Week in Review
Math 152

Week 05

Trigonometric Integrals

Trigonometric Substitution

Integration by Partial Fractions

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Trigonometric Integrals

Compute $\int_0^{\pi/2} \sin^2(\theta)\cos^3(\theta)\,d\theta$.

(a) $\frac{2}{5}$

(b) $\frac{4}{5}$

(c) $\frac{2}{15}$

(d) $\frac{8}{5}$

(e) None of the above

$\int \sin^2\theta \cos^3\theta \, d\theta$

$= \int \sin^2\theta \cos^2\theta \,(\cos\theta \, d\theta)$: odd man out

$= \int \sin^2\theta \,(1-\sin^2\theta)(\cos\theta \, d\theta)$

$u = \sin\theta \;\Rightarrow$

$du = \cos\theta d\theta$

$\int_{x=0}^{x=\pi/2} \Rightarrow$

$\int_{u=0}^{u=1}$

$\int_0^1 u^2(1-u^2)du$

$= \int_0^1 (u^2 - u^4)du$

$= \left[\frac{1}{3}u^3 - \frac{1}{5}u^4\right]_0^1$

$= \frac{1}{3} - \frac{1}{5} = \frac{2}{15}$

Trigonometric Integrals

Compute $\int \cos^2(x)\sin^2(x)\,dx$

Recall $\sin 2x = 2\sin x\cos x$

$$\begin{aligned}\cos 2x &= \cos^2 x - \sin^2 x\\ &= 2\cos^2 x - 1\\ &= 1 - 2\sin^2 x\end{aligned}$$

From $\sin x\cos x = \frac{\sin 2x}{2}$

$$\begin{aligned}\int \cos^2 x\sin^2 x\,dx &= \int \left[\frac{\sin 2x}{2}\right]^2 dx\\ &= \frac{1}{4}\int \sin^2 2x\,dx\\ &= \frac{1}{4}\int \left[\frac{1-\cos 2x}{2}\right] dx\\ &= \frac{1}{8}\left[x - \frac{1}{2}\sin 2x\right] + C\end{aligned}$$

Trigonometric Integrals

The region bounded by $y = \cos x$ and the x-axis on the interval $\left[0, \frac{\pi}{2}\right]$ is rotated about the x-axis. Find the volume of the resulting solid.

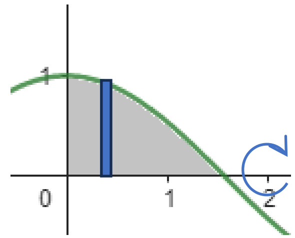
(a) $\frac{\pi}{4}$

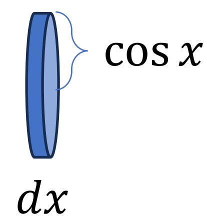
(b) $\frac{\pi^2}{4}$

(c) 1

(d) $\frac{\pi^2}{2}$

(e) $\frac{\pi}{2}$





$$V(\,\text{disk}\,) = \pi \cos^2 x \, dx$$

$$\begin{aligned} V &= \int_0^{\pi/2} \pi \cos^2 x \, dx \\ &= \int_0^{\pi/2} \pi \left[\frac{1+\cos 2x}{2}\right] dx \\ &= \frac{\pi}{2}\left[x + \frac{1}{2}\sin 2x\right]_0^{\pi/2} \\ &= \frac{\pi}{2}\left[\frac{\pi}{2}\right] \\ &= \frac{\pi^2}{4} \end{aligned}$$

Trigonometric Integrals

Which of the following is equal to $\int_0^{\pi/4} \tan^2(\theta)\sec^4(\theta)\,d\theta$?

(a) $\int_0^{\pi/4} u^2(u^2-1)\,du$

(b) $\int_0^{\pi/4} u^2(1+u^2)\,du$

(c) $\int_0^{\sqrt{2}/2} u^2(1+u^2)\,du$

(d) $\int_0^{1} u^2(u^2-1)\,du$

(e) $\int_0^{1} u^2(1+u^2)\,du$

$$\int_0^{\pi/4} \tan^2\theta \sec^2\theta\,(\sec^2\theta\,d\theta)$$

$$= \int_0^{\pi/4} \tan^2\theta\,(\tan^2\theta + 1)(\sec^2\theta\,d\theta)$$

$$u = \tan\theta$$

$$\Rightarrow du = \sec^2\theta$$

$$\int_{x=0}^{x=\pi/4} \Rightarrow$$

$$\int_{u=0}^{u=1}$$

$$\int_0^1 u^2(u^2+1)du$$

sec out; odd man out

Trigonometric Integrals

Compute $\int \tan^3(x)\sec^3(x)\ dx$

(a) $\frac{1}{5}\sec^5(x) - \frac{1}{3}\sec^3(x) + C$

(b) $-\frac{1}{5}\sec^5(x) + \frac{1}{3}\sec^3(x) + C$

(c) $\frac{1}{5}\tan^5(x) - \frac{1}{3}\tan^3(x) + C$

(d) $-\frac{1}{5}\tan^5(x) + \frac{1}{3}\tan^3(x) + C$

(e) $-\sec^4(x) + \sec^2(x) + C$

$$\int \tan^2 x \sec^2 x\,(\sec x \tan x)dx$$
$$= \int (\sec^2 x - 1)\sec^2 x\,(\sec x \tan x)dx$$

u −sub : $u = \sec x$

$du = \sec x\ \tan x dx$

$$= \int (u^4 - u^2)du$$
$$= \frac{1}{5}u^5 - \frac{1}{3}u^3 + C$$
$$= \frac{1}{5}\sec^5 x - \frac{1}{3}\sec^3 x + C$$

sec out; odd man out

Trigonometric Integrals (Exercise)

Compute $\int \cos^4(x)\sin^5(x)\, dx$

(a) $-\frac{1}{5}\cos^5(x) + \frac{1}{9}\cos^9(x) + C$

(b) $\frac{1}{6}\sin^6(x) - \frac{1}{4}\sin^8(x) + \frac{1}{10}\sin^{10}(x) + C$

(c) $\frac{1}{6}\sin^6(x) - \frac{1}{10}\sin^{10}(x) + C$

(d) $-\frac{1}{5}\cos^5(x) + \frac{2}{7}\cos^7(x) - \frac{1}{9}\cos^9(x) + C$

(e) None of these.

Compute $\int \cos^3(2x)\, dx$

(a) $-\sin(2x) + \frac{1}{3}\sin^3(2x) + C$

(b) $\frac{-1}{2}\sin(2x) + \frac{1}{6}\cos^3(2x) + C$

(c) None of these.

(d) $\sin(2x) - \frac{1}{3}\sin^3(2x) + C$

(e) $\frac{1}{2}\sin(2x) - \frac{1}{6}\sin^3(2x) + C$

Evaluate $\int \tan^3(x)\sec^5(x)\, dx$.

(a) $\frac{1}{7}\tan^7 x - \frac{1}{5}\sec^5 x + C$

(b) $\frac{1}{7}\sec^7 x - \frac{1}{5}\tan^5 x + C$

(c) $\frac{1}{4}\sec^6 x - \frac{1}{6}\tan^{10} x + C$

(d) $\frac{1}{4}\sec^4 x - \frac{1}{6}\tan^6 x + C$

(e) $\frac{1}{7}\sec^7 x - \frac{1}{5}\sec^5 x + C$

Compute $\int_0^{\pi/4} \sec^4(x)\, dx$

(a) $\frac{2}{3}$

(b) $\frac{32}{5}$

(c) $\frac{4}{3}$

(d) $\frac{4\sqrt{2}}{5}$

(e) $\frac{2\sqrt{2}-1}{3}$

Trigonometric Integrals (Exercise)

Evaluate $\int \sin^2(x)\, dx$.

(a) $\frac{1}{2}x - \frac{1}{4}\sin(2x) + C$
(b) $\frac{1}{3}\sin x \cos x + C$
(c) $\frac{1}{2}x + \frac{1}{2}\sin x + C$
(d) $\frac{1}{2}x - \frac{1}{4}\sin x + C$
(e) $\frac{1}{3}\cos^3 x + C$

Compute $\int \sin^7\theta \cos^5\theta\, d\theta$.

Compute $\int \cos^2(x)\sin^2(x)\, dx$

Find $\int \cos^4 x\, dx$

Trigonometric Integrals (Exercise)

Compute $\int 2\sin^2(2\theta)\,d\theta$

(a) $\theta - \frac{1}{2}\sin(2\theta) + C$

(b) $\theta - \frac{1}{4}\sin(4\theta) + C$

(c) $\theta + \frac{1}{2}\sin(2\theta) + C$

(d) $\theta + \frac{1}{4}\sin(4\theta) + C$

(e) None of the above

Compute $\int_0^{\pi/3} \tan^3(\theta)\sec(\theta)\,d\theta$

(a) $\frac{4}{3}$

(b) $\frac{16 - 9\sqrt{3}}{24}$

(c) $\frac{2}{3}$

(d) $\frac{-3\sqrt{3}}{8}$

(e) None of the above

$\int_0^{\pi/2} \cos^3 x \sin^3 x\,dx =$

(a) $\frac{1}{12}$

(b) $\frac{2}{15}$

(c) $\frac{5}{12}$

(d) $\frac{-1}{12}$

(e) $\frac{-2}{15}$

$\int \tan^4 x \sec^4 x\,dx =$

(a) $\frac{\tan^7 x}{7} + \frac{\tan^5 x}{5} + C$

(b) $\frac{\tan^7 x}{7} - \frac{\tan^5 x}{5} + C$

(c) $\frac{\tan^9 x}{9} + \frac{\tan^5 x}{5} + C$

(d) $\frac{\tan^9 x}{9} - \frac{\tan^5 x}{5} + C$

(e) None of these

Trigonometric Integrals (Exercise)

Compute $\int_0^{\pi/2} \sin(2x)\cos x \; dx$.

(a) $\frac{3}{2}$
(b) $\frac{2}{3}$
(c) 0
(d) 1
(e) $\frac{1}{2}$

$\int_0^{\pi/4} \sin^2(x)\, dx =$

(a) $\frac{\pi}{8} - \frac{1}{4}$
(b) $\frac{\pi}{8}$
(c) $\frac{\pi}{8} - \frac{1}{2}$
(d) $\frac{2}{\sqrt{2}} - \frac{\pi}{2\sqrt{2}}$
(e) $\frac{\pi}{8} + \frac{1}{4}$

Trigonometric Substitution

Evaluate $\int \frac{1+x}{1+x^2}\,dx$.

(a) $\frac{1}{2}\ln(1+x^2)+C$
(b) $\frac{3}{2}\ln(1+x^2)+C$
(c) $\ln(1+x^2)+C$
(d) $\frac{1}{2}\ln(1+x^2)+\arctan x+C$
(e) $\arctan x+\arcsin(x^2)+C$

$$\int \frac{1}{1+x^2}+\frac{x}{1+x^2}\,dx$$
$$=\int \frac{1}{1+x^2}\,dx+\int \frac{x}{1+x^2}\,dx$$
$$\operatorname{atan} x+\frac{1}{2}\ln(1+x^2)+C$$

Trigonometric Substitution

After an appropriate substitution, the integral $\int \sqrt{x^2+x}\,dx$ is equivalent to which of the following?

(a) $\int \tan^2\theta \sec\theta \, d\theta$

(b) $\frac{1}{4}\int \sec^3\theta \, d\theta$

(c) $-\frac{1}{4}\int \sin^2\theta \, d\theta$

(d) $\frac{1}{4}\int \tan^2\theta \sec\theta \, d\theta$

(e) $\int \cos^2\theta \, d\theta$

$$\int \sqrt{x^2+x+\frac{1}{4}-\frac{1}{4}}\,dx$$

$$= \int \sqrt{\left(x-\frac{1}{2}\right)^2-\frac{1}{4}}\,dx$$

$$= \int \sqrt{\frac{1}{4}[(2x-1)^2-1]}\,dx$$

$$(2x-1) = \sec\theta$$

$$2dx = \sec\theta\tan\theta\,d\theta$$

$$= \frac{1}{2}\int \sqrt{(\sec^2\theta-1)}\left(\frac{1}{2}\sec\theta\tan\theta\,d\theta\right)$$

$$= \frac{1}{4}\int \sqrt{\tan^2\theta}\,(\sec\theta\tan\theta\,d\theta)$$

$$= \frac{1}{4}\int \sec\theta\tan^2\theta\,d\theta$$

Trigonometric Substitution

Compute the following integral showing all necessary work clearly.

$$\int \frac{1}{(x^2+9)^{5/2}}\,dx$$

$$\int \frac{dx}{\left[9\left(\left[\frac{x}{3}\right]^2+1\right)\right]^{5/2}}$$

$$\frac{x}{3} = \tan\theta$$

$$\frac{1}{3}dx = \sec^2\theta\, d\theta$$

$$= \int \frac{3\sec^2\theta d\theta}{[3^2(\tan^2\theta+1)]^{5/2}}$$

$$= \int \frac{3\sec^2\theta d\theta}{[3^2\sec^2\theta]^{5/2}}$$

$$= \int \frac{3\sec^2\theta d\theta}{3^5\sec^5\theta}$$

$$= \frac{1}{3^4}\int \frac{1}{\sec^3\theta}\,d\theta$$

$$= \frac{1}{3^4}\int \cos^3\theta\, d\theta$$

$$= \frac{1}{3^4}\int \cos^2\theta\cos\theta\, d\theta$$

$$= \frac{1}{3^4}\int (1-\sin^2\theta)\cos\theta\, d\theta$$

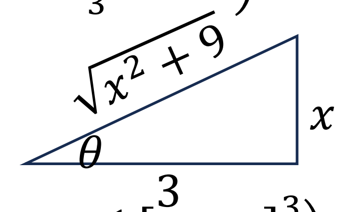
Let $u = \sin\theta$

$$\Rightarrow du = \cos\theta\, d\theta$$

$$= \frac{1}{3^4}\int (1-u^2)du$$

$$= \frac{1}{3^4}\left(u - \frac{1}{3}u^3\right) + C$$

$$= \frac{1}{3^4}\left(\sin\theta - \frac{1}{3}\sin^3\theta\right) + C$$



$$= \frac{1}{3^4}\left(\frac{x}{\sqrt{x^2+9}} - \frac{1}{3}\left[\frac{x}{\sqrt{x^2+9}}\right]^3\right) + C$$

Trigonometric Substitution

Which of these substitutions would be used to evaluate $\int x^2\sqrt{x^2+4x+13}\,dx$?

(a) $x+4=\sqrt{13}\sec\theta$

(b) $x+2=3\tan\theta$

(c) $x^2+4x=\sqrt{13}\tan\theta$

(d) none of these.

(e) $x+2=3\sec\theta$

$$\int x^2\sqrt{(x+2)^2+3^2}dx$$

Trig sub: $(x+2)=3\tan u$

$$dx=3\sec^2 u\,du$$

$$x^2=(3\tan u-2)^2$$

$$\int x^2\sqrt{(x+2)^2+3^2}dx$$

$$=\int(3\tan u-2)^2[3\sec u]\,(3\sec^2 u\,du)$$

Trigonometric Substitution

After an appropriate trigonometric substitution,

$\int_{2\sqrt{2}}^{4} \frac{\sqrt{x^2-4}}{x}\,dx$ is equivalent to

(a) $2\int_{\pi/4}^{\pi/3} \tan^2\theta\,d\theta$

(b) $\int_{\pi/4}^{\pi/3} \sin(\theta)\,d\theta$

(c) $2\int_{\pi/4}^{\pi/6} \tan^2\theta\,d\theta$

(d) $\int_{\pi/4}^{\pi/6} \sin(\theta)\,d\theta$

(e) None of the above

Trig sub $x = 2\sec u$

$dx = 2\sec u \tan u\, du$

$x^2 - 4 = 4\sec^2 u - 4 = 4\tan^2 u$

$\int_{x=2\sqrt{2}}^{x=4} \Rightarrow \int_{u=\frac{\pi}{4}}^{u=\frac{\pi}{3}}$

$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{2\tan u}{2\sec u}(2\sec u \tan u\, du)$

$= 2\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \tan^2 u\, du$

Trigonometric Substitution (Exercise)

After an appropriate substitution, the integral $\int x^2\sqrt{9-x^2}\,dx$ is equivalent to which of the following?

(a) $9\int \cos^2\theta\,d\theta$

(b) $81\int \sin^2\theta\cos^2\theta d\theta$

(c) $27\int \sin^2\theta\cos\theta\,d\theta$

(d) $81\int \sec^3\theta\tan^2\theta\,d\theta$

(e) $27\int \sec^2\theta\tan\theta\,d\theta$

Which of the following is an appropriate substitution to use when solving the integral $\int \sqrt{16x^2-9}\,dx$?

(a) $x=\frac{3}{4}\sin\theta$

(b) $x=\frac{4}{3}\sec\theta$

(c) $x=\frac{4}{3}\sin\theta$

(d) $x=\frac{3}{4}\sec\theta$

(e) $x=\frac{3}{4}\tan\theta$

Compute $\int_0^4 \frac{x+2}{x^2+4}\,dx$.

(a) $\frac{1}{2}(\ln 20-\ln 4)+\arctan(2)$

(b) $\ln 6-\ln 2$

(c) $\ln 20-\ln 4$

(d) $\frac{1}{2}(\ln 20-\ln 4)+2\arctan(4)$

(e) $\ln 20-\ln 4+2\arctan(4)$

Which of the following integrals is equivalent to $\int \sqrt{4x^2-9}\,dx$?

(a) $2\int \sec\theta\tan^2\theta\,d\theta$

(b) $\frac{9}{2}\int \tan\theta\,d\theta$

(c) $\frac{9}{2}\int \sec\theta\tan^2\theta\,d\theta$

(d) $\frac{9}{2}\int \sec^2\theta\tan\theta\,d\theta$

(e) $2\int \sec^2\theta\tan\theta\,d\theta$

Trigonometric Substitution (Exercise)

Compute $\displaystyle\int \frac{1}{x^4\sqrt{x^2-4}}\,dx$. In your final answer, any trig or inverse trig expressions that can be rewritten algebraically must be.

After an appropriate trigonometric substitution,

$\displaystyle\int \frac{dx}{\sqrt{x^2+8x+41}}$ is equivalent to which of the following?

(a) $\displaystyle\frac{1}{5}\int \cos(\theta)\,d\theta$

(b) $\displaystyle\int \sec(\theta)\,d\theta$

(c) $\displaystyle\int \sec^2(\theta)\,d\theta$

(d) $\displaystyle\int \tan(\theta)\,d\theta$

(e) $\displaystyle\frac{1}{5}\int \sin(\theta)\,d\theta$

After an appropriate trigonometric substitution,

$\displaystyle\int_{2\sqrt{2}}^{4} \frac{\sqrt{x^2-4}}{x}\,dx$ is equivalent to

(a) $\displaystyle 2\int_{\pi/4}^{\pi/3} \tan^2\theta\,d\theta$

(b) $\displaystyle\int_{\pi/4}^{\pi/3} \sin(\theta)\,d\theta$

(c) $\displaystyle 2\int_{\pi/4}^{\pi/6} \tan^2\theta\,d\theta$

(d) $\displaystyle\int_{\pi/4}^{\pi/6} \sin(\theta)\,d\theta$

(e) None of the above

Trigonometric Substitution (Exercise)

After an appropriate substitution, the integral $\int \sqrt{9-x^2}\,dx$ is equivalent to which of the following?

(a) $9\int \sec\theta\tan^2\theta\,d\theta$

(b) $3\int \cos\theta\,d\theta$

(c) $9\int \sec^3\theta\,d\theta$

(d) $9\int \cos^2\theta\,d\theta$

(e) $3\int \tan\theta\,d\theta$

Which of the following integrals is equivalent to $\int \frac{1}{(x^2-4x+5)^{3/2}}\,dx$?

(a) $\frac{1}{9}\int \cos\theta\,d\theta$

(b) $\int \cos^3\theta\,d\theta$

(c) $\frac{1}{27}\int \cos^3\theta\,d\theta$

(d) $\int \sec\theta\,d\theta$

(e) $\int \cos\theta\,d\theta$

If we use the appropriate trigonometric substitution to evaluate $\int_1^{2/\sqrt{3}} \left(\frac{\sqrt{x^2-1}}{x}\right) dx$, which of the following is the correct result?

(a) $\int_0^{\pi/6} \tan^2\theta\,d\theta$

(b) $\int_0^{\pi/6} \frac{\tan\theta}{\sec\theta}\,d\theta$

(c) $\int_0^{\pi/3} \tan^2\theta\,d\theta$

(d) $\int_{\pi/2}^{\pi/6} \sin^2\theta d\theta$

(e) $\int_{\pi/2}^{\pi/3} \sin^2\theta d\theta$

Trigonometric Substitution (Exercise)

Find $\int \frac{x^2}{\sqrt{4-x^2}}\,dx$

Evaluate $\int \frac{1}{x^2\sqrt{x^2+4}}\,dx.$

Integration by Partial Fractions

Which of the following is the form of the partial-fraction decomposition for the rational function?

$$\frac{1}{(x+1)(x^2-2x-3)(x^2-2x+2)}$$

(a) $\frac{A}{x+1}+\frac{Bx+C}{x^2-2x-3}+\frac{Dx+E}{x^2-2x+2}$

(b) $\frac{A}{x+1}+\frac{Bx+C}{(x+1)^2}+\frac{D}{x-3}+\frac{Ex+F}{x^2-2x+2}$

(c) $\frac{A}{x+1}+\frac{B}{(x+1)^2}+\frac{C}{x-3}+\frac{D}{x^2-2x+2}$

(d) $\frac{A}{x+1}+\frac{B}{(x+1)^2}+\frac{C}{x-3}+\frac{Dx+E}{x^2-2x+2}$

(e) $\frac{A}{x+1}+\frac{B}{x^2-2x-3}+\frac{C}{x^2-2x+2}$

$$\frac{1}{(x+1)(x-3)(x+1)(x^2-2x+2)}$$

$$=\frac{1}{(x+1)^2(x-3)(x^2-2x+2)}$$

$$=\frac{A}{(x+1)}+\frac{B}{(x+1)^2}+\frac{C}{x-3}+\frac{Dx+E}{x^2-2x+2}$$

Integration by Partial Fractions

Compute the following integral showing all necessary work clearly.

$$\int \frac{4x^2 - 5x + 11}{(x+1)(x-1)(x^2+4)}\,dx$$

- $\int \frac{1}{x^2+4} dx = \frac{1}{4}\int \frac{1}{\left(\frac{x}{2}\right)^2+1} dx$
  - $\tan\theta = \frac{x}{2}$
  - $2\sec^2\theta\, d\theta = dx$

$= \frac{1}{4}\int \frac{2\sec^2\theta}{(\tan\theta)^2+1} d\theta$

$= \frac{1}{2}\int \frac{\sec^2\theta}{\sec^2\theta} d\theta$

$= \frac{1}{2}\theta + C$

$= \frac{1}{2}\arctan\left(\frac{x}{2}\right) + C$

$\int \frac{4x^2-5x+11}{(x+1)(x-1)(x^2+4)} dx =$

$2\ln|x+1| + \ln|x-1|$

$-\frac{1}{2}\ln(x^2+4) + \frac{1}{2}\arctan\left(\frac{x}{2}\right) + C$

$$\frac{4x^2-5x+11}{(x+1)(x-1)(x^2+4)} = \frac{\frac{4+5+11}{(-2)(1+4)}}{(x+1)} + \frac{\frac{4-5+11}{(2)(1+4)}}{(x-1)} + \frac{Cx+D}{x^2+4}$$

$$= \frac{\frac{20}{-10}}{(x+1)} + \frac{\frac{10}{10}}{(x-1)} + \frac{Cx+D}{x^2+4}$$

$$= \frac{-2}{(x+1)} + \frac{1}{(x-1)} + \frac{Cx+D}{x^2+4}$$

$$\frac{4x^2-5x+11}{(x+1)(x-1)} = \left[\frac{-2}{(x+1)} + \frac{1}{(x-1)}\right](x^2+4) + Cx + D$$

Let $x = 2i$

$$\frac{-16-10i+11}{5} = 2Ci + D$$

$-1 - 2i = 2Ci + D$

$C = -1, D = -1$

$$\int \left(\frac{2}{(x+1)} + \frac{1}{(x-1)} - \frac{x}{x^2+4} - \frac{1}{x^2+4}\right) dx$$

- $\int \frac{2}{(x+1)} dx = 2\ln|x+1| + C$
- $\int \frac{1}{(x-1)} dx = \ln|x-1| + C$
- $\int \frac{x}{x^2+4} dx = \frac{1}{2}\ln(x^2+4) + C$
  - $u = x^2 + 4 \Rightarrow du = 2x dx$
  - $\int \frac{x}{x^2+4} dx = \frac{1}{2}\int \frac{1}{u} du$

Integration by Partial Fractions

$$\int \frac{x^3+x}{x-1}\,dx =$$

(a) $\frac{x^3}{3}+\frac{x^2}{2}+2x-2\ln|x-1|+C$

(b) $\frac{x^3}{3}-\frac{x^2}{2}+2x+2\ln|x-1|+C$

(c) $\frac{x^3}{3}+\frac{x^2}{2}+2x+2\ln|x-1|+C$

(d) $\frac{x^3}{3}+\frac{x^2}{2}-2x+2\ln|x-1|+C$

(e) $\frac{x^3}{3}-\frac{x^2}{2}+2x-2\ln|x-1|+C$

Long division

$$\frac{x^3+x}{x-1} = \frac{x^3-x^2+x^2-x+2x-2+2}{x-1}$$

$$= \frac{x^2(x-1)+x(x-1)+2(x-1)+2}{x-1}$$

$$= x^2+x+2+\frac{2}{x-1}$$

$$\int\left(x^2+x+2+\frac{2}{x-1}\right)dx$$

$$\frac{x^3}{3}+\frac{x^2}{2}+2x+2\ln|x-1|+C$$

Integration by Partial Fractions

Find $\int \frac{x+2}{x^2(x^2+1)}\,dx$

Partial fraction

$$\frac{x+2}{x^2(x^2+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+1}$$

Multiply by x^2 and let $x = 0$

$$\frac{0+2}{(0^2+1)} = B \Rightarrow B = 2$$

Simplifying $\frac{x+2}{x^2(x^2+1)} - \frac{2}{x^2} = \frac{A}{x} + \frac{Cx+D}{x^2+1}$

$$\frac{x+2-2x^2-2}{x^2(x^2+1)} = \frac{A}{x} + \frac{Cx+D}{x^2+1} \Rightarrow \frac{1-2x}{x(x^2+1)} = \frac{A}{x} + \frac{Cx+D}{x^2+1}$$

Multiply by $x$and let $x = 0$

$$\frac{1-2\cdot 0}{(0^2+1)} = A \Rightarrow A = 1$$

Simplifying $\frac{1-2x}{x(x^2+1)} - \frac{1}{x} = \frac{Cx+D}{x^2+1}$

$$\frac{1-2x-x^2-1}{x(x^2+1)} = \frac{Cx+D}{x^2+1} \Rightarrow \frac{-2-x}{x^2+1} = \frac{Cx+D}{x^2+1}$$

$$\int \frac{x+2}{x^2(x^2+1)}\,dx = \int \left[\frac{1}{x} + \frac{2}{x^2} + \frac{-x-2}{x^2+1}\right] dx$$

$$= \int \left[\frac{1}{x} + \frac{2}{x^2} - \frac{x}{x^2+1} - \frac{2}{x^2+1}\right] dx$$

$$= \ln|x| - \frac{2}{x} - \frac{1}{2}\ln(x^2+1) - 2\,\mathrm{atan}\,x + C$$

Integration by Partial Fractions (Exercise)

Write out the form of the partial fraction decomposition of the function

$$f(x) = \frac{x^3 - 2x^2 - 5x + 4}{(x+2)^2(x^2-1)(x^2+5x+7)}$$

(a) $\frac{A}{x+2} + \frac{B}{(x+2)^2} + \frac{C}{x-1} + \frac{D}{x+1} + \frac{Ex+F}{x^2+5x+7}$

(b) $\frac{A}{x+2} + \frac{B}{(x+2)^2} + \frac{Cx+D}{x^2-1} + \frac{Ex+F}{x^2+5x+7}$

(c) $\frac{A}{(x+2)^2} + \frac{B}{x-1} + \frac{C}{x+1} + \frac{Dx+E}{x^2+5x+7}$

(d) $\frac{A}{(x+2)^2} + \frac{B}{x^2-1} + \frac{Cx+D}{x^2+5x+7}$

(e) $\frac{Ax+B}{(x+2)^2} + \frac{Cx+D}{x^2-1} + \frac{Ex+F}{x^2+5x+7}$

$$\int \frac{3-x}{x^2+3x-4}\,dx =$$

(a) $\frac{-7}{5}\ln|x+4| + \frac{2}{5}\ln|x-1| + C$

(b) $\frac{7}{2}\ln|x+4| + \frac{2}{5}\ln|x-1| + C$

(c) $\frac{1}{5}\ln|x-4| - \frac{4}{5}\ln|x+1| + C$

(d) $\frac{-1}{5}\ln|x-4| + \frac{4}{5}\ln|x+1| + C$

(e) $\frac{2}{5}\ln|x+4| - \frac{7}{5}\ln|x-1| + C$

Integration by Partial Fractions (Exercise)

Which of the following is a proper Partial Fraction Decomposition for $\frac{x+1}{(x^2-16)(x-3)^2(x^2+1)}$?

(a) $\frac{x+1}{(x^2-16)(x-3)^2(x^2+1)} = \frac{Ax+B}{x^2-16} + \frac{C}{x-3} + \frac{D}{(x-3)^2} + \frac{Ex+F}{x^2+1}$

(b) $\frac{x+1}{(x^2-16)(x-3)^2(x^2+1)} = \frac{A}{x-4} + \frac{B}{x+4} + \frac{C}{x-3} + \frac{D}{(x-3)^2} + \frac{Ex+F}{x^2+1}$

(c) $\frac{x+1}{(x^2-16)(x-3)^2(x^2+1)} = \frac{Ax+B}{x^2-16} + \frac{Cx+D}{(x-3)^2} + \frac{Ex+F}{x^2+1}$

(d) $\frac{x+1}{(x^2-16)(x-3)^2(x^2+1)} = \frac{A}{x-4} + \frac{B}{x+4} + \frac{C}{x-3} + \frac{D}{(x-3)^2} + \frac{E}{x^2+1}$

(e) $\frac{x+1}{(x^2-16)(x-3)^2(x^2+1)} = \frac{A}{x-4} + \frac{B}{(x-4)^2} + \frac{C}{x-3} + \frac{D}{(x-3)^2} + \frac{Ex+F}{x^2+1}$

Which of the following is a proper Partial Fraction Decomposition for the rational function

$$\frac{5x+1}{(x+3)(x^2+4x+3)(x^2+4)}$$

(a) $\frac{A}{x+3} + \frac{B}{(x+3)^2} + \frac{C}{x+1} + \frac{Dx+E}{x^2+4}$

(b) $\frac{A}{x+3} + \frac{Bx+C}{x^2+4x+3} + \frac{Dx+E}{x^2+4}$

(c) $\frac{A}{x+3} + \frac{Bx+C}{(x+3)^2} + \frac{D}{x+1} + \frac{Ex+F}{x^2+4}$

(d) $\frac{A}{x+3} + \frac{B}{(x+3)^2} + \frac{C}{x+1} + \frac{D}{x+2} + \frac{D}{x-2}$

(e) None of these.

Integration by Partial Fractions (Exercise)

Compute $\displaystyle\int \frac{2x^2+5x-5}{(x+1)(x+3)^2}\,dx$

Evaluate $\displaystyle\int \frac{-2x+4}{(x^2+1)(x-1)}\,dx$

Evaluate $\displaystyle\int_0^1 \frac{4x^2+5}{2x+1}\,dx$

(a) $2\ln 3$

(b) $3\ln 3$

(c) $4\ln 3$

(d) $6\ln 3$

(e) None of these

Find $\displaystyle\int \frac{2x^3-4x-8}{(x^2-x)(x^2+4)}\,dx$ showing all necessary work.