TEXAS A&M UNIVERSITY Mathematics

Math 140 - Spring 2025
WEEK IN REVIEW ♯10 - APR. 08, 2025

SECTION 5.5: PIECEWISE-DEFINED FUNCTIONS

Pr 1. Let $f(x) = \begin{cases} 3x-1 & x<-3 \\ x^2-1 & -3 \le x < 3 \\ 7 & 3 \le x \le 5 \\ \frac{1}{x-5} & x > 6 \end{cases}$. Compute the following function values.

(a) $f(-5)$. $x=-5$ T

Step 1: determine which condition is satisfied / true

is $-5<-3$? Yes, use 1st rule

$f(-5) = 3(-5)-1 = -15-1 = -16$

(b) $f(-3)$. is $(-3)<-3$? No $\Rightarrow$ skip rule 1

is $-3 \le (-3) < 3$? Yes $\to$ use 2nd rule

$f(-3) = (-3)^2 - 1 = 9-1 = 8$

$f(-3) = -3^2-1 = -9-1 = -10$

(c) $f(0)$. is $0<-3$? No, $-3 \le 0 < 3$ Yes

$f(x) = (0)^2 - 1 = 0^2 - 1 = 0-1 = -1$

(d) $f(3)$. is $3<-3$? No, is $-3 \le 3 < 3$? No. is $3 \le 3 \le 5$?

Yes $\to$ use 3rd rule

$f(3) = 7$.

reminder: if $g(x) = 2028$
then $g(\sqrt{2}) = 2028$

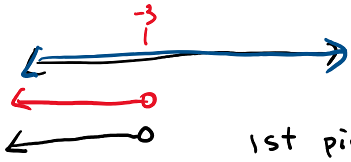
6 (e) $f(6)$. is $6>6$? No ? If no condition is true for $x=c$, then $f(c)$ D.N.E

$f(6)$ is undefined / does not exist

(f) $f(7)$. $x=7$, is $7>6$? Yes. use last rule

$f(7) = \frac{1}{(7)-5} = \frac{1}{7-5} = \frac{1}{2}$ ✓

Pr 2. State the domain of $g(x)$.

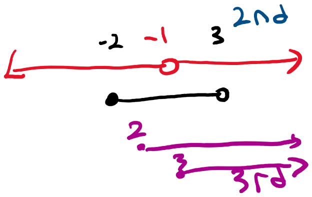


$$g(x)=\begin{cases} x+1 & x<-3 \\ \frac{1}{x+1} & -2\le x<3 \\ \sqrt{x-2} & x\ge 3 \end{cases}$$

Two stages

for each piece, I find the corresponding part of the domain.

1st piece: $(-\infty,-3) \cap$ domain of $f_1(x)=x+1$ $= (-\infty,-3)\cap(-\infty,\infty)$ $= (-\infty,-3)$



2nd piece: $[-2,3) \cap$ domain of $f_2(x)=\frac{1}{x+1}$

$= [-2,3) \cap \big((-\infty,-1)\cup(-1,\infty)\big)$

$= [-2,-1)\cup(-1,3)$ $\to$ $[-2,-1)\cup(-1,3)$

$x+1\neq 0 \to x\neq -1$

3rd piece: $[3,\infty) \cap$ domain of $\sqrt{x-2}$ $\to$ $\to$ $[3,\infty)$

$[2,\infty)$

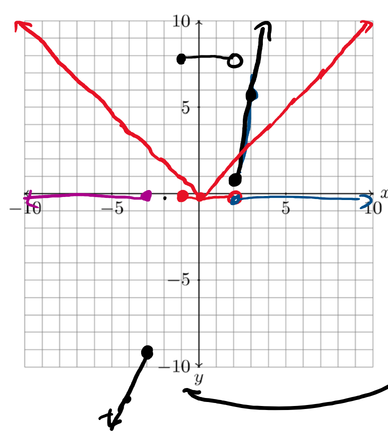
$x-2\ge 0 \to x\ge 2$

$(-\infty,-3)\cup[-2,-1)\cup(-1,3)\cup[3,\infty)$

$= \boxed{(-\infty,-3)\cup[-2,-1)\cup(-1,\infty)}$

Pr 3. Sketch the graph of $h(x)$ as well as the graph of $k(x)=|x|$.

$$h(x)=\begin{cases} 2x-3 & x\le -3 \\ 8 & -1\le x<2 \\ x^2-3 & x\ge 2 \end{cases}$$



$k(x)=|x|$

part 1: $(-\infty,-3]$ rule $2x-3$

set $x=-3$

$2(-3)-3=-6-3=-9$

Since -3 is in domain for rule 1, the hole should be filled

$(-3,-9)$

part 2: $[-1,2)$ rule $y=8$

part 3: $[2,\infty)$ $y=x^2-3$

set $x=2$

$y=2^2-3=4-3=1$

$(2,1)$

$x=3$ $y=3^2-3=9-3=6$

$(3,6)$

$$|x| = \begin{cases} -x & \text{if } x \le 0 \\ x & \text{if } x > 0 \end{cases}$$

Pr 4. Rewrite $f(x) = |5 - 3x|$ as a piecewise-defined function.

$$|5-3x| = \begin{cases} -(5-3x) & \text{if } 5-3x \le 0 \\ 5-3x & \text{if } 5-3x > 0 \end{cases}$$

rewrite the conditions

$5-3x > 0$, -5 from both sides: $-3x > -5$, divide by -3: $x < \frac{5}{3}$ ✓

$$= \begin{cases} -5+3x & \text{if } x \ge 5/3 \\ 5-3x & \text{if } x < 5/3 \end{cases}$$

$$= \begin{cases} 5-3x & \text{if } x < 5/3 \\ -5+3x & \text{if } x \ge 5/3 \end{cases}$$

$5-3x \le 0 \to x \ge \frac{5}{3}$

$x < a$

$x > a$

Not correct: $\begin{cases} -5+3x & \text{if } x < 5/3 \\ 5-3x & \text{if } x \ge 5/3 \end{cases}$

Pr 5. Suppose that you have a channel on a social media server. The social media company pays you \$.01 for each view for the first 1000 views on a video, and then \$.05 for each additional view, up to the first 5000 views. After that, each additional view is \$.10. Let $R(v)$ be the revenue you make for having v views on one of your videos. Write the corresponding function.

$$R(v) = \begin{cases} .01v & \text{if } v \le 1000 \\ .05v - 40 & \text{if } 1000 < v \le 5000 \\ .10v - 290 & \text{if } v > 5000 \end{cases}$$

not $.05v$ ✗ $\quad .05(\#\text{ additional views}) + \text{amount } R(1000)$

$= .05(v - 1000) + R(1000)$

$= .05v - .05 \times 1000 + .01 \times 1000$

$= .05v - .04 \times 1000$

$= .05v - 40$

$v > 1000$: $\quad R(v) = .10(\text{additional views over } 5000) + R(5000)$

$= .10(v - 5000) + R(5000)$

$= .1v - 500 + (.05 \times 5000 - 40)$

$.1v - 500 + 250 - 40$

Section 5.6: Exponential Functions

- Exponential Function
- Exponential Growth and Exponential Decay
- Common Base Property of Exponents: For $b \neq 1$, $b^S = b^T$ if and only if $S = T$.
- Finance Applications

$f(x) = a \cdot b^x$ $b =$ base

$b \neq 1$, $0 < b$

Pr 1. Rewrite each exponential expression as a single equivalent expression in the stated base.

(a) $125 \cdot 5^{x+3}$, base 5.

$125 \times 5^{x+3} = 125 \times 5^x \times 5^3$

$= 125 \times 5^3 \times 5^x = 125 \times 125 \times 5^x$

$= 15625 \cdot 5^x$

$4 = 2^2$

(b) $\left(\frac{1}{2}\right)^x \frac{8}{4^x}$, base 2.

$\frac{1}{2} = 2^{-1}$

$= (2^{-1})^x \times 8 \times (4^x)^{-1} = 2^{-x} \times 8 \times 4^{-x}$

$= 2^{-x} \cdot 8 \times (2^2)^{-x}$

$(a^b)^c = a^{bc}$

$= 8 \times 2^{-x} \, 2^{-2x}$

$= 8 \times 2^{-x-2x} = 8 \times 2^{-3x}$

Pr 2. Determine if each function is an exponential function. If the function is an exponential function, determine whether the function represents exponential growth or decay.

$b > 1$ $b < 1$

(a) 7^{-x} is exponential

$7^{-x} = (7^{-1})^x = \left(\frac{1}{7}\right)^x$ base $= \frac{1}{7} < 1$

(b) $-3x^{17}$ is not exponential

variable is in the base

this is a power function.

(c) $\frac{3}{4} 2^{x/2+4} = \frac{3}{4} 2^{x/2} \cdot 2^4 = \frac{3}{4} 2^4 (2^{1/2})^x$

$= \frac{3}{4} \times 16 \times (2^{1/2})^x$

$= 12 (\sqrt{2})^x$ is $\sqrt{2} > 1$? Yes

this is exponential. growth

Pr 3. State the domain of each function, and match each function with the graph of the parent function.

(a) $f(x) = \left(\frac{5}{3}\right)^{x+2}$

domain of $f(x) = ab^x$

$= (-\infty, \infty)$ ✓

domain: $(-\infty, \infty)$

range of $f(x) = (0, \infty)$

when $a > 0$

$$\left(\frac{5}{3}\right)^{x+2} = \left(\frac{5}{3}\right)^x \cdot \left(\frac{5}{3}\right)^2 = \left(\frac{5}{3}\right)^2 \cdot \underline{\left(\frac{5}{3}\right)^x}$$

is $\frac{5}{3} > 1$? Yes

growth

parent function is blue

(b) $g(x) = 5^{4-x}$

domain: $(-\infty, \infty)$

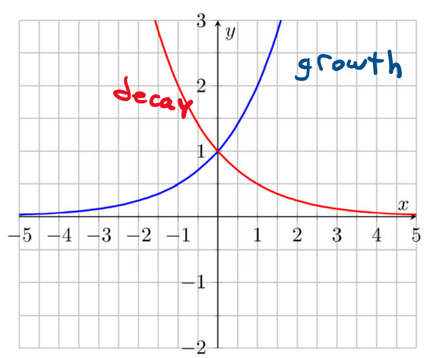
$$g(x) = 5^{4-x} = 5^4 \cdot 5^{-x} = 5^4 \cdot \left(\frac{1}{5}\right)^x$$

$a^{b-c} = a^b a^{-c}$

$1/5 < 1$

decay

parent function is red



range: $(0, \infty)$ if $a > 0$

or $(-\infty, 0)$ if $a < 0$

$b^x = 0$ has no solution

$b^x = -1$ has no solution

$b^0 = 1$

Pr 4. State the domain of each function. Write your answer using interval notation.

(a) $f(x) = 5^{\frac{2x}{x-4}}$

$f(x) = 5^{\frac{2x}{x-4}}$ $\quad f(x) = b^{\overline{g(x)}}$

then the domain of $f(x)$

= domain of $g(x)$

Find the domain of $\frac{2x}{x-4}$ $\rightarrow$ need $x-4 \neq 0$

need non-zero denominator

$x \neq 4$

$\boxed{(-\infty, 4) \cup (4, \infty)}$

(b) $g(x) = e^{\sqrt{1-4x}}$

domain $g(x)$ = domain of $\sqrt{1-4x}$

e is the natural number

$\sqrt{u} \leftarrow u \geq 0$

need $1-4x \geq 0$

$-1 \quad\quad -1$

$-4x \geq -1$

$\frac{-4x}{-4} \quad \frac{-1}{-4}$

$x \leq \frac{1}{4}$

$\rightarrow$ domain: $(-\infty, \frac{1}{4}]$

(c) $h(a) = \frac{\sqrt[3]{2x-5}}{3^{x+2}}$ $\leftarrow$ ratios

domain restrictions:

denominator $\neq 0$

numerator: $\sqrt[3]{2x-5}$ $\rightarrow$ odd root $(-\infty, \infty)$

$3^{x+2} \neq 0$

$3^x \neq 0$

$3^2 \cdot 3^x \neq 0$

$3^x = 0$ has no solution

$3^x \neq \frac{0}{9}$.

$\boxed{(-\infty, \infty)}$

Pr 5. Algebraically solve each equation for x.

if $b^x = b^y$, then $x=y$.

(a) $3^{5x+1} = 3^{2x-3}$. base 3

$3^{5x+1} = 3^{2x-3} \longrightarrow 5x+1 = 2x-3$

$-2x \quad -2x$

$3x+1 = -3$

$-1 \quad -1$

$\frac{3x}{3} = \frac{-4}{3}$

$\boxed{x = -4/3}$

(b) $7^{x^2} 7^{2x+1} = 1$.

$1 = b^0$

$7^{x^2} 7^{2x+1} = 7^{x^2+2x+1} = 1 = 7^0$

$x^2 + 2x+1 = 0$

$(x+1)^2 = 0$

$x+1=0$ or $x+1=0$

$\boxed{x=-1}$

common mistake → $x^2 \cdot (2x+1) = 1 \to \ldots$

(c) $\left(\frac{1}{4}\right)^{2x} = 8^{x-5}$. "easy" base is $b=2$

$4 = 2^2$

$8 = 2^3$

$\left(\frac{1}{4}\right)^{2x} = \left(\frac{1}{2^2}\right)^{2x} = (2^{-2})^{2x} = 2^{-2\cdot 2x} = 2^{-4x}$

$8^{(x-5)} = (2^3)^{(x-5)} = 2^{3(x-5)} = 2^{3x-15}$ (not 2^{3x-5})

$8 \wedge (x-5)$

Thus $2^{-4x} = 2^{3x-15}$ → $-4x = 3x-15$ and solve

$-3x \quad -3x$

$\frac{-7x}{-7} = \frac{-15}{-7}$

$\boxed{x = \frac{15}{7}}$

(d) $(\frac{1}{25})^{3x} \cdot 5^x - 1 = 0$

common base: $b=5$

$25 = 5^2$

$1 = 5^0$

$\left(\frac{1}{25}\right)^{3x} \cdot 5^x = 1$

$\left(\frac{1}{5^2}\right)^{3x} \cdot 5^x - 5^0 = 0$

$(5^{-2})^{3x} \cdot 5^x - 5^0 = 0$

$a^b + a^c = a^?$

no rule

$5^{-6x} \cdot 5^x - 5^0 = 0$

$5^{-6x+x} - 5^0 = 0$

$5^{-5x} - 5^0 = 0 \rightarrow 5^{-5x} = 5^0 \rightarrow -5x = 0 \rightarrow \boxed{x=0}$

$+5^0 \quad +5^0$

Pr 6. If you invest \$2000 in an account that earns interest at a rate of 3.16% per year, compounded monthly, how much will be in the account after 10 years? If the annual interest is compounded continuously instead of monthly, how much more will be in the account after 10 years compared to your previous answer?

Formula: $A(t) = P(1+\frac{r}{n})^{nt}$ → often included on test

P = initial amount

r = interest rate (as a decimal)

n = compounding period

t = time (in years)

continuous compounding (must memorize)

$A(t) = Pe^{rt}$

$A(10) = 2000\, e^{.0316 \times 10}$

≈ 2743.26

$P = 2000$ $\quad r = .0316$ $\quad n = 12$

$t = 10$ years

Difference $\approx \$1$

$\approx \$1.14$

$A(10) = 2000\left(1 + \frac{.0316}{12}\right)^{12\times10}$

$= \$2742.12$

Pr 7. If a company opens in 2018, and the company's revenue grows at an annual rate of 125% per year, the revenue function would be $\boxed{R(t) = R_0(\frac{5}{4})^t}$, where R_0 represents the initial revenue earned in 2018, and t represents the number of years since 2018. How much money did the company bring in, in revenue, in 2020, if the company's revenue is \$850,000 in 2023?

$R(t) = R_0\left(\frac{5}{4}\right)^t$

R_0 = initial revenue

t = number of years since 2018

2020: want $R(2) = R_0\left(\frac{5}{4}\right)^2$ what is R_0?

$t=2$ years after 2018

2023 corresponds to $t=5$

$850000 = R(5) = R_0\left(\frac{5}{4}\right)^5 = 850000$

$\rightarrow R_0 = \dfrac{850000}{(5/4)^5}$

$R(2) = R_0 (5/4)^2 = \dfrac{850000}{(5/4)^5}\cdot(5/4)^2$

$= 850000\left(\frac{5}{4}\right)^2\left(\frac{4}{5}\right)^5$

$\dfrac{1}{(a/b)^c} = \left(\dfrac{b}{a}\right)^c$

$\boxed{\$435,200} = 850000\,\dfrac{\cancel{5^2}\cdot 4^{\cancel{5}3}}{\cancel{4^2}\cdot 5^{\cancel{5}3}} = 850000 \times \dfrac{4^3}{5^3}$

next week

SECTION 5.7: COMBINING AND TRANSFORMING FUNCTIONS

$f(x) = |x|$ or x^2 or 2^x

Pr 1. Let $g(x) = -2|x-5|+7$. Identify the parent function, $f(x)$, and describe the series of transformations that need to preformed to transform $f(x)$ into $g(x)$.

$\pm a f(x \pm h) + k$

$x - h$ right

$x + h$ left

$|a| > 1$ stretch by a

$0 < |a| < 1$ shrink by $1/a$

$a < 0 \rightarrow$

$f(x) = |x|$

- horizontal shift right by 5 units
- vertical stretch by a factor of 2
- reflect across x-axis
- vertical shift up by 7 units

order: 1) horizontal shift 2) stretch/shrink 3) reflection 4) vertical shift

Pr 2. Let $p(x) = \sqrt{x}$. Write the function that results from performing the following transformations on $p(x)$:

(i) a horizontal shift left 3 units,
(ii) a vertical compression by a factor of 2,
(iii) and a vertical shift up 4 units.

$\sqrt{x+3}$

$\rightarrow \frac{1}{2}\sqrt{x+3}$

$\frac{1}{2}\sqrt{x+3} + 4$

compression = shrink

(trickier: compression by 7/5 $\rightarrow \frac{5}{7}\sqrt{x+3}$)

$f(x) = \frac{1}{2}\sqrt{x+3} + 4$

under $\sqrt{\ }$ sign

Pr 3. Let $f(x) = x^2$. Draw the graph of $g(x)$ obtained from $f(x)$ by the following series of transformations:

(i) a horizontal shift right 2 units,
(ii) reflection across the $x-$axis,
(iii) and a vertical shift down 5 units.

$x^2 \rightarrow \quad a(x-h)^2 + k$

vertex form of the parabola