Exam 1 review

Tuesday, February 4, 2025 7:24 PM

Exam 1 Review over Chapters 1 and 2

- Basic Matrix Operations
- Matrix Multiplication
- Review of Lines
- Modeling with Linear Functions
- Systems of Two Equations in Two Unknowns
- Setting Up and Solving Systems of Linear Equations

E = # of Earth shares
W = # of Wind shares
F = # of Fire shares

Pr 1. Consider the following scenario:

Maurice has $\boxed{\$12{,}000}$ to invest. He decides to invest in three different companies. The Earth company costs $\boxed{\$150}$ per share and pays dividends of \$2 per share each year. The Wind company costs $\boxed{\$75}$ per share and pay dividends of \$1.00 per share each year. The Fire company costs $\boxed{\$180}$ per share and pays \$4.00 per share per year in dividends. Maurice wants to have twice as much money in the Wind company as in the Fire company. Maurice also wants to earn \$200 in dividends per year. How ~~much~~ should Maurice invest in Earth, Wind, and Fire to meet his goals?

many shares

(a) Set up the problem as a system of linear equations.

$150E + 75W + 180F = 12000$ (total invested)

$2E + 1W + 4F = 200$ (total dividends per year)

$1 \cdot W = 2F$ (ratio)

$\boxed{2F = W}$ or $2W = F$

F=1 W=2

2·1 = 2

$2 \cdot 2$ 1

F = 1 → W = 2

$W - 2F = 0$

(b) Write down the corresponding augmented matrix.

$$\begin{array}{cccc} E & W & F & con \\ \end{array}$$
$$\left[\begin{array}{ccc|c} 150 & 75 & 180 & 12000 \\ 2 & 1 & 4 & 200 \\ 0 & 1 & -2 & 0 \end{array}\right]$$

(c) Suppose that the reduced row-echelon form for our system is

$$\begin{array}{ccc} E & W & F \\ \end{array}$$
$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 25 \\ 0 & 1 & 0 & 50 \\ 0 & 0 & 1 & 25 \end{array}\right].$$

→ $E + 0W + 0F = 25 \to E = 25$

$W = 50$

$F = 25$

What is the corresponding solution to our original problem?

Maurice should buy 25 shares of Earth, 50 shares of Wind, and 25 shares of Fire.

cost → $25 \times 150 = \ldots$

Pr 2. Write the augmented matrix corresponding to the given system of linear equations. Solve the system, using technology.

$$\begin{aligned} x - 5y &= 4 \\ -2x + 2y &= 6z - 4 \\ 3z - 2 &= -4y \end{aligned}$$

$1x - 5y + 0z = 4$
$-2x + 2y - 6z = -4$
$0x + 4y + 3z = 2$

$$\begin{array}{cccc} x & y & z & \text{cons} \end{array}$$
$$\left[\begin{array}{ccc|c} 1 & -5 & 0 & 4 \\ -2 & 2 & -6 & -4 \\ 0 & 4 & 3 & 2 \end{array}\right] \xrightarrow{\text{RREF}} \left[\begin{array}{ccc|c} 1 & 0 & 15/4 & 0 \\ 0 & 1 & 3/4 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right] \to 0 = 1$$

The system is inconsistent / There is no solution / DNE

Pr 3. Determine the value of k so that the following system of linear equations has exactly one solution.

$$\begin{aligned} -x + ky &= 24 \\ 2x - 4y &= 30 \end{aligned}$$

$-x = -ky + 24$
$x = ky - 24$

$2(ky - 24) - 4y = 30$
$2ky - 48 - 4y = 30$
$2ky - 4y = 30 + 48 = 78$
$(2k - 4)y = 78$

$y = \frac{78}{2k-4} \to 2k - 4 \neq 0$

$2k - 4 \neq 0$
$\frac{2k}{2} \neq \frac{4}{2} \to \boxed{k \neq 2}$

$ax = 6$
$x = 6/a$

$10 + 15$
$2 \cdot 5 + 3 \cdot 5 = (2+3)5$

There is a unique solution for any real number k except $k = 2$

Pr 4. Solve the following system using substitution or the addition method.

$$\begin{aligned} -2x + 5y &= 19 \\ 7x + 2y &= 30 \end{aligned}$$

$7(-2x + 5y = 19)$
$2(7x + 2y = 30)$

$-14x + 35y = 133$
$+ 14x + 4y = 60$
$0x + 39y = 193$

$\boxed{y = 193/39}$ ✓

$7x + 2\left(\frac{193}{39}\right) = 30$
$7x = 30 - 2\left(\frac{193}{39}\right)$
$\frac{7x}{7} = \frac{1784}{7 \cdot 39}$
$\boxed{x = \frac{784}{273}}$

Pr 5. Determine if the following augmented matrix is in reduced row-echelon form or not.

$$\left[\begin{array}{cccc|c} 1 & -2 & 0 & 3 & -4 \\ 0 & 0 & 1 & 1 & -6 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

✓ leading non-zero entries are 1
✓ leading 1 is only non-zero in its column
zero rows at bottom
leading 1s go right as we go down

$$\left[\begin{array}{cccccc} 0 & 0 & \boxed{1} & 2 & 0 & 1 \\ 0 & 0 & 0 & 0 & \boxed{1} & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \to \text{not RREF}$$

Pr 6. Perform the following row operations $R_1 + R_2 \to R_2$ and $-2R_1 + R_3 \to R_3$ in order on the given matrix.

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 7 \\ -1 & 1 & 1 & 3 \\ 2 & 2 & -3 & 8 \end{array}\right]$$

$R_1 + R_2 \to R_2$
$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 7 \\ 0 & 1 & 3 & 10 \\ 2 & 2 & -3 & 8 \end{array}\right]$$

$-2R_1 + R_3 \to R_3$
$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 7 \\ 0 & 1 & 3 & 10 \\ 0 & 2 & -7 & -6 \end{array}\right]$$

$$\begin{array}{cccc} 1 & 0 & 2 & 7 \\ +\,-1 & 1 & 1 & 3 \\ \hline 0 & 1 & 3 & 10 \end{array}$$

$$\begin{array}{cccc} -2\cdot 1 & -2\cdot 0 & (-2)\,2 & (-2)\,7 \\ +\,2 & 2 & -3 & 8 \\ \hline -2+2 & 0+2 & -4-3 & -14+8 \\ 0 & 2 & -7 & -6 \end{array}$$ ← new R_3

Pr 7. Use row operations to transform the matrix into reduced row-echelon form.

$$\left[\begin{array}{cc|c} 1 & -2 & 5 \\ 3 & 2 & 12 \end{array}\right]$$

$c\cdot 1 + 3 = 0$?

$-3R_1 + R_2 \to R_2$
$$\left[\begin{array}{cc|c} 1 & -2 & 5 \\ 0 & 8 & -3 \end{array}\right]$$

$$\begin{array}{ccc} -3\cdot 1 & (-3)(-2) & (-3)\,5 \\ 3 & 2 & 12 \\ \hline -3+3 & 6+2 & -15+12 \\ 0 & 8 & -3 \end{array}$$

$\frac{1}{8}R_2 \to R_2$
$$\left[\begin{array}{cc|c} 1 & -2 & 5 \\ 0 & 1 & -3/8 \end{array}\right]$$

$2R_2 + R_1 \to R_1$
$$\left[\begin{array}{cc|c} 1 & 0 & 17/4 \\ 0 & 1 & -3/8 \end{array}\right]$$

$$\begin{array}{ccc} 1 & -2 & 5 \\ 2\cdot 0 & 2\cdot 1 & 2(-3/8) \\ \hline 1+0 & -2+2 & 5-3/4 \\ 1 & 0 & 5\,3/4 \end{array}$$ $5-3/4$

Pr 8. Solve the system of linear equations, using technology.

$$2x + 2y - 4z = 24$$
$$x + z = -9$$
$$-y + 3z = -21$$

$2x + 2y - 4z = 24$
$x + 0y + z = -9$
$0x - y + 3z = -21$

$$\begin{array}{ccc|c} x & y & z & \text{con} \\ \hline 2 & 2 & -4 & 24 \\ 1 & 0 & 1 & -9 \\ 0 & -1 & 3 & -21 \end{array} \xrightarrow{\text{RREF}} \begin{array}{ccc|c} x & y & z & \text{con} \\ \hline 1 & 0 & 1 & -9 \\ 0 & 1 & -3 & 21 \\ 0 & 0 & 0 & 0 \end{array}$$

→ $x + z = -9$
$y - 3z = 21$
$x = -z - 9$ ($z = t$)
$y = 3z + 21$
$z = t$

Pr 9. Assume your solution to a real-world application problem was $(x, y, z) = (5 - t, -6 + 2t, t)$. If x, y, and z represent the number of whole items produced, how many solutions does the problem actually have?

if RREF(A) $\begin{matrix} 1 & 0 \\ 0 & 0 & 1 \end{matrix}$ → $y = t$ would be parameter

$(-t - 9,\ 3t + 21,\ t)$, t is any real #

$x \ge 0$ $5 - t \ge 0$ → $5 \ge t$ or $t \le 5$
$y \ge 0$ $-6 + 2t \ge 0$ → $\frac{2t}{2} \ge \frac{6}{2}$ → $t \ge 3$
$z \ge 0$ $t \ge 0$

$3 \le t \le 5$

$t = 3, 4,$ or 5

There are only three solutions

Pr 10. Consider the equation $2x + 3y = 5$. If x decreases by 6 units, what is the corresponding change in y?

$\Delta x = -6$ Solve for Δy

$m = \frac{\Delta y}{\Delta x}$

$$3y = -2x + 5$$
$$y = -\frac{2}{3}x + \frac{5}{3}$$

$$(-6)\left(-\frac{2}{3}\right) = \frac{\Delta y}{\Delta x} = \frac{\Delta y}{\cancel{-6}} \cdot \cancel{(-6)} = \Delta y$$

$$\frac{(-6)(-2)}{3} = \frac{12}{3} = 4 = \Delta y$$

$\boxed{\text{y increases by 4 units.}}$

Pr 11. An automobile purchased by the manager of a firm at a price of \$24,950 depreciates to a scrap value of \$2,000 at the end of ten years. Using a linear model, determine the rate of depreciation of the car.

scrap value

$V(t) = mt + b$ $\quad |m| =$ rate of depreciation $\quad$ \$24,950

$$-\frac{9}{6} = \frac{9}{-6}$$

$$m = \frac{24950 - 2000}{0 - 10} = \frac{22950}{-10} = -2295$$

$t =$ time in years

$\boxed{\$2,295 \text{ per year}}$

Pr 12. Dave sells lemonade at his lemonade stand. He makes the lemonade at a production cost of \$.15 per cup. When he sells 30 cups in a day, then his profit is \$27. When he sells 50 cups in a day, then his cost for that day is \$21.

profit $\quad$ cost

(a) Determine the linear cost function $C(x)$, in dollars, for making x cups of lemonade.

$C(x) = mx + F \quad F = ? \quad m = .15$

$\boxed{C(x) = .15x + F}$

$$21 = C(50) = .15(50) + F$$

$$21 = .15 \times 50 + F$$
$$21 = 7.5 + F$$
$$-7.5 \quad -7.5$$
$$F = 13.5$$

$\boxed{C(x) = .15x + 13.5}$

(b) Determine the linear revenue function, $R(x)$, in dollars, for selling x cups of lemonade.

Know cost function

profit @ 30 cups

$P = R - C \to R = P + C$

$$R(30) = P(30) + C(30)$$
$$= 27 + \boxed{.15 \times 30 + 13.5}$$
$$= 27 + 4.5 + 13.5 = 45$$

$R(x) = px \quad$ price

$$\frac{45}{30} = \frac{p \cdot 30}{30}$$
$$p = \frac{45}{30} = 1.5$$

$\boxed{R(x) = 1.5x}$

(c) Determine the linear profit function, $P(x)$, in dollars, for making and selling x cups of lemonade.

$$P = R - C = 1.5x - (.15x + 13.5)$$
$$= 1.5x - .15x - 13.5$$
$$= (1.5 - .15)x - 13.5$$

$\boxed{P(x) = 1.35x - 13.5}$

(d) Determine and interpret the break-even point.

where $R(x) = C(x)$

$$1.5x = .15x + 13.5$$
$$-.15x \quad -.15x$$
$$\frac{1.35x}{1.35} = \frac{13.5}{1.35}$$

$x = 10$ cups

$$R(10) = 1.5 \times 10$$
$$= \$15$$

$\boxed{(10, 15)}$

If Dave sells 10 cups of lemonade, his revenue of \$15 will meet/equal his total cost.

Demand

Pr 13. When a company sells a smartphone at \$500, they sell 2000 phones daily. If the price increases by \$100, then the company sells 400 less phones daily. The producer will provide 1500 phones if the price is \$250 and will provide 2100 phones when the price is \$350. We assume that supply and demand are linear. → $\Delta y = 100$

Demand

Supply

(a) Determine the Supply function, $S(x)$, in dollars, as a function of the supplier providing x phones.

$$S(x) = mx + b \qquad m = \frac{350-250}{2100-1500} = \frac{100}{600} = +\frac{1}{6}$$

$$S(x) - 250 = \frac{1}{6}(x - 1500)$$

$$S(x) = \frac{1}{6}x - \frac{1}{6} \times 1500 + 250$$

$$= \frac{1}{6}x - 250 + 250$$

$$\boxed{S(x) = \tfrac{1}{6}x}$$

(b) Determine the Demand function, $D(x)$, in dollars, as a function of customers demanding x phones.

(2000, \$500)

~~(400, \$100)~~

$\Delta y = 100$, then $\Delta x = -400$

$$m = \frac{\Delta y}{\Delta x} = \frac{100}{-400} = -\frac{1}{4} \qquad \boxed{D(x) = -\frac{1}{4}x + 1000}$$

$$D(x) = mx + b = -\frac{1}{4}x + b$$

$$\$500 = D(2000) = -\frac{1}{4}(2000) + b$$

$$500 = -\frac{1}{4} \times 2000 + b \rightarrow 500 = -500 + b$$

$$+500 \quad +500$$

$$b = 1000$$

(c) Determine and interpret the equilibrium point.

Supply = Demand

$$6 \cdot \frac{1}{6}x = \left(-\frac{1}{4}x + 1000\right)6 \qquad S(2400) = \frac{1}{6} \times 2400$$

$$= \$400$$

$$4 \cdot x = \left(-\frac{6}{4}x + 6000\right)4$$

$$4x = -6x + 24000 \qquad \boxed{(2400, 400)}$$

$$+6x \quad +6x$$

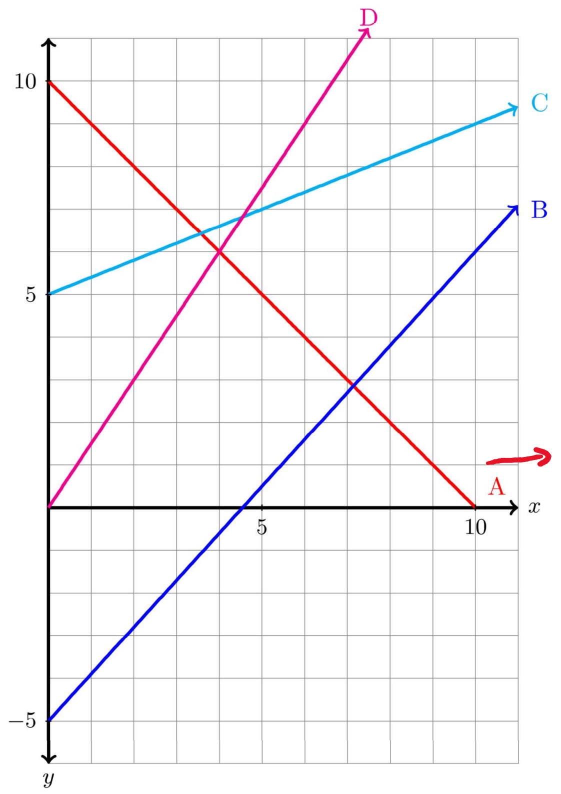
$$\frac{10x}{10} = \frac{24000}{10}$$

$x = 2400$ phones

If we sell the phones for \$400, then 2400 phones will be supplied and demanded.

Pr 14. Which of the following lines graphed below could be the graphs of a supply, demand, cost, revenue, or profit function? Explain your answer.

n.



negative slope
Demand

(a) Lines that could be graphs of Cost functions: C or D (pos. slope, $b \geq 0$)

(b) Lines that could be graphs of Revenue functions: only D (positive slope, $b=0$)

$R = px$

(c) Lines that could be graphs of Profit functions: B or D (positive slopes) (non-positive b)

(d) Lines that could be graphs of demand functions: A → negative slope

(e) Lines that could be graphs of supply functions: C or D (pos. slope; $b \geq 0$)

Pr 15. State the dimensions of the matrix $A = \begin{bmatrix} 6 & 3x & -y \\ 4w & 2 & -9 \\ -2y & 0 & 1 \\ 3x & 7 & 12w \end{bmatrix}$.

4 rows 3 columns

$\boxed{4 \times 3}$

Pr 16. State the value of b_{32} given $B = \begin{bmatrix} -2 & 4 & w \\ -2 & 9 & 13 \\ 3y & \boxed{0} & 8 \\ 0 & -7 & -2 \end{bmatrix}$.

3rd row
2nd column

$b_{32} = 0$

Pr 17. If A is a 2×3 matrix, B is a 2×3 matrix, and C is a 3×2 matrix, determine the size of $4C + (2B + 3A)^T$, if possible.

4C has size 3x2

$\left.\begin{matrix} 2B \\ 3A \end{matrix}\right\}$ size 2x3

2B + 3A is 2x3

$(2B+3A)^T$ is 3x2

$4C + (2B+3A)^T$ is $\boxed{3 \times 2}$

Pr 18. Determine the value of w, x, and y given $\begin{bmatrix} 2 & w \\ 2 & 4x \end{bmatrix} - \begin{bmatrix} y+2 & -6 \\ -8 & 12 \end{bmatrix}^T = 2\begin{bmatrix} -1 & 9 \\ 4 & -4 \end{bmatrix}$

$$\begin{bmatrix} 2 & w \\ 2 & 4x \end{bmatrix} - \begin{bmatrix} y+2 & -8 \\ -6 & 12 \end{bmatrix} = \begin{bmatrix} 2\cdot(-1) & 2\cdot 9 \\ 2\cdot 4 & 2\cdot(-4) \end{bmatrix}$$

$$\begin{bmatrix} 2-y-2 & w+8 \\ 2+6 & 4x-12 \end{bmatrix} = \begin{bmatrix} -2 & 18 \\ 8 & -8 \end{bmatrix} \rightarrow \quad -y=-2 \qquad w+8=18$$

$$8=8 \qquad 4x-12=-8$$

$$4x=4$$

$$x=1$$

$y=2$, $w=10$, $x=1$

Pr 19. If A is a 2×4 matrix, B is a 2×4 matrix, and C is a 3×2 matrix, determine the size of CAB^T, if possible.

$3\times 2 \quad 2\times 4 \quad (2\times 4)^T$

$3\times 2 \quad 2\times 4 \quad 4\times 2$

$3\times 4 \qquad 4\times 2$

match

3×2

$\boxed{3\times 2}$

Pr 20. Compute $\begin{bmatrix} -2 & 3x & 3 \\ 6w & 0 & 2y \end{bmatrix}\begin{bmatrix} -6 & 3m & 3 \\ 3n & 4 & 0 \\ -p & 0 & -1 \end{bmatrix}$.

$$\begin{bmatrix} (-2)(-6)+3x(3n)+3(-p) & -2(3m)+3x(4)+3(0) & -2(3)+3x(0)+3(-1) \\ 6w(-6)+0(3n)+2y(-p) & 6w(3m)+0(4)+2y(0) & 6w(3)+0(0)+2y(-1) \end{bmatrix}$$

$$= \begin{bmatrix} 12+9xn-3p & -6m+12x+0 & -6+0-3 \\ -36w+0-2py & 18wm+0+0 & 18w+0-2 \end{bmatrix}$$

$$= \begin{bmatrix} 12+9xn-3p & -6m+12x & -9 \\ -36w-2py & 18wm & 18w-2 \end{bmatrix}$$

Pr 21. There are three food trucks in town which sell chicken. Last week, the east store sold 115 chicken fingers, 42 baskets of fries, 63 chicken sandwiches, and 60 cans of soda. The west store sold 105 chicken fingers, 72 baskets of fries, 23 chicken sandwiches, and 140 cans of soda. The north store sold 60 chicken fingers, 43 baskets of fries, 50 cans of soda, but no chicken sandwiches. Use a 4×3 matrix to express the sales information for these three food trucks last week. → 0

items → 3 stores

(a) Then if sales at the food trucks are expected to decrease by 13% next week, use a matrix to show the expected sales for next week. → .13

$$\begin{array}{cc} \begin{array}{c} \text{fin.} \\ \text{fri.} \\ \text{Sand.} \\ \text{Soda} \end{array} & \begin{bmatrix} 115 & 105 & 60 \\ 42 & 72 & 43 \\ 63 & 23 & 0 \\ 60 & 140 & 50 \end{bmatrix} = A \\ & \begin{array}{ccc} E & W & N \end{array} \end{array}$$

$$B = \text{expected sales} = A - .13A$$

use calculator

$$= (1 - .13)A$$

$$= \underline{.87\,A}$$

$$= \begin{bmatrix} 100 & 91 & 52 \\ 36 & 62 & 37 \\ 54 & 20 & 0 \\ 52 & 121 & 43 \end{bmatrix}$$

(b) If all three trucks sell chicken fingers for $1.50, a basket of fries for $1, a can of soda for $.50, and a chicken sandwich for $2, how much did each food truck bring in last week?

$$A \begin{bmatrix} 1.5 \\ 1 \\ 2 \\ .5 \end{bmatrix} \begin{array}{l} \text{fin} \\ \text{frie} \\ \text{Sand} \\ \text{sodas} \end{array}$$

$$B = A \begin{bmatrix} 1.5 \\ 1 \\ 2 \\ .5 \end{bmatrix} =$$

wrong sizes

$$B = [1.5 \quad 1 \quad 2 \quad .5]\, A =$$

$$[\$370.50 \quad \$345.50 \quad \$158]$$

Pr 22. You have a line which passes through the points $(3, -4)$ and $\left(\frac{1}{2}, 1\right)$.

(a) Find the equation of the line in point-slope form.

$$m = \frac{1-(-4)}{\frac{1}{2}-3} = \frac{5}{-5/2} = \frac{5}{1} \div \frac{-5}{2} = \frac{5}{1} \times \frac{-2}{5} = -2$$

$$y-1 = -2\left(x-\tfrac{1}{2}\right)$$

(b) Compute the slope and the $x-$ and $y-$ intercepts. Graph the line.

x-intercept: $x=0 \rightarrow$ $y-1 = -2(0-\tfrac{1}{2}) = 1$ $\rightarrow y=2$ $(0,2)$

$+1$ $\quad +1$

y-intercept: $y=0$ $\quad 0-1 = -2(x-\tfrac{1}{2}) = -2x+1$

-1 $\quad -1$

$$\left.\frac{-2}{-2} = \frac{-2x}{-2}\right\} x=1 \qquad (1,0)$$