Math 308: Week-in-Review 14 (FINAL EXAM REVIEW)

1. Find the solution. Where is the solution defined?

$$y' - x^2 y = x^2, \quad y(0) = 3.$$

2. Find the general solution.

$$(\cos(x)y + x) + (\sin(x) + y^2)y' = 0$$

Leave your solution in implicit form.

3. Without solving the equation, determine where a unique solution is guaranteed to exist.

$$\ln(t)y'' - y' + \frac{1}{t-3}y = \sqrt{8-t}, \quad y(2) = 3.$$

4. Consider the differential equation

$$y' = y^3 - 4y^2 + 3y.$$

   (a) Find the equilibrium solutions.
   (b) Plot the direction field.
      - Draw a few example solutions on the direction field.
   (c) Draw the phase line diagram.
   (d) Determine the stability of each equilibrium point.

5. An object is initially at 100°C in a room with a constant temperature of 20°C. It cools according to Newton's law of cooling with a cooling constant $k = 0.05$ per minute. Find the time when the object's temperature reaches 50°C.

6. Find the general solution of the equation

$$u'' + 2u' + u = 0$$

7. Find the general solution. Prove that it is indeed the general solution.

$$u'' + 6u' + 10u = 0$$

8. Find a particular solution to

$$t^2y'' - 3ty' + 3y = 5t^2, \quad t > 0,$$

given that t and t^3 are solutions to the corresponding homogeneous equation.

9. Find a particular solution.

$$y'' - 3y' + 2 = 4e^t + 5$$

10. Suppose there is a 30 N mass hanging on a spring. When the mass was attached to the spring, the spring stretched by 40 cm. When the mass is moving 5 m/s, it experiences a damping force of 15 N. There is an external upward force of 10 N acting on the mass for the first 10 seconds, after which there is no external force. Initially the mass is sent into motion with a downward velocity of 50 cm/s from the equilibrium position. (Use $g = 10\,\text{m/s}^2$.)

    (a) Write down an initial value problem that describes the motion of the mass.

    (b) Is the system over, under, or critically damped?

11. Solve the initial value problem.

$$y'' + y = u_1(t), \quad y(0) = 0, \quad y'(0) = 0.$$

12. Using the **definition** of the Laplace transform, show that $\mathcal{L}\{t^2\} = \dfrac{2}{s^3}$.

13. Find the general solution in the form of a power series centered at $x = 0$.

$$y'' + xy = 0.$$

14. Find the general solution to the system of differential equations.

$$x_1' = 3x_1 + x_2$$
$$x_2' = -x_1 + x_2$$