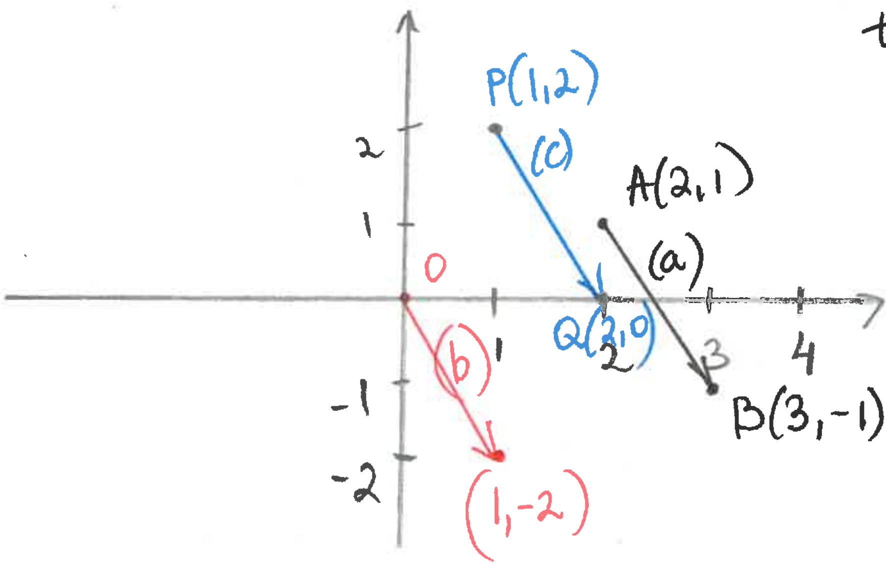
1. Find a vector **a** with representation given by a directed line segment $\overrightarrow{AB}$, where $A(2,1)$ and $B(3,-1)$.

   (a) Draw the vector $\overrightarrow{AB}$.

   (b) Draw the equivalent representation for **a** that starts at the origin.

   (c) Draw the equivalent representation starting at the point $P(1,2)$.




the components of $\overrightarrow{AB}$ are [end] − [start]

$$\boxed{\begin{aligned}\overrightarrow{AB} &= \langle 3-2, -1-1 \rangle \\ &= \langle 1, -2 \rangle.\end{aligned}}$$

If a vector starts @ $P(1,2)$, then the end point will be

$Q(1+1, 2-2) = Q(2,0)$.

(add the components of the vector to the coordinates of P)

$a=\langle 3,-4\rangle,\ \vec{b}=\langle 1,3\rangle,\ \vec{c}=\langle 2,1\rangle$

2. For the vectors $\mathbf{a} = 3\mathbf{i} - 4\mathbf{j}$, $\mathbf{b} = \mathbf{i} + 3\mathbf{j}$, $\mathbf{c} = 2\mathbf{i} + \mathbf{j}$, find:

(a) $|-4\mathbf{a} + 3\mathbf{b}|$

(b) a unit vector in the direction opposite to $\mathbf{c}$

(c) a vector of length 3 in the direction of $\mathbf{b}$

(d) constants $s$ and $t$ such that $\mathbf{c} = s\mathbf{a} + t\mathbf{b}$

(a) $-4\vec{a}+3\vec{b}=-4\langle 3,-4\rangle+3\langle 1,3\rangle=\langle +3(-4)+3,\ -4(-4)+3(3)\rangle$

$=\langle -9, 25\rangle$

$|-4\vec{a}+3\vec{b}|=\sqrt{(-9)^2+25^2}=\sqrt{81+625}=\boxed{\sqrt{706}}$

(b) $\vec{u}=-\dfrac{\vec{c}}{|\vec{c}|}=-\dfrac{\langle 2,1\rangle}{\sqrt{2^2+1^2}}=-\dfrac{\langle 2,1\rangle}{\sqrt{5}}=\boxed{\left\langle -\frac{2}{\sqrt{5}},-\frac{1}{\sqrt{5}}\right\rangle}$

(c) the vector is $\dfrac{3\vec{b}}{|\vec{b}|}=3\dfrac{\langle 1,3\rangle}{\sqrt{1^2+3^2}}=3\dfrac{\langle 1,3\rangle}{\sqrt{1+9}}$

$=\dfrac{\langle 3,9\rangle}{\sqrt{10}}=\boxed{\left\langle \frac{3}{\sqrt{10}},\ \frac{9}{\sqrt{10}}\right\rangle}$

d) $\vec{c}=s\vec{a}+t\vec{b}$ or $\langle 2,1\rangle=s\langle 3,-4\rangle+t\langle 1,3\rangle$

$\langle 2,1\rangle=\langle 3s+t,\ -4s+3t\rangle$

match up the components: $\begin{cases}2=3s+t \\ 1=-4s+3t\end{cases}$ $\Rightarrow$ $t=2-3s$ plug into the 2nd eqn.

$1=-4s+3\underbrace{(2-3s)}^{t}$

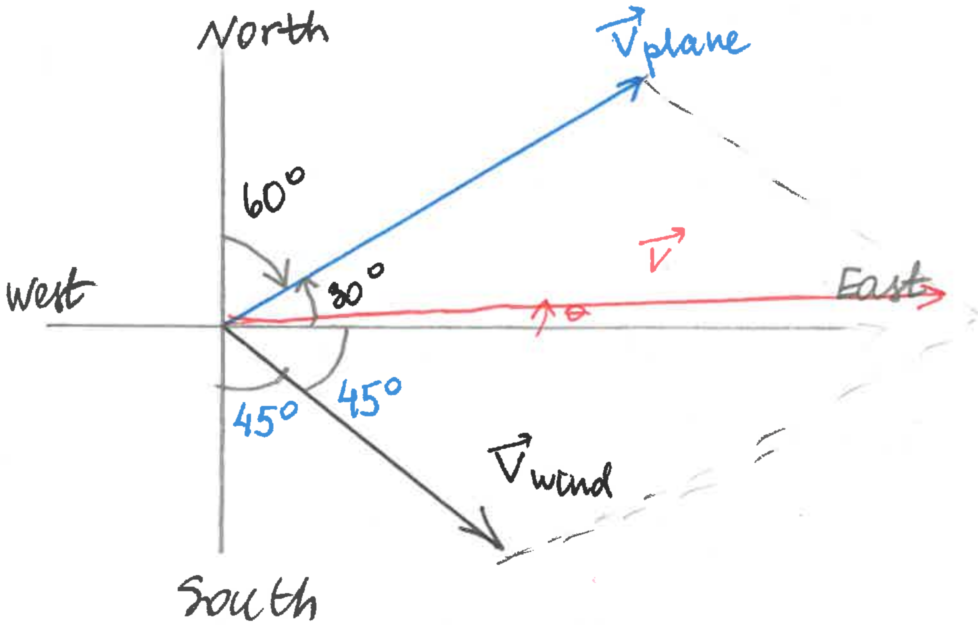
$1=-4s+6-9s \Rightarrow -13s+6=1$

$-13s=1-6$

$-13s=-5 \rightarrow \boxed{s=\frac{5}{13}}$

$t=2-3\cdot\frac{5}{13}=2-\frac{15}{13}=\frac{26-15}{13}=\boxed{\frac{11}{13}=t}$

3. Suppose that a wind is blowing in the direction S45°E at a speed of 60 km/h. A pilot is steering a plane in the direction N60°E at an airspeed (speed in still air) of 100 km/h. The *true course*, or *track*, of the plane is the direction of the resultant of the velocity vectors of the plane and the wind. The *ground speed* of the plane is the magnitude of the resultant. Find the true course and the ground speed of the plane.




Ground velocity of the plane

$\vec{V} = \vec{V}_{wind} + \vec{V}_{plane}$

ground speed $= |\vec{V}|$

$|\vec{V}_{wind}| = 60$

$|\vec{V}_{plane}| = 100$

$$\vec{V}_{wind} = 60 \langle \cos 45°, -\sin 45° \rangle$$
$$= 60 \langle \frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} \rangle = \langle 30\sqrt{2}, -30\sqrt{2} \rangle$$

$$\vec{V}_{plane} = 100 \langle \cos 30°, \sin 30° \rangle = 100 \langle \frac{\sqrt{3}}{2}, \frac{1}{2} \rangle$$
$$= \langle 50\sqrt{3}, 50 \rangle .$$

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$$\vec{V} = \vec{V}_{wind} + \vec{V}_{plane} = \langle 30\sqrt{2}, -30\sqrt{2} \rangle + \langle 50\sqrt{3}, 50 \rangle$$
$$= \langle 30\sqrt{2} + 50\sqrt{3}, -30\sqrt{2} + 50 \rangle \approx \langle 129, 7.6 \rangle$$

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$$\text{ground speed} = |\vec{V}| \approx \sqrt{129^2 + (7.6)^2} \approx \boxed{129.25 (km/h)}$$

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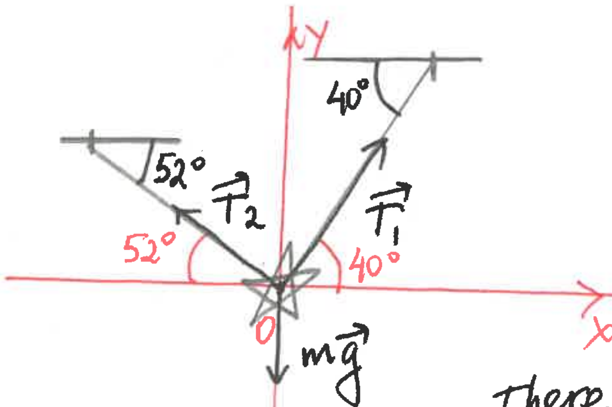
The true course is ~~N θ E~~ N(90°−θ)E where $\tan\theta = \frac{7.6}{129}$

$$\theta = \arctan \frac{7.6}{129} \approx 3.37° .$$

true course is $N(90° - 3.37°)E \Rightarrow \boxed{N\ 86.63°\ E}$

4. Ropes 3 m and 5 m in length are fastened to a holiday decoration that is suspended over a town square. The decoration has a mass of 5 kg. The ropes, fastened at different heights, make angles of 52° and 40° with the horizontal. Find the magnitude of the tension in each wire.




$|\vec{T_1}| = ?$

$|\vec{T_2}| = ?$

$\vec{T_1}$ is the tension in wire 1

$\vec{T_2}$ is the tension in wire 2.

There are three forces acting on the decoration: $T_1$, $T_2$, and the gravity $mg$, $m = 5\ kg$, $g = 9.8\ m/sec^2$.

The decoration is in the equilibrium, so the resultant force on zero.

$$\boxed{\vec{T_1} + \vec{T_2} + m\vec{g} = \vec{0}}$$

$$\vec{T_1} = |\vec{T_1}| < \cos 40°, \sin 40° >$$

$$\vec{T_2} = |\vec{T_2}| < -\cos 52°, \sin 52° >$$

$$m\vec{g} = < 0, -mg > = < 0, -49 >.$$

$$|\vec{T_1}| < \cos 40°, \sin 40° > + |\vec{T_2}| < -\cos 52°, \sin 52° > + < 0, -49 > = < 0, 0$$

Match up the components:

$$\begin{cases} |\vec{T_1}|\cos 40° + |\vec{T_2}|(-\cos 52°) = 0 \\ |\vec{T_1}|\sin 40° + |\vec{T_2}|\sin 52° - 49 = 0. \end{cases}$$

$\Longrightarrow |\vec{T_2}| = \dfrac{|\vec{T_1}|\cos 40°}{\cos 52°}$, plug into the 2nd equation.

$$|\vec{T_1}|\sin 40° + \frac{|\vec{T_1}|\cos 40°}{\cos 52°}\sin 52° - 49 = 0.$$

$$|\vec{T_1}|(\sin 40° + \cos 40° \tan 52°) = 49$$

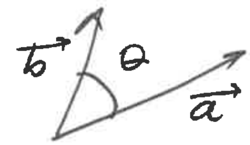
$$|\vec{T_1}| = \frac{49}{\sin 40° + \cos 40° \tan 52°} \approx \boxed{30.25\,(N) = |\vec{T_1}|}$$

#4 continued. $|\vec{F}_2| = \dfrac{|\vec{T}_1|\cos 40^\circ}{\cos 52^\circ} \approx \dfrac{30.25\cdot\cos 40^\circ}{\cos 52^\circ} \approx \boxed{37.6(N) = |\vec{T}_2|}$

5. Find $\mathbf{a}\cdot\mathbf{b}$

(a) $|\mathbf{a}| = 2$, $|\mathbf{b}| = 5$ and the angle between $\mathbf{a}$ and $\mathbf{b}$ is $150^\circ$

(b) $\mathbf{a} = -3\mathbf{i} + \mathbf{j}$, $\mathbf{b} = 2\mathbf{i} + 4\vec{\mathbf{j}}$




(a) $\vec{a}\cdot\vec{b} = |\vec{a}|\cdot|\vec{b}|\cos\theta$

$\vec{a} = \langle a_1, a_2\rangle$, $\vec{b} = \langle b_1, b_2\rangle$, then

$\vec{a}\cdot\vec{b} = a_1b_1 + a_2b_2$

$(150^\circ = 180^\circ - 30^\circ)$

(a) $\vec{a}\cdot\vec{b} = 2\cdot 5\cos 150^\circ$

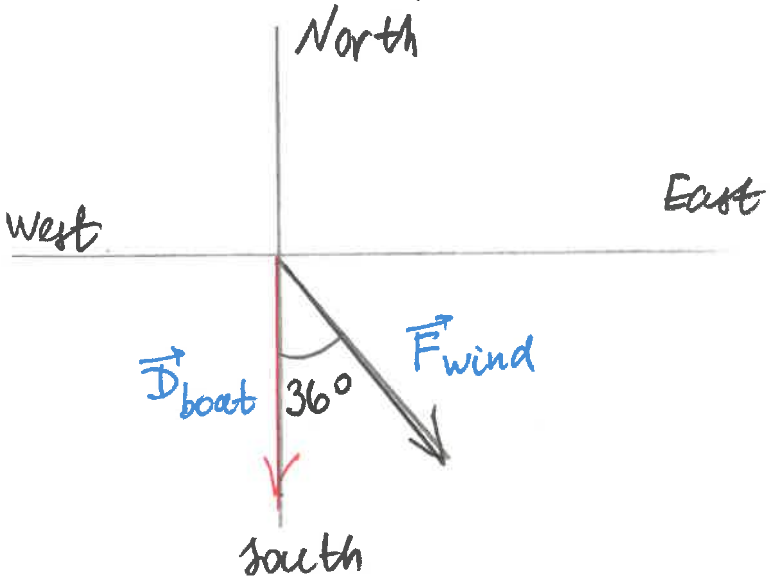
$= 2\cdot 5(-\cos 30^\circ)$

$= -10\cdot\dfrac{\sqrt{3}}{2} = \boxed{-5\sqrt{3}}$

(b) $\vec{a} = \langle -3, 1\rangle$, $\vec{b} = \langle 2, 4\rangle$

$\vec{a}\cdot\vec{b} = -3(2) + 1(4) = -6+4 = \boxed{-2}$

6. A boat sails south with the help of a wind blowing in the direction S36°E with magnitude 400 lb. Find the work done by the wind as the boat moves 110 ft. (Round your answer to the nearest whole number.)




$\vec{W} = \vec{F} \cdot \vec{D} = |\vec{F}| \cdot |\vec{D}| \cos\theta$

$|\vec{F}| = 400$ lb

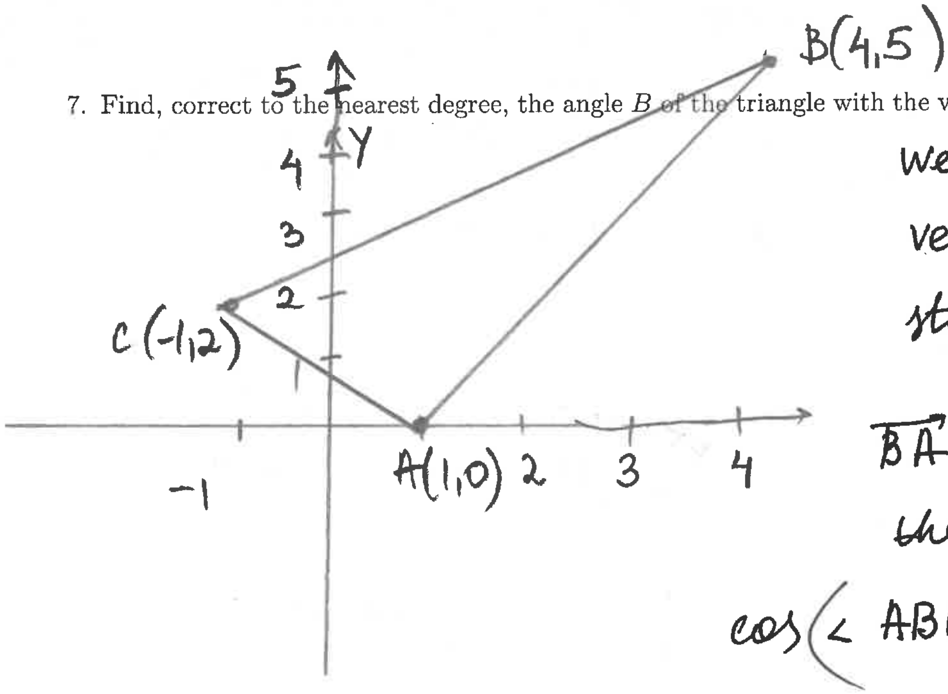
$|\vec{D}| = 110$ ft.

$W = |\vec{F}| \cdot |\vec{D}| \cos\theta$

$= (110)(400) \cos 36°$

$\approx \boxed{35{,}597 \text{ (lb-ft)}}$

7. Find, correct to the nearest degree, the angle $B$ of the triangle with the vertices $A(1,0)$, $B(4,5)$, $C(-1,2)$




We make two vectors that start at B

$\overrightarrow{BA}$ and $\overrightarrow{BC}$, then

$$\cos(\angle ABC) = \frac{\overrightarrow{BA}\cdot\overrightarrow{BC}}{|\overrightarrow{BA}|\,|\overrightarrow{BC}|}$$

$$\overrightarrow{BA} = \langle -4+1, -5+0\rangle = \langle -3, -5\rangle$$

$$\overrightarrow{BC} = \langle -1-4,\ 2-5\rangle = \langle -5, -3\rangle$$

$$\cos(\angle ABC) = \frac{\langle -3,-5\rangle \cdot \langle -5,-3\rangle}{|\langle -3,-5\rangle|\,|\langle -5,-3\rangle|}$$

$$= \frac{15+15}{\sqrt{(-3)^2+(-5)^2}\cdot\sqrt{(-5)^2+(-3)^2}} = \frac{30}{9+25} = \frac{30}{34} = \frac{15}{17}.$$

$$\angle ABC = \arccos\left(\frac{15}{17}\right) \approx \boxed{28^\circ}$$

if $\vec{a} = \langle a_1, a_2 \rangle$, then its orthogonal complement is $\vec{a}^{\perp} = \langle -a_2, a_1 \rangle$ or $\vec{a}^{\perp} = \langle a_2, -a_1 \rangle$

8. Find a unit vector orthogonal to the vector $< -2, 4 >$.

$\vec{a} = \langle 2, -4 \rangle$

the orthogonal complement to $\vec{a}$

$\vec{a}^{\perp} = \langle 4, 2 \rangle$

$$\vec{u} = \frac{\vec{a}^{\perp}}{|\vec{a}^{\perp}|} = \frac{\langle 4, 2 \rangle}{\sqrt{4^2+2^2}} = \frac{\langle 4, 2 \rangle}{\sqrt{16+4}} = \frac{\langle 4, 2 \rangle}{\sqrt{20}}$$

$$= \frac{\langle 4, 2 \rangle}{2\sqrt{5}} = \left\langle \frac{4}{2\sqrt{5}}, \frac{2}{2\sqrt{5}} \right\rangle$$

$$\boxed{\vec{u} = \left\langle \frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}} \right\rangle}$$

9. Find the value(s) of $x$ such that the vectors $x\mathbf{i} + 3x\mathbf{j}$ and $x\mathbf{i} - 4\mathbf{j}$ are orthogonal.

$\langle x, 3x \rangle$ , $\langle x, -4 \rangle$.

Find $x$ such that

$$\langle x, 3x \rangle \cdot \langle x, -4 \rangle = 0.$$

$$x^2 - 12x = 0$$

$$x(x-12) = 0$$

$$\boxed{x_1 = 0} \quad \boxed{x_2 = 12}$$