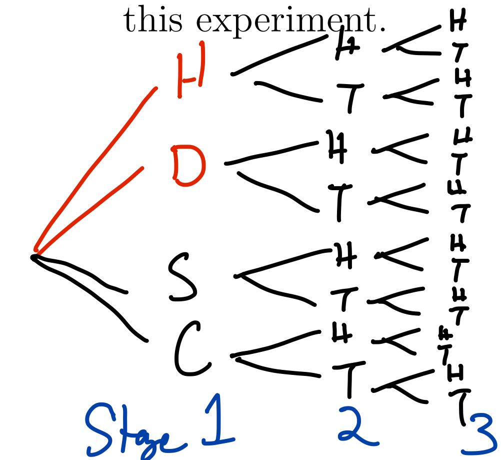
1 Week 7 HOGU: 4.1 - 4.4, Exam 2 Review

Problem 1. A student draws a card from a standard deck of 52 cards, noting the suit, then flips *two* coins, noting which side of the coin lands facing up.

(a) Make a tree diagram that details each element in the sample space of this experiment.



(b) How many *total possible events* are there in this experiment?

16 outcomes (16 branches on last stage)

$\rightarrow 2^{16} = \boxed{65536}$ total possible events

Problem 2. A veterinary network records the number of households with only a single pet in each of the Houston, Austin, and BCS regions. Out of 300 total households, the network provides the following data:

	Cats	**Dogs**	**Parrots**
Houston	75	35	10
~~**Austin**~~	~~40~~	~~40~~	~~20~~
~~**BCS**~~	~~30~~	~~45~~	~~5~~

A travelling salesman picks one of these households at random to visit. What is the probability that the salesman visits a Houston household that does *not* own a parrot?

$$\frac{75+35}{300} = \boxed{\frac{110}{300}}$$

Problem 3. A Math Learning Center tutor rolls two six-sided dice, one green and one blue, noting the side facing up when they land.

Let E be the event "the sum of the two dice is even". Let F be the event "a 4 is rolled on the blue die". Let G be the event "the green die shows a number greater than 7".

(a) How many outcomes are there in G^C?

There are 36 outcomes in total!

There are 0 outcomes in G!

So G^C must have $\boxed{36}$ outcomes!

(b) Verbally describe the outcomes in the event $E \cap F$.

The sum of the two dice is even AND a 4 is rolled on the blue die

(c) List the outcomes in $E \cap F$.

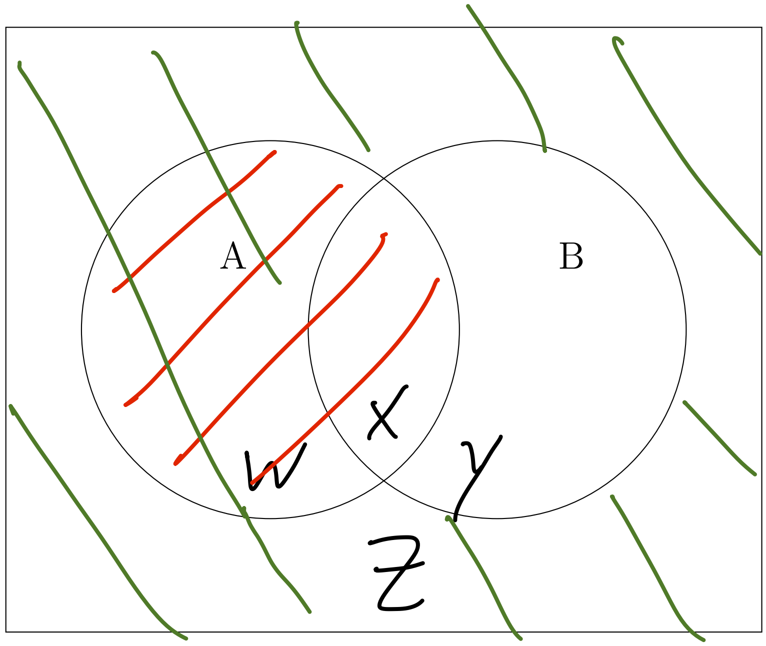
$\{(2,4),(4,4),(6,4)\}$

sum is 6 AND 4 is rolled | sum is 8 AND 4 is rolled | sum is 10 AND 4 is rolled

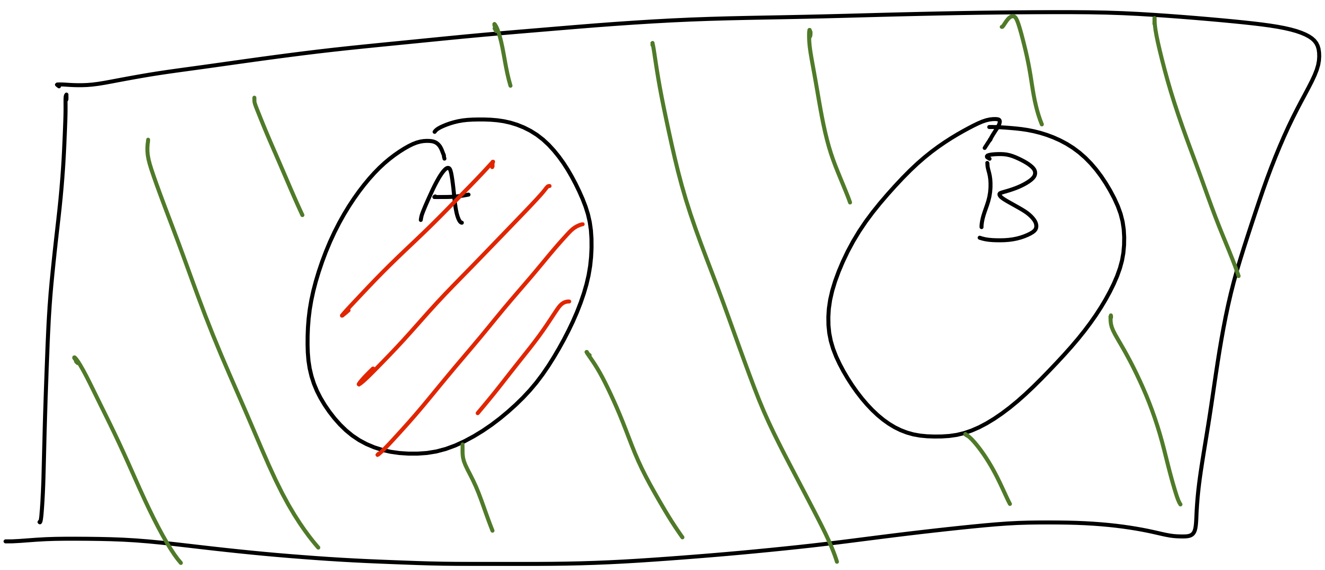
Problem 4. Shade in the given set in each Venn diagram.

(a) $A \cup B^C$

"Or" means "combine"



(b) $A \cup B^C$, where A and B are **mutually exclusive** (Draw the Venn diagram!)



Problem 5. Let A and B be two events such that $P(A) = 0.4$, $P(B) = 0.3$, and $P(A \cup B) = 0.5$. Compute $P(A \cap B)$. (Hint: you can use Venn diagrams... or you can use a rule from Section 4.3 notes.)

Union Rule: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

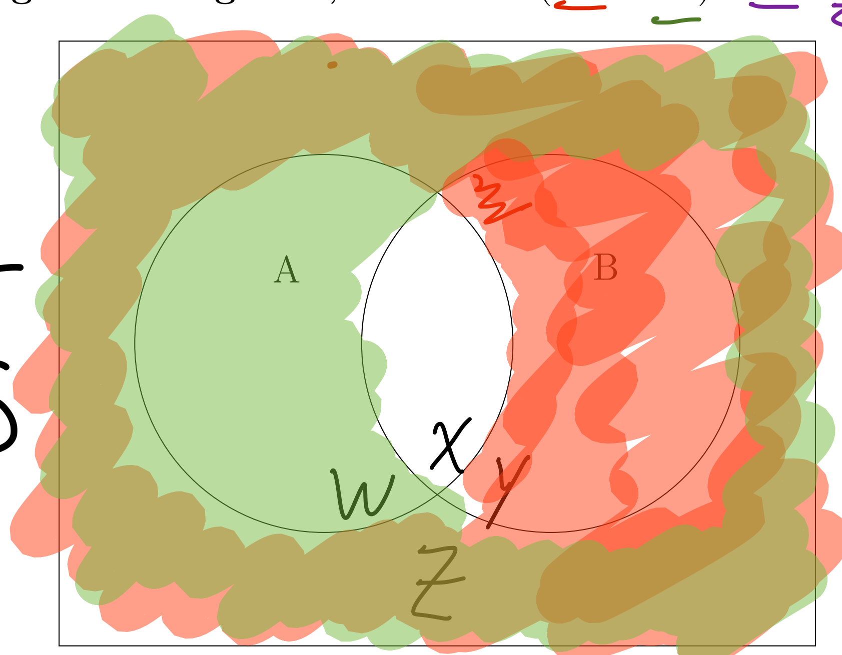
$$0.5 = 0.4 + 0.3 - x$$

$$0.5 = 0.7 - x$$

$$+x \quad -0.5 \quad ; \quad -0.5 \quad +x$$

$$P(A \cap B) = x = 0.2$$

Problem 6. Let A and B be two events such that $P(A^C) = 0.3$, $P(B^C) = 0.4$, and $P(A \cap B) = 0.5$. **Using Venn diagrams,** calculate $P(A^C \cap B^C)$. $= z$!



$P(A^c) = y + z = 0.3$

$P(B^c) = w + z = 0.4$

$P(A \cap B) = x = 0.5$

$w + x + y + z = 1$

RREF!

$$\begin{array}{ccccc} w & x & y & z & \text{constant} \end{array}$$
$$\left[\begin{array}{cccc|c} 0 & 0 & 1 & 1 & 0.3 \\ 1 & 0 & 0 & 1 & 0.4 \\ 0 & 1 & 0 & 0 & 0.5 \\ 1 & 1 & 1 & 1 & 1 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0.2 \\ 0 & 1 & 0 & 0 & 0.5 \\ 0 & 0 & 1 & 0 & 0.1 \\ 0 & 0 & 0 & 1 & \boxed{0.2} \end{array}\right]$$

Problem 7. A fair standard four-sided die is rolled, noting the number shown. Then, a spinner divided into 4 equal regions - red, green, blue, and yellow - is spun, noting the color. (Hint: is there a type of diagram that is useful to draw when given this type of experiment?)

(a) What is the probability that the spinner lands on blue?

$$\frac{4}{16}$$

(b) What is the probability that a 6 is rolled on the die *and* that the spinner lands on blue?

$$\frac{0}{16}$$

(careful... no "6"'s on a 4-sided die!)

Complement Rule!

(c) What is the probability that the die shows a 2 OR the spinner does not land on blue?

Union Rule!

A = die shows a 2

B = spinner lands on blue

$1 - P(B) = \frac{16}{16} - \frac{4}{16}$

$$P(A \cup B^c) = P(A) + P(B^c) - P(A \cap B)$$

$$= \frac{4}{16} + \frac{12}{16} - \frac{1}{16} = \frac{15}{16}$$

Problem 8. The probability distribution given below is missing a value:

X	1	2	3	4	5
$P(X)$	$\frac{1}{20}$	$\frac{3}{20}$	$\frac{5}{20}$	$\frac{4}{20}$	$\frac{x}{20}$

(a) Compute the missing value in the distribution.

Probabilities add to 1!

$$\frac{1}{20}+\frac{3}{20}+\frac{5}{20}+\frac{4}{20}+\frac{x}{20}=\frac{20}{20}$$

$$13+x=20$$

$$x=7!$$

$$\boxed{\frac{7}{20}}$$

(b) Calculate $P(X > 2)$.

$$P(X>2)=P(X=3)+P(X=4)+P(X=5)$$

$$=\frac{5}{20}+\frac{4}{20}+\frac{7}{20}=\boxed{\frac{16}{20}}$$

(c) Find the expected value of the random variable X in the first distribution table above.

$$1\cdot\frac{1}{20}+2\cdot\frac{3}{20}+3\cdot\frac{5}{20}+4\cdot\frac{4}{20}+5\cdot\frac{7}{20}$$

$$=\frac{1}{20}+\frac{6}{20}+\frac{15}{20}+\frac{16}{20}+\frac{35}{20}=\boxed{\frac{73}{20}}$$

Problem 9. *Set up, but do not solve*, the following linear programming problem.

> You have at most \$24,000 to invest in bonds and stocks. You have decided that the amount of money invested in bonds must be at least twice as much as that in stocks, but the money invested in bonds must not be greater than \$18,000. If you receive 6% profit on bonds and 8% profit on stocks, how much money should you place in each type of investment to maximize your profit?

Variables: b - amount of money, in dollars, invested in bonds

s - amount of money, in dollars, invested in stocks

P - profit, in dollars, from investing in stocks and bonds

Maximize/Minimize (circle one):

$$P = .06b + .08s$$

Subject to:

$$b + s \leq 24000$$
$$b \geq 2s$$
$$b \leq 18000$$

$$b \geq 0 \quad s \geq 0$$

Problem 10. *Set up but do not solve* the following linear programming problem:

A baker has 600 pounds of chocolate, 100 pounds of nuts, and 50 pounds of fruit, with which to make three types of candy. The following table details how much it takes to make each box of candy:

Candy Type	**Chocolate (lbs)**	**Nuts (lbs)**	**Fruit (lbs)**	**Selling Price ($)**
A	3	1	1	8
B	4	0	1/2	5
C	5	3/4	1	6

How many boxes of each type of candy should be made from the inventory available and sold in order to maximize revenue?

Variables:

x - number of boxes of Candy Type A made

y - number of boxes of Candy Type B made

z - number of boxes of Candy Type C made

R - the revenue, in dollars, the baker makes from candy boxes

Maximize:

$$R = 8x + 5y + 6z$$

Subject to:

actual available

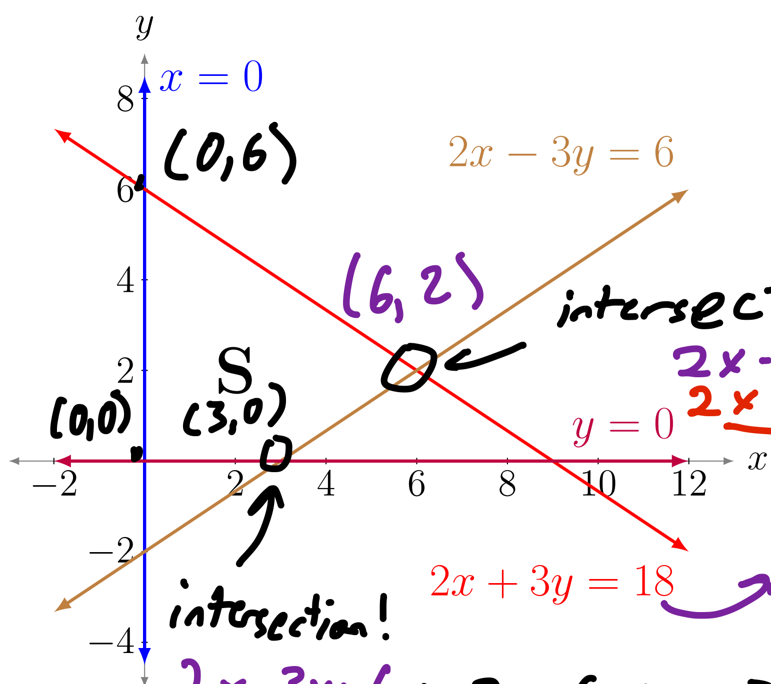
$$3x + 4y + 5z \le 600$$

$$x + \frac{3}{4}z \le 100$$

$$x + \frac{1}{2}y + z \le 50$$

$$x \ge 0, \quad y \ge 0, \quad z \ge 0$$

Problem 11. Consider the following solution set for a system of inequalities:



$2x - 3y = 6$
$2x + 3y = 18$
$4x = 24$
$x = 6$
$2(6) - 3y = 6$
$-3y = -6$
$y = 2$

$2x - 3y = 6$, $y = 0$: $2x = 6 \rightarrow x = 3$

The objective function for this system is $P = 4x + 5y$.
Does this objective function have a maximum in this solution set?

Yes! Feasible region is bounded.

Does this objective function have a minimum in this solution set?

Yes! Feasible region is bounded.

Find the maximum and minimum, whichever exist, of this objective function in this solution set using the Method of Corners.

Corner Points	$P = 4x + 5y$
(0,0)	$P = 0 + 0 = 0$
(3,0)	$P = 12 + 0 = 12$
(0,6)	$P = 0 + 30 = 30$
(6,2)	$P = 24 + 10 = 34$

minimum of $P = 0$ at (0,0)

maximum of $P = 34$ at (6,2)

Problem 12. Consider the following scenario:

> A Swix factory is busy making left Swix and right Swix. Every left Swix requires 3 minutes to manufacture and 1 minute to package. Each right Swix requires 4 minutes to manufacture and 2 minutes to package. There are 1.5 hours available for manufacturing and 0.5 hours available for packaging. If the profit the factory makes from each left Swix is \$0.50 and the profit from each right Swix is \$0.60, determine how many of each kind of Swix they should produce to maximize profit.

90 minutes

30 minutes

Let x be the "number of left Swix made by the Swix factory", and let y be the "number of right Swix made by the Swix factory". If P is the "profit the factory makes from making Swix", then these are the initial and final tableaus for this scenario (with slack variables s_1 and s_2):

Manufacturing

$3x+4y \le 90$

$x+2y \le 30$

Packaging

$$
\begin{array}{cccccc}
x & y & s_1 & s_2 & P & \text{const}
\end{array}
\qquad\qquad
\begin{array}{cccccc}
x & y & s_1 & s_2 & P & \text{const}
\end{array}
$$

$$
\left[\begin{array}{ccccc|c}
3 & 4 & 1 & 0 & 0 & 90 \\
1 & 2 & 0 & 1 & 0 & 30 \\
\hline
-.50 & -.60 & 0 & 0 & 1 & 15
\end{array}\right]
\to
\left[\begin{array}{ccccc|c}
0 & -2 & 1 & -3 & 0 & 0 \\
1 & 2 & 0 & 1 & 0 & 30 \\
\hline
0 & 2/5 & 0 & 1/2 & 1 & 15
\end{array}\right]
$$

Manufacturing: s_1 leftovers

s_2 leftovers

Packaging:

Given this **final tableau**, fill in the table below with how many resources are leftover in this solution:

Resource	Available	Used	Leftover
Manufacturing Minutes	90 minutes	90 min	90-90 = $\boxed{0 \text{ min}}$
Packaging Minutes	30 minutes	30 min	30-30 = $\boxed{0 \text{ min}}$

Solution: $x = 30$, $y = 0$

Manufacturing: Used: $\overset{3}{3}(\overset{x}{30}) \overset{+}{+} \overset{4}{4}(\overset{y}{0}) = 90$

Packaging: Used: $(\overset{x}{30}) \overset{+}{+} \overset{2}{2}(\overset{y}{0}) = 30$