Math 150 - Week-In-Review 7

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Exam 2 Review

1. The number of bacteria y in a culture after t days is given by the function $y(t) = 100e^{t/8}$.

   (a) What is the initial number of bacteria in the culture?

   $t=0$

   $y(0) = 100\, e^{\frac{0}{8}} = 100$ bacteria

   (b) After how many days will there be 4000 bacteria?

   $4000 = 100\, e^{\frac{t}{8}} \rightarrow 40 = e^{t/8}$

   $\ln(40) = \frac{t}{8}$ $\quad t = 8\ln(40)$ days

2. The sound intensity level L (in decibels, dB), is related to the intensity of the sound I (in watts per square meter), by the equation $L = 10\log\left(\frac{I}{I_0}\right)$, where $I_0 = 1\times 10^{-12}\ W/m^2$ is the threshold of human hearing. Determine the intensity I of a sound that registers $L = 85\ dB$.

   $85 = 10\log\left(\frac{I}{I_0}\right)$

   $8.5 = \log\left(\frac{I}{I_0}\right)$

   $\frac{I}{I_0} = 10^{8.5}$

   $I = I_0 \times 10^{8.5}$ W/m^2

   $= 10^{-12} \times 10^{8.5} = 10^{-3.5}$ W/m^2

3. If you invest \$2000 in an account with an annual interest rate of 4%, compounded annually, find the time it takes for an investment of \$2000 to grow to \$3000.

$n=1$

$t=?$ P $A(t)$

$$A(t) = P\left(1+\frac{r}{n}\right)^{nt}$$

P: initial principal, r: annual interest rate

n: # of compounds per year t: # of years

A(t): final amount

$$3000 = 2000\left(1+\frac{0.04}{1}\right)^{1(t)}$$

$$\frac{3}{2} = (1.04)^t$$

$$\ln(3/2) = t \cdot \ln(1.04)$$

$$t = \frac{\ln(3/2)}{\ln(1.04)} \overset{\text{or}}{=} \frac{\ln(3)-\ln(2)}{\ln(1.04)} \overset{\text{or}}{=} \frac{\ln(1.5)}{\ln(1.04)} \text{ years}$$

4. A population of rabbits can be modeled using the logistic equation

$$N(t) = \frac{1000}{1-24e^{-0.18t}}$$

How long does it take for population of rabbits to grow to 4200?

$t=?$ $N(t)$

$$42\cancel{00} = \frac{10\cancel{00}}{1-24e^{-0.18t}} \xrightarrow[\text{multiply}]{\text{Cross}} 42(1-24e^{-0.18t}) = 10$$

$$1-24e^{-0.18t} = \frac{\cancel{10}\,5}{\cancel{42}\,21}$$

$$-24e^{-0.18t} = \frac{5}{21}-1$$

$$-24e^{-0.18t} = \frac{-16}{21}$$

$$e^{-0.18t} = \frac{\cancel{16}\,2}{21\times\cancel{24}\,3}$$

$$-0.18t = \ln\left(\frac{2}{63}\right)$$

$$t = \frac{\ln\left(\frac{2}{63}\right)}{-0.18} = -\frac{\cancel{100}\,50\ln\left(\frac{2}{63}\right)}{\cancel{18}\,9} \overset{\text{or}}{=} -\frac{50(\ln 2-\ln 63)}{9}$$

$\ln$

5. A cup of coffee cools from 80°C to 70°C in 5 minutes. If the room temperature is 25°C, what will be the temperature of the coffee after 15 minutes?

Newton's Law of Cooling $T(t) = T_a + (T_0 - T_a)e^{-kt}$

T_0 : initial temp. T_a : surrounding temp. $k>0$ constant of porp.

find k: $70 = 25 + (80-25)e^{-k(5)}$

$$45 = (55)e^{-k(5)}$$

$$\frac{45}{55} = e^{-5k}$$

$$\ln\left(\frac{9}{11}\right) = -5k$$

$$k = \frac{\ln(9/11)}{-5} = -\frac{\ln(9/11)}{5}$$

$$T(15) = 25 + (80-25)e^{\frac{\ln(9/11)}{5}\cdot 15} = 25 + (55)e^{3\ln(9/11)}$$

$$= 25 + 55\left(e^{\ln(9/11)}\right)^3$$

$$T(15) = 25 + 55\left(\frac{9}{11}\right)^3$$

6. Solve for for x using the techniques discussed in class.

(a) $\sqrt{x^4+9} = \sqrt{6}\,x$

$x^4 + 9 = 6x^2$

$x^4 - 6x^2 + 9 = 0$

let $t = x^2$ $\quad t^2 - 6t + 9 = 0$

$(t-3)^2 = 0$

$t = 3$

$x^2 = 3 \rightarrow x = \pm\sqrt{3}$

Check:

$t = \sqrt{3}$

$\sqrt{(\sqrt{3})^4 + 9} = \sqrt{6} \times \sqrt{3}$

$\sqrt{9+9} = \sqrt{18}$ ✓

$\cancel{t = -\sqrt{3}}$

$\sqrt{(-\sqrt{3})^4 + 9} = \sqrt{3} \times (-\sqrt{3})$ Not a solution

(b) $\log_5(10-x) - \log_5(x+4) = 1$

$\log_5\left(\frac{10-x}{x+4}\right) = 1$

$\iff \frac{10-x}{x+4} = 5^1$

$10 - x = 5x + 20$

$-10 = 6x$

$-\frac{5}{3} = x$

Check: $x = -\frac{5}{3}$

$\log_5\left(10 - \left(-\frac{5}{3}\right)\right) - \log_5\left(-\frac{5}{3} + 4\right) \overset{?}{=} 1$

$\log_5\left(\frac{30}{3} + \frac{5}{3}\right) - \log_5\left(-\frac{5}{3} + \frac{12}{3}\right) \overset{?}{=} 1$

$\log_5\left(\frac{35}{3}\right) - \log_5\left(\frac{7}{3}\right) \overset{?}{=} 1$

$\log_5\left(\frac{\frac{35}{3}}{\frac{7}{3}}\right) \overset{?}{=} 1$

$\log_5 5 \overset{\checkmark}{=} 1$

(c) $\ln(2x+4)=5$

$\Longleftrightarrow \quad 2x+4=e^5$

$2x=e^5-4$

$x=\dfrac{e^5-4}{2}$

Check:

$\ln(e^5-4+4)=5$

$\ln(e^5)=5 \checkmark$

(d) $\dfrac{15}{100-e^{2x}}=3$ Note: $100-e^{2x}\neq 0$ $100\neq e^{2x}$ $\rightarrow$ $\ln(100)\neq 2x$

$\dfrac{\ln(100)}{2}\neq x$

$15=3(100-e^{2x})$

$15=300-3e^{2x}$

$-285=-3e^{2x}$

$\dfrac{285}{3}=e^{2x}$

$e^{2x}=95$

$2x=\ln(95)$ $\quad \boxed{x=\frac{1}{2}\ln(95)}$

Check:

$\dfrac{15}{100-e^{\ln(95)}}=\dfrac{15}{100-95}=3 \checkmark$

(e) $9 \cdot 3^{x^2-1} = 27^x$

$3^2 \times 3^{x^2-1} = 3^{3x}$

$x^2+1 = 3x$

$x^2 - 3x + 1 = 0$

$x = \dfrac{3 \pm \sqrt{9-4}}{2}$

$x = \dfrac{3+\sqrt{5}}{2}$

$x = \dfrac{3-\sqrt{5}}{2}$

Check:

$x = \dfrac{3+\sqrt{5}}{2}$

$9 \times 3^{\left(\frac{3+\sqrt{5}}{2}\right)^2 - 1} \stackrel{?}{=} 27^{\frac{3+\sqrt{5}}{2}}$

$3^{\frac{(3+\sqrt{5})^2}{4} - 1 + 2} \stackrel{?}{=} 3^{3\left(\frac{3+\sqrt{5}}{2}\right)}$

$3^{\frac{(3+\sqrt{5})^2}{4} + 1} \stackrel{?}{=} 3^{\frac{9+3\sqrt{5}}{2}}$

$3^{\frac{9+5+6\sqrt{5}}{4} + \frac{4}{4}}$

$3^{\frac{18+6\sqrt{5}}{4}} \stackrel{\checkmark}{=} 3^{\frac{9+3\sqrt{5}}{2}}$

$x = \dfrac{3-\sqrt{5}}{2}$

$9 \times 3^{\left(\frac{3-\sqrt{5}}{2}\right)^2 - 1} \stackrel{?}{=} 27^{\frac{3-\sqrt{5}}{2}}$

$3^{\frac{(3-\sqrt{5})^2}{4} - 1 + 2} \stackrel{?}{=} 3^{3\left(\frac{3-\sqrt{5}}{2}\right)}$

$3^{\frac{(3-\sqrt{5})^2}{4} + 1} \stackrel{?}{=} 3^{\frac{9-3\sqrt{5}}{2}}$

$3^{\frac{9+5-6\sqrt{5}}{4} + \frac{4}{4}}$

$3^{\frac{18-6\sqrt{5}}{4}} \stackrel{\checkmark}{=} 3^{\frac{9-3\sqrt{5}}{2}}$

(f) $e^{2x} + 7e^x - 18 = 0$

$(e^x)^2 + 7(e^x) - 18 = 0$

let $t = e^x$

$t^2 + 7t - 18 = 0$

$(t+9)(t-2) = 0$

$t = -9$

$e^x = -9$

(no solution)

$t = 2$

$e^x = 2$

$\boxed{x = \ln 2}$

Check:

$x = \ln 2$

$e^{2\ln 2} + 7e^{\ln 2} - 18$

$= e^{\ln 4} + 7(2) - 18$

$= 4 - 4 = 0$ ✓

(g) $\log_5(4x) = 3$

$\iff 4x = 5^3$

$x = \frac{5^3}{4} = \frac{125}{4}$

Check:

$\log_5\left(\cancel{4}\left(\frac{125}{\cancel{4}}\right)\right) = 3$ ✓

(h) $\log_3(x-1) + \log_3(x+4) = 0$

$\log_3((x-1)(x+4)) = 0$

$\iff (x-1)(x+4) = 1$

$x^2 + 3x - 4 = 1$

$x^2 + 3x - 5 = 0$

$x = \frac{-3 \pm \sqrt{9 - 4(-5)}}{2} = \frac{-3 \pm \sqrt{29}}{2}$

Check:

✓ $\boxed{x = \frac{-3+\sqrt{29}}{2}}$ $> \frac{-3+5}{2} = 1$

all arguments remain positive

$\log_3\left(\frac{-3+\sqrt{29}}{2} - 1\right) + \log_3\left(\frac{-3+\sqrt{29}}{2} + 4\right)$

$= \log_3\left(\frac{-3+\sqrt{29}-2}{2}\right) + \log_3\left(\frac{-3+\sqrt{29}+8}{2}\right)$

$= \log_3\left(\frac{-5+\sqrt{29}}{2}\right) + \log_3\left(\frac{5+\sqrt{29}}{2}\right)$

$= \log_3\left(\frac{\sqrt{29}-5}{2} \cdot \frac{\sqrt{29}+5}{2}\right) = \log_3\left(\frac{29-25}{4}\right) = \log_3\left(\frac{4}{4}\right) = \log_3 1 = 0$

$x = \frac{-3-\sqrt{29}}{2} < \frac{-3-5}{2} = -4$

$\log_3\left(\underbrace{\frac{-3-\sqrt{29}}{2} - 1}_{<0}\right) + \log_3\left(\underbrace{\frac{-3-\sqrt{29}}{2} + 4}_{<0}\right) = 0$

not in domain

not a solution

(i) $\dfrac{2}{x-1} - \dfrac{5}{x+2} = \dfrac{10}{x^2 + \underbrace{x-2}}$ $(x+2)(x-1)$

Note: $x \neq 1$, $x \neq -2$

$2(x+2) - 5(x-1) = 10$

$2x + 4 - 5x + 5 = 10$

$-3x + 9 = 10$

$-3x = 1$

$x = -\frac{1}{3}$

Check: $x = -\frac{1}{3}$

$$\frac{2}{-\frac{1}{3} - 1} - \frac{5}{-\frac{1}{3} + 2} \stackrel{?}{=} \frac{10}{(-\frac{1}{3})^2 + (-\frac{1}{3}) - 2}$$

RHs: $\dfrac{2}{\frac{-1-3}{3}} - \dfrac{5}{\frac{-1+6}{3}} = \dfrac{6}{-4} - \dfrac{15}{5}$

$= -\frac{3}{2} - 3 = -\frac{3-6}{2}$

$= \frac{-9}{2}$

LHs: $\dfrac{10}{\frac{1}{9} - \frac{1}{3} - 2} = \dfrac{10}{\frac{1-3-18}{9}}$

$= \dfrac{10 \times 9}{-20} = \boxed{-\frac{9}{2}}$

(j) $\sqrt[5]{x-2} - 1 = 0$

$\sqrt[5]{x-2} = 1$

$x - 2 = 1$

$x = 3$

Check: $x = 3$

$\sqrt[5]{3-2} - 1 \stackrel{\checkmark}{=} 0$

(k) $\left|\dfrac{3x}{x^2-9}\right| = \left|\dfrac{1}{x-3}\right|$

Note: $x \neq 3$ & $x \neq -3$

$$\frac{3x}{x^2-9} = \frac{1}{x-3}$$

$$(x^2-9)\left(\frac{3x}{x^2-9}\right) = \left(\frac{1}{x-3}\right)(x^2-9)$$

$$3x = x+3$$

$$2x = 3$$

$$x = \frac{3}{2}$$

$$\frac{3x}{x^2-9} = -\frac{1}{x-3}$$

$$(x^2-9)\left(\frac{3x}{x^2-9}\right) = \left(\frac{-1}{x-3}\right)(x^2-9)$$

$$3x = -(x+3)$$

$$3x = -x-3$$

$$4x = -3$$

$$x = -\frac{3}{4}$$

Check:

$x = \frac{3}{2}$

$$\left|\frac{3\left(\frac{3}{2}\right)}{\left(\frac{3}{2}\right)^2-9}\right| \overset{?}{=} \left|\frac{1}{\frac{3}{2}-3}\right|$$

$$\left|\frac{\frac{9}{2}}{\frac{9}{4}-9}\right| \overset{?}{=} \left|\frac{1}{\frac{-3}{2}}\right|$$

$$\left|\frac{\frac{9}{2}}{\frac{9-36}{4}}\right| \overset{?}{=} \left|-\frac{2}{3}\right|$$

$$\left|\frac{18}{-27}\right| \overset{?}{=} \frac{2}{3} \checkmark$$

$x = -\frac{3}{4}$

$$\left|\frac{3\left(-\frac{3}{4}\right)}{\frac{9}{16}-9}\right| \overset{?}{=} \left|\frac{1}{-\frac{3}{4}-3}\right|$$

LHS $\left|\dfrac{\frac{-9}{4}}{\frac{9-144}{16}}\right| = \left|\dfrac{-\cancel{9}}{\cancel{4}} \times \dfrac{\cancel{16}^{4}}{\cancel{135}_{15}}\right| = \dfrac{4}{15}$

RHS $\left|\dfrac{1}{\frac{-3-12}{4}}\right| = \dfrac{4}{15}$

(l) $16 = \dfrac{2^{3x-5}}{4^{2x+1}}$

$$2^4 = \frac{2^{3x-5}}{(2^2)^{2x+1}}$$

$$2^4 = 2^{3x-5-2(2x+1)}$$

$$2^4 = 2^{3x-5-4x-2}$$

$$2^4 = 2^{-7-x}$$

$$4 = -7-x$$

$$\boxed{x = -11}$$

Check:

$x = -11$

$$\frac{2^{3(-11)-5}}{4^{2(-11)+1}} = \frac{2^{-33-5}}{4^{-22+1}}$$

$$= \frac{2^{-38}}{4^{-21}} = \frac{2^{-38}}{2^{-42}}$$

$$= 2^{-38-(-42)} = 2^4 = 16 \checkmark$$

7. Use properties of logarithms to write the following as a single logarithm.

(a) $2\left(\log_5(x) + 2\log_5(y) - 3\log_5(z)\right) = 2\log_5(x) + 4\log_5(y) - 6\log_6(z)$

$$= \log_5(x)^2 + \log_5(y)^4 - \log_5(z)^6$$

$$= \log_5\left(\frac{x^2 \cdot y^4}{z^6}\right)$$

(b) $\dfrac{1}{3}\log(x+2)^3 + \dfrac{1}{2}\left(\log(x)^4 - \log(x^2 - x - 6)^2\right)$

$$= \log\left((x+2)^3\right)^{\frac{1}{3}} + \frac{1}{2}\log(x)^4 - \frac{1}{2}\log(x^2 - x - 6)^2$$

$$= \log\left((x+2)^3\right)^{\frac{1}{3}} + \log\left((x)^4\right)^{\frac{1}{2}} - \log\left((x^2 - x - 6)^2\right)^{\frac{1}{2}}$$

$$= \log(x+2) + \log(x)^2 - \log(x^2 - x - 6)$$

$$= \log\left(\frac{(x+2) \cdot x^2}{x^2 - x - 6}\right) = \log\left(\frac{x^2}{x-3}\right)$$

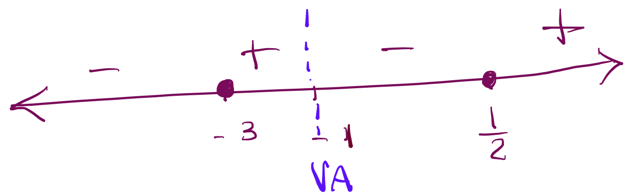
8. Find the intervals where the inequalities are true.

(a) $\dfrac{2x^2+5x-3}{x+1} \geq 0$

$2x^2+5x-3=0$ $\qquad$ $x+1 \neq 0$

$(2x-1)(x+3)=0$ $\qquad$ $x \neq -1$

$x=\frac{1}{2}$ & $x=-3$

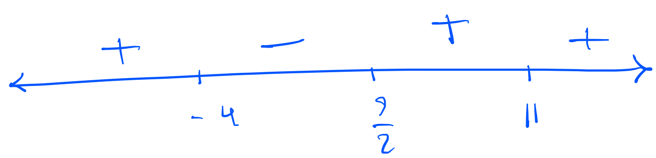


$x \in [-3,-1) \cup [\frac{1}{2}, +\infty)$

(b) $\underbrace{(2x-9)(11-x)^6(x+4)^3}_{y} < 0$

$y=0$ when:

$x=\frac{9}{2}$ $\qquad$ $x=11$ $\qquad$ $x=-4$



$x \in (-4, \frac{9}{2})$

(c) $2x(2x-3)^{-2} \leq 4(2x-3)^{-3}$

$\dfrac{2x}{(2x-3)^2} \leq \dfrac{4}{(2x-3)^3}$

$\underbrace{\dfrac{2x}{(2x-3)^2} - \dfrac{4}{(2x-3)^3}}_{y} \leq 0$

where $y \overset{?}{=} 0$ $\qquad$ $\dfrac{2x}{(2x-3)^2} - \dfrac{4}{(2x-3)^3} = 0$

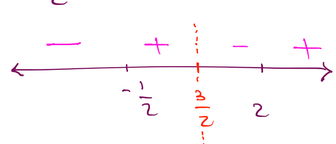
$2x(2x-3) - 4 = 0$

$4x^2 - 6x - 4 = 0$

$2(\underbrace{2x^2-3x-2}) = 0$

$(2x+1)(x-2)=0$

$x=-\frac{1}{2}$, $x=2$

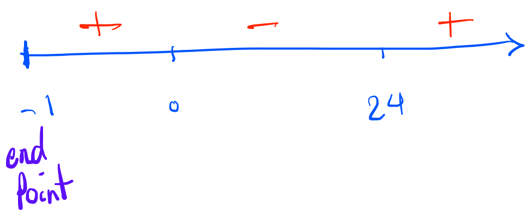


$x \in (-\infty, -\frac{1}{2}] \cup (\frac{3}{2}, 2]$

(d) $t\sqrt{t+1} \geq 5t$

Note: $t+1 \geq 0 \to t \geq -1$

$\underbrace{t\sqrt{t+1} - 5t}_{y} \geq 0$



$y=0 \to t\sqrt{t+1} - 5t = 0$

$t(\sqrt{t+1} - 5) = 0$

$t = 0$

$\sqrt{t+1} = 5$

$t+1 = 25$

$t = 24$

$x \in [-1, 0] \cup [24, \infty)$

9. Rewrite (expand) the following logarithmic expressions as a sum and/or difference of logarithms with linear arguments.

(a) $\log\left(\dfrac{10x}{(x+17)^2(x-9)}\right) = \log(10x) - \log\left((x+17)^2(x-9)\right)$

$= \log(10x) - \left(\log(x+17)^2 + \log(x-9)\right)$

$= \log(10) + \log x - 2\log(x+17) - \log(x-9)$

$= 1 + \log x - 2\log(x+17) - \log(x-9)$

(b) $\ln\left(\dfrac{x^5 \cdot (y+1)^{-2}}{a^{-3} \cdot (p-2)^4}\right)$ $= \ln\left(x^5 \cdot (y+1)^{-2}\right) - \ln\left(a^{-3} \cdot (p-2)^4\right)$

$= \ln(x)^5 + \ln(y+1)^{-2} - \left[\ln(a)^{-3} + \ln(p-2)^4\right]$

$= 5\ln(x) - 2\ln(y+1) - \left[-3\ln(a) + 4\ln(p-2)\right]$

$= 5\ln(x) - 2\ln(y+1) + 3\ln a - 4\ln(p-2)$

(c) $\log_2\left(\sqrt[3]{\dfrac{x^2}{x^2-8x-20}}\right)$

$= \log_2\left(\dfrac{x^2}{x^2-8x-20}\right)^{\frac{1}{3}} = \frac{1}{3}\log_2\left(\dfrac{x^2}{x^2-8x-20}\right)$

$= \frac{1}{3}\left[\log_2 (x)^2 - \log_2 (x^2-8x-20)\right] = \frac{1}{3}\left[2\log_2 x - \log_2((x-10)(x+2))\right]$

$= \frac{1}{3}\left[2\log_2 x - \left(\log_2(x-10) + \log_2(x+2)\right)\right] = \frac{2}{3}\log_2 x - \frac{1}{3}\log_2(x-10) - \frac{1}{3}\log_2(x+2)$

10. State domain of the following functions.

(a) $h(t) = 5^{\frac{3x+5}{x+1}}$

restriction: denominator

$x+1 \neq 0 \rightarrow x \neq -1$

domain $x \in (-\infty, -1) \cup (-1, \infty)$

(b) $h(x) = \dfrac{\sqrt[4]{5x+1}}{\sqrt{e^x - 1}}$.

restriction: even root & denom.

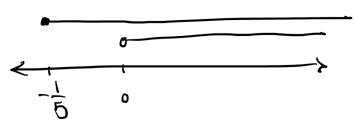
① $\sqrt[4]{5x+1}$: $5x+1 \geq 0 \rightarrow x \geq -\frac{1}{5}$

② $\sqrt{e^x - 1}$: $e^x - 1 \geq 0$

③ $\sqrt{e^x - 1}$ in denominator: $\sqrt{e^x - 1} \neq 0 \rightarrow e^x - 1 \neq 0$

$\Big\}$ $e^x - 1 > 0 \rightarrow e^x > 1$

$\rightarrow x > \ln 1 = 0$



$x \in (0, \infty)$

(c) $f(x) = \log_{11}(a - x) + 4x^2$

restriction: log

$a - x > 0 \rightarrow a > x$

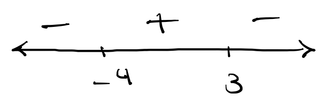
$x \in (-\infty, a)$

(d) $f(x) = \log\left(\dfrac{9-3x}{x+4}\right)$

$\dfrac{9-3x}{x+4} > 0$

$9-3x=0 \rightarrow x=3$

$x+4 \neq 0 \rightarrow x \neq -4$



$x \in (-4, 3)$

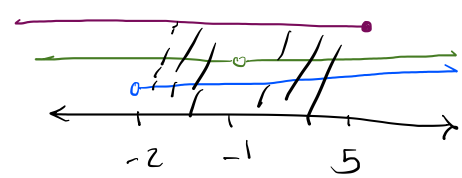
(e) $f(x) = \dfrac{\sqrt{5-x}+e^{3x}}{\log_3(x+2)}$

restrictions: even root, denom, log

① $\sqrt{5-x}$: $5-x \geq 0 \longrightarrow 5 \geq x$

② $\log_3(x+2)$: $x+2>0 \longrightarrow x>-2$

③ denom: $\log_3(x+2) \neq 0 \rightarrow x+2 \neq 1 \rightarrow x \neq -1$



$x \in (-2,-1) \cup (-1, 5]$

11. Given $f(t) = -5(1+t)^{\frac{3}{2}} + 2$, evaluate the following.

even root

$1+t \geq 0 \longrightarrow t \geq -1$

Domain: $[-1, +\infty)$

Vertical asymptote(s): None!

$\underbrace{-5\ \overbrace{(1+t)^{\frac{3}{2}}}^{\to +\infty}}_{\to -\infty} + 2 \to -\infty$

as $x \to +\infty$, $f(t) \longrightarrow -\infty$

End behavior: as $x \to -1$, $f(t) \to 2$

end point: $(-1, 2)$

Horizontal asymptotes: None!

Intercept(s): x-int.: $\left(\left(\frac{2}{5}\right)^{\frac{2}{3}} - 1, 0\right)$

$-5(1+t)^{\frac{3}{2}} + 2 = 0$

$-5(1+t)^{\frac{3}{2}} = -2$

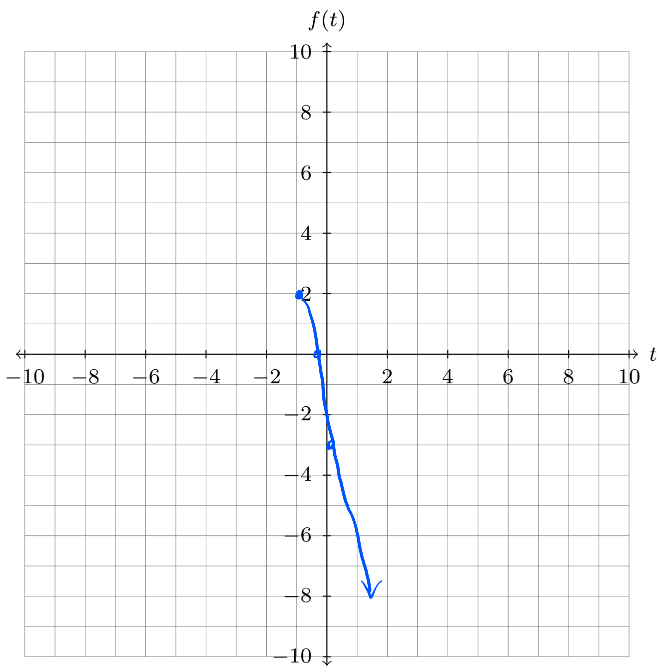
$(1+t)^{\frac{3}{2}} = \frac{2}{5}$

$\left((1+t)^{\frac{3}{2}}\right)^{\frac{2}{3}} = \left(\frac{2}{5}\right)^{\frac{2}{3}}$

$1+t = \left(\frac{2}{5}\right)^{2/3}$

$t = \left(\frac{2}{5}\right)^{2/3} - 1$

y-int: $(0, -3)$



12. Given the function $f(x) = \dfrac{(x+4)(2x+1)}{(2x+1)(x-5)}$, evaluate the following.

$x \neq 5$ & $x \neq -\frac{1}{2}$

Domain: $x \in (-\infty, -\frac{1}{2}) \cup (-\frac{1}{2}, 5) \cup (5, \infty)$

$f(x) = \dfrac{(x+4)\cancel{(2x+1)}}{\cancel{(2x+1)}(x-5)} = \dfrac{x+4}{x-5}$ (for $x \neq -\frac{1}{2}$)

Hole(s): $\left(-\frac{1}{2}, -\frac{7}{11}\right)$

Vertical asymptote(s): $x = 5$

End behavior: as $x \to +\infty$, $f(x) \to 1$
as $x \to -\infty$, $f(x) \to 1$

$\dfrac{x+4}{x-5} = \dfrac{\frac{x}{x} + \frac{4}{x}}{\frac{x}{x} - \frac{5}{x}} = \dfrac{1 + \frac{4}{x}}{1 - \frac{5}{x}} \to \dfrac{1 + 0^+}{1 - 0^+}$

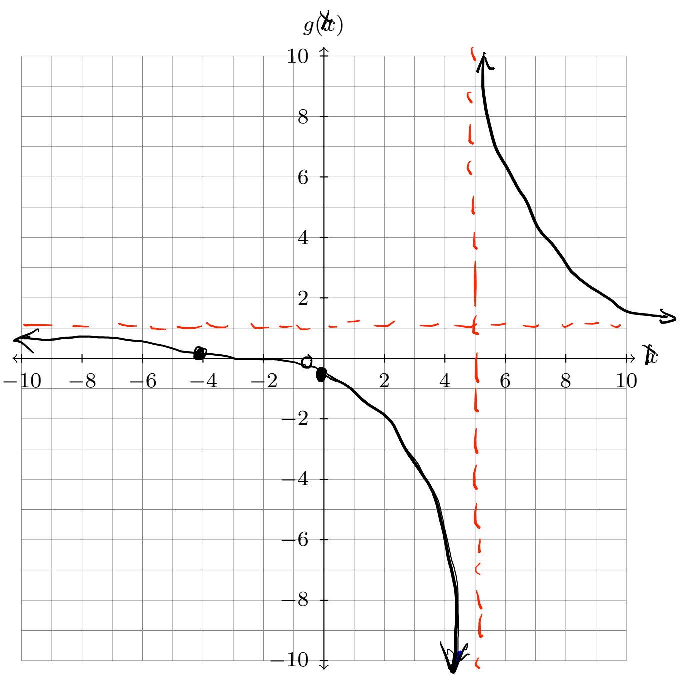
Horizontal asymptotes: $y = 1$

Intercept(s): x-int.: $(-4, 0)$ y-int.: $(0, -\frac{4}{5})$

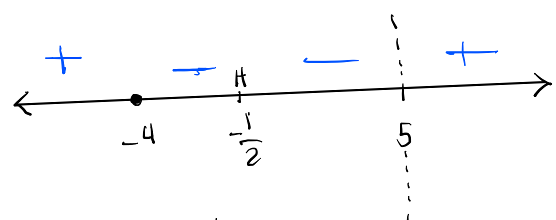
$\dfrac{x+4}{x-5} = 0$

$x = -4$

$\dfrac{0+4}{0-5} = -\dfrac{4}{5}$



Sign chart



behavior around V.A.:

as $x \to 5^-$, $f(x) \to +\infty$

as $x \to 5^-$, $f(x) \to -\infty$

13. Given $g(x) = 2\,(e)^{8-2x} + 5$, evaluate the following.

Domain: $(-\infty, \infty)$

Vertical asymptote(s): None

End behavior: $x \to +\infty$, $f(x) \to 5$; $x \to -\infty$, $f(x) \to \infty$

$$2\,\underbrace{(e)^{\overbrace{8-2x}^{\to -\infty}}}_{\to 0} + 5 \to 0 + 5 = 5$$

$$2\,\underbrace{(e)^{\overbrace{8-2x}^{\to \infty}}}_{\to \infty} + 5 \to \infty$$

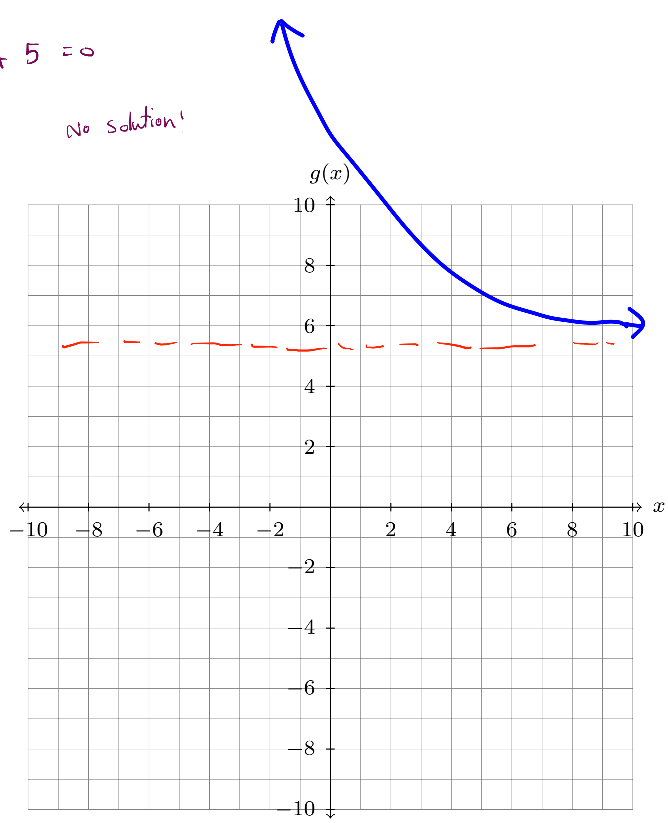
Horizontal asymptotes: $y = 5$

Intercept(s): x-int: None. y-int: $(0, 2e^8 + 5)$

$2e^{8-2x} + 5 = 0$

$e^{8-2x} = -\frac{5}{2}$ No solution!

$g(0) = 2e^{8-0} + 5 = 2e^8 + 5$



14. Given $h(x) = -\log_3(4-2x)+2$, evaluate the following.

Domain: $(-\infty, 2)$

$4-2x>0 \rightarrow x<+2$

argument $=0$

Vertical asymptote(s): $x=+2$

$\underbrace{-\log_3(\overbrace{4-2x}^{\to 0^+})}_{\to +\infty}+2 \to +\infty$

End behavior: as $x \to 2^-$ (from left), $h(x) \to +\infty$

as $x \to -\infty$, $h(x) \to -\infty$

$\underbrace{-\log_3(\underbrace{4-2x}_{\to\infty})}_{\to -\infty} + 2 \to -\infty$

Horizontal asymptotes: None!

x-int: $(-\frac{5}{2}, 0)$

y-int: $(0, -\log_3 4 + 2)$

Intercept(s): $\cancel{-}\log_3(4-2x) = \cancel{-}2$

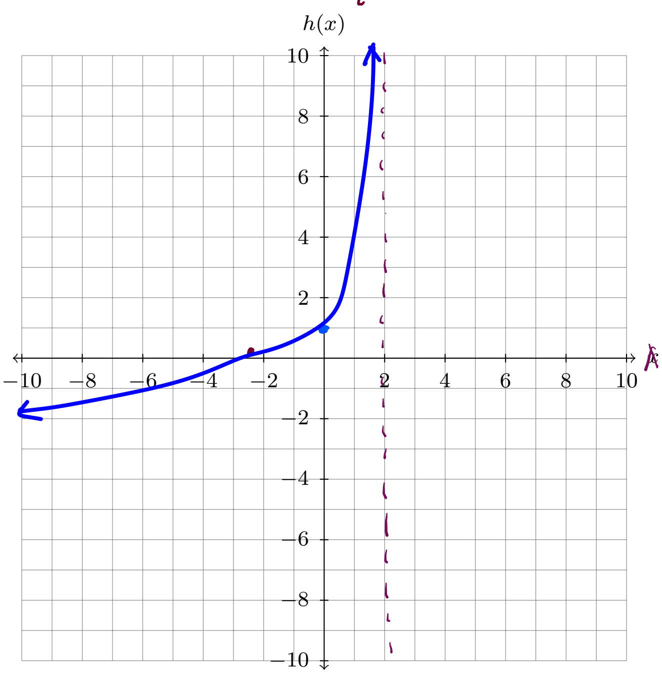
$4-2x=9$

$2x=-5$

$x=-\frac{5}{2}$

$-\log_3(4-2(0))+2 = -\underbrace{\log_3 4}_{>1}+2$

$0<$ <1



15. Compute and completely simplify the difference quotient for $f(x) = \sqrt{1-5x}$ using the techniques discussed in class.

① $f(x+h) = \sqrt{1-5x-5h}$

② $f(x+h) - f(x) = \sqrt{1-5x-5h} - \sqrt{1-5x}$

③ $\dfrac{f(x+h)-f(x)}{h} = \dfrac{\sqrt{1-5x-5h} - \sqrt{1-5x}}{h} \times \dfrac{\sqrt{1-5x-5h} + \sqrt{1-5x}}{\sqrt{1-5x-5h} + \sqrt{1-5x}}$

$= \dfrac{\cancel{1}-\cancel{5x}-5h-(\cancel{1}-\cancel{5x})}{h\left(\sqrt{1-5x-5h}+\sqrt{1-5x}\right)} = \dfrac{-5\cancel{h}}{\cancel{h}\left(\sqrt{1-5x-5h}+\sqrt{1-5x}\right)}$

$= \dfrac{-5}{\sqrt{1-5x-5h}+\sqrt{1-5x}}$

16. Compute and completely simplify the difference quotient for $g(x) = \dfrac{2}{1-x^2}$ using the techniques discussed in class.

① $g(x+h) = \dfrac{2}{1-(x+h)^2} = \dfrac{2}{1-x^2-h^2-2xh}$

Common denom.

② $g(x+h) - g(x) = \dfrac{2}{1-x^2-h^2-2xh} - \dfrac{2}{1-x^2} = \dfrac{2(1-x^2) - 2(1-x^2-h^2-2xh)}{(1-x^2-h^2-2xh)(1-x^2)}$

$= \dfrac{\cancel{2}-\cancel{2x^2}-\cancel{2}+\cancel{2x^2}+2h^2+4xh}{(1-x^2-h^2-2xh)(1-x^2)} = \dfrac{2h^2+4xh}{(1-x^2-h^2-2xh)(1-x^2)}$

③ $\dfrac{g(x+h)-g(x)}{h} = \dfrac{\dfrac{2h^2+4xh}{(1-x^2-h^2-2xh)(1-x^2)}}{h} = \dfrac{2h^2+4xh}{h(1-x^2-h^2-2xh)(1-x^2)} = \dfrac{\cancel{h}(2h+4x)}{\cancel{h}(1-x^2-h^2-2xh)(1-x^2)}$

$= \dfrac{2h+4x}{(1-x^2-h^2-2xh)(1-x^2)}$

17. simplify the following

(a) $(2^5)^{\log_2(3)}$

$= 2^{5\log_2 3} = 2^{\log_2 (3)^5} = (3)^5 = \boxed{243}$

(b) $\log_{2^3}(2^8)$

$= 8 \log_{2^3}(2) = 8 \log_{2^3}(2^3)^{\frac{1}{3}} = \frac{8}{3} \log_{2^3}(2)^3 = \boxed{\frac{8}{3}}$