integral diverges because  $\int_{1}^{\infty} \frac{\sin x + 2}{x(x+1)} dx \geq \int_{1}^{\infty} \frac{1}{x} dx$ , which diverges.

(e) The integral diverges by oscillation.



Let  $s = \sum_{n=1}^{\infty} \frac{3}{n^4}$ . Using the Remainder Estimate for the Integral Test, find the smallest value of n that

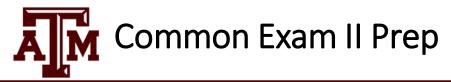
will ensure that  $R_n = s - s_n \le \frac{1}{100}$ .

- (a) n = 2
- (b) n = 3
- (c) n = 4
- (d)  $n = 5 \leftarrow \text{correct}$
- (e) n = 6



The series 
$$\sum_{n=1}^{\infty} (3^{1/n} - 3^{1/(n+1)})$$

- (a) converges to 3
- (b) converges to  $2 \leftarrow \text{correct}$
- (c) converges to 1
- (d) converges to  $\frac{1}{3}$
- (e) diverges



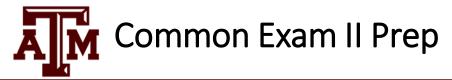
Which of the following is true for the three sequences below?

(II) 
$$a_n = \frac{\ln(n^2)}{n}$$
 (III)  $a_n = n \sin\left(\frac{1}{n}\right)$ 

- (a) Only (I) and (II) converge.
- (b) Only (II) converges.

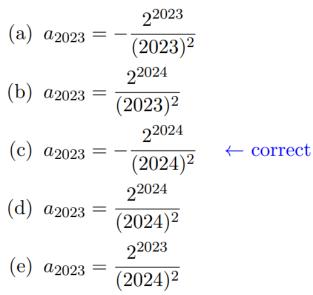
(I)  $a_n = \ln(2n+3) - \ln n$ 

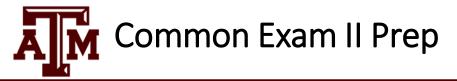
- (c) Only (II) and (III) converge.
- (d) Only (I) and (III) converge.
- (e) All three converge.  $\leftarrow$  correct



Find the 2023rd term,  $a_{2023}$ , of the sequence  $\{a_n\}$ , assuming that the pattern of the first few terms continues beginning with n = 1.

$$\left\{-\frac{4}{4}, \frac{8}{9}, -\frac{16}{16}, \frac{32}{25}, -\frac{64}{36}, \cdots\right\}$$



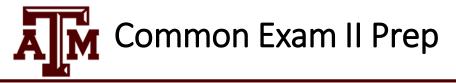


Find the sum of series 
$$\sum_{n=1}^{\infty} a_n$$
 if

$$a_1 + a_2 + \dots + a_n = \frac{2}{n(n+1)}$$

(a) 
$$0 \leftarrow \text{correct}$$

- (b)  $\frac{1}{2}$
- (c) 1
- (d) 2
- (e) This series diverges



Suppose a sequence  $\{a_n\}$  is increasing and bounded above by 5 for all positive integers n. Determine the convergence if the sequence satisfies

$$a_1 = 3$$
 and  $a_{n+1} = 5 - \frac{4}{a_n}$ 

- (a) convergent to 4  $\leftarrow$  correct
- (b) convergent to 1
- (c) convergent to 5
- (d) convergent to  $\frac{2}{9}$
- (e) divergent



Find TRUE statements in the following.

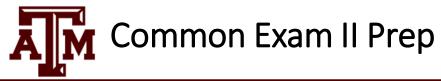
I. If  $\lim_{n \to \infty} a_n = 0$ , then  $\sum_{n=1}^{\infty} a_n$  converges.

II. If the *n*th partial sum,  $\{s_n\}$ , converges, then  $\sum_{n=1}^{\infty} a_n$  converges.

III. The geometric series  $\sum_{n=1}^{\infty} ar^{n-1}$  converges if |r| < 1.

IV. If  $\{a_n\}$  is decreasing and  $a_n \ge 0$  for all n, then  $\lim_{n \to \infty} a_n = 0$ .

- (a) I and II only
- (b) II and III only  $\leftarrow$  correct
- (c) III and IV only
- (d) I, II, and III only
- (e) II, III, and IV only



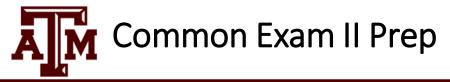
Which of the following series diverges by the Test for Divergence?

(I) 
$$\sum_{n=1}^{\infty} \cos\left(\frac{n}{2n+3}\right)$$

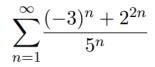
(II) 
$$\sum_{n=1}^{\infty} \frac{2}{3+2^{4n}}$$

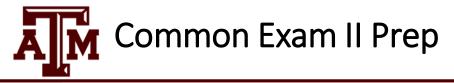
(III) 
$$\sum_{n=1}^{\infty} \frac{1}{\arctan n}$$

- (a) I only
- (b) II only
- (c) III only
- (d) I and II only
- (e) I and III only  $\leftarrow$  correct



(8 points) Determine whether the series converges or diverges. If it converges, find the sum. Simplify your final answer.





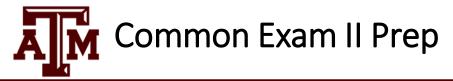
(11 points) Compute the following integral showing all necessary work clearly.

$$\int \frac{1}{(x^2+9)^{5/2}} \, dx$$



(11 points) Compute the following integral showing all necessary work clearly.

$$\int \frac{4x^2 - 5x + 11}{(x+1)(x-1)(x^2+4)} \, dx$$



(10 points) Compute the following integral or show it diverges. Correct mathematical notations must be shown throughout the work for full credit.

 $\int_0^1 x^2 \ln x \, dx$