Example 1 (14.1). *Find and sketch the domain of the function.*

(a) $f(x, y) = \sqrt{xy} + e^{\sqrt{2-x}} + 3.$

(b) $f(x, y) = \dfrac{\sqrt{4 - x^2 - y^2}}{x + y}.$

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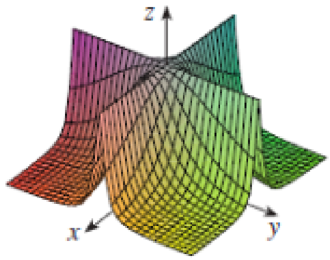
Example 2 (14.1)**.** *Draw the level curves* $z = k$ *when* $k = 1, 2, 3$ *of the function*

$$f(x, y) = \ln(x^2 + y^2 - 9),$$

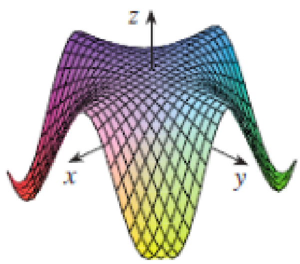
and sketch its graph.

Example 3 (14.1)**.** *Match the graph with its function.*

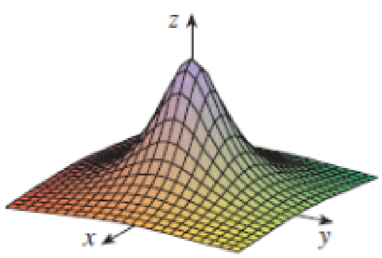
I.



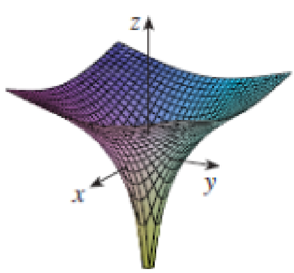
II.



III.



IV.



(A) $f(x,y) = \dfrac{1}{1+x^2+y^2}$

(B) $f(x,y) = \dfrac{1}{1+x^2y^2}$

(C) $f(x,y) = \ln(x^2+y^2)$

(D) $f(x,y) = \cos\sqrt{x^2+y^2}$

(E) $f(x,y) = \cos(xy)$

Example 4 (14.3). *Find the first partial derivative of the function.*

(a) $f(x,y) = \dfrac{\ln x}{(x+y)^2}$

(b) $W(p,q) = \displaystyle\int_p^q \sqrt{1+t^5}\,dt$

(a) $F(r,s,t) = r\tan(s+3t) + 2t.$

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Example 5 (14.3)**.** *Suppose* $f(x, y) = \sqrt{9 - x^2 - 9y^2}$*. Find* $f_x(2, 0)$ *and* $f_y(2, 0)$ *and interpret these values as slopes.*

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Example 6 (14.3/14.4). *Suppose* $f(x, y) = e^{xy} - 3x^2y$.

(a) Find $f_x(0, -3)$ *and* $f_y(0, -3)$.

(b) Is f *differentiable at* $(0, -3)$? *Explain.*

(c) Find all the second order partial derivatives. Is f_{xy} *equal to* f_{yx}*? Explain why.*

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Tangent Planes *Suppose f has continuous partial derivatives. An equation of the tangent plane to the surface $z = f(x, y)$ at the point $P(x_0, y_0, z_0)$ is*

$$z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

or

$$z = f(x_0, y_0) + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

The linear function

$$L(x, y) = f(a, b) + f_x(a, b)(x - a) + f_y(a, b)(y - b)$$

is called the linearization of f at (a, b), and the approximation

$$f(x, y) \approx f(a, b) + f_x(a, b)(x - a) + f_y(a, b)(y - b)$$

is called the linear approximation or the **tangent plane approximation** *of f at (a, b).*

Example 7 (14.4)**.** *Find an equation of the tangent plane at the point $P(4, 1, 2)$ on the surface $z = \ln(x - 3y) + 2y$.*

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Example 8 (14.4)**.** *Consider the function*

$$f(x, y, z) = \sqrt{x^2 + y^2 + z^2}.$$

(a) Find the differential of the function.

(b) Find the linearization of the function at the point $(2, 1, 2)$.

(c) Use the differential or linearization to estimate $\sqrt{1.9^2 + 1.1^2 + 2.01^2}$.

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Example 9 (14.4)**.** *If* $z = x^2 + 2y^2 - x$ *and* (x, y) *changes from* $(2, 3)$ *to* $(1.9, 3.01)$, *compare the values* Δz *and* dz.

The Chan Rule Suppose that $z = f(x, y)$ *is a differentiable function of* x *and* y, *where* $x = g(t)$ *and* $y = h(t)$ *are both differentiable functions of* t. *Then* z *is a differentiable function of* t *and*

$$\frac{dz}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$$

Suppose that $z = f(x, y)$ *is a differentiable function of* x *and* y, *where* $x = g(s, t)$ *and* $y = h(s, t)$ *are differentiable functions of* s *and* t. *Then*

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s}$$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t}$$

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Example 10 (14.5)**.** *The length l, width w, and height h of a rectangular box are changing with respect to time t. At a certain instance the dimensions are $l = 5$ cm, $w = 4$ cm and $h = 10$ cm, and length and width are increasing at a rate of 4 cm/s while the height is decreasing at a rate of 3 cm/s. At that instant find the rate at which the volume of the box is changing.*

Example 11 (14.5)**.** *If $z = \tan^{-1}(x^2 + y^2)$, $x = s \ln t$, $y = te^s$, find $\dfrac{\partial z}{\partial s}$ when $s = 2$ and $t = 1$.*

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Example 12 (14.5)**.** *Find* $\frac{\partial z}{\partial x}$ *and* $\frac{\partial z}{\partial y}$ *if* $x^3 + y^3 + z^3 = 3 - xyz$.

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