TEXAS A&M UNIVERSITY
Math Learning Center

Problem 1

1. A sampling distribution is the probability distribution for which one of the following:
   a. A sample
   b. A sample statistic
   c. A population
   d. A population parameter
   e. None of the above

Problem 2

THE BLUE NUMBERS ARE DIRECTLY FROM Z TABLE

The average number of acres burned by all wildfires in the United States is 780 acres with a standard deviation 500 acres. The distribution of acres burned by wildfires is bell shaped.

2. What is the probability that a random wildfire burns more than 800 acres?

$P(X > 800) = P(Z > \frac{800-780}{500}) = P(Z > .04) = 1 - P(Z < .04) = 1 - 0.5160 = 0.484$

The average number of acres burned by all wildfires in the United States is 780 acres with a standard deviation 500 acres. Of course, some wildfires burn thousands of acres, so the distribution of acres burned by wildfires is strongly right skewed.

A simple random sample of 200 wildfires is to be taken from this population and the sample mean acres burned calculated. Use this to answer the next two questions.

3. What is the probability to have a sample mean that is higher than 800 acres? Find the closest answer.
   a. 0.391
   b. 0.484
   c. 0.516
   d. 0.286
   e. none of above

$P(\text{Sample Mean} > 800) = P(Z > \frac{800-780}{500/\sqrt{(200)}}) = P(Z > .5657) = 1 - P(Z < .5657) =$
$1 - 0.7142 = 0.2858$

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4. What is the third quartile (Q3) of the sampling distribution of sample mean acres burned? Find the closest answer.
   a. 1116 acres
   b. 756 acres
   c. 804 acres
   d. 782 acres
   e. 815 acres

P(Z < ?) = .75 (use z table to solve for ?), find that ? = .674
BUT WE'RE NOT DONE YET! We have to convert it back to the sample mean using the z-transformation formula:
$Z = \frac{X - MEAN}{SD/\sqrt{(n)}}$ => $.674 = \frac{X - 780}{500/\sqrt{200}}$ => X = .674(500/√(200)) + 780 = 803.83

Problem 3

The distribution of the number of eggs laid by a certain species of hen during their breeding period is 35 eggs with a standard deviation of 18.2. Suppose a group of researchers randomly samples 45 hens of this species, counts the number of eggs laid during their breeding period, and records the sample mean.

5. Define the random variable of interest.
   X : number of eggs laid by a certain species of hen.

6. Which are the parameters of the population distribution of X?
   $\mu = 35$ and $\sigma = 18.2$

7. Which is the shape of the population distribution of X?
   Unknown.

8. Suppose the researchers take all the possible samples of size 45 and estimate the sample mean for each sample. Which is the name of the distribution that they obtain by plotting all the estimated sample means?
   Answer: The sampling distribution of $\bar{X}$

9. Would you expect the shape of this distribution to be symmetric, right skewed, or left skewed? Explain your reasoning.
   Symmetric bell shape due to Central Limit Theorem, as $n \geq 30$ and observations are independent.

10. Calculate the variability of the sampling distribution and state the appropriate term used to refer to this value.
    Standard deviation of $(\bar{X}) = \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{18.2}{\sqrt{45}}$

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11. Suppose the researchers' budget is reduced and they are only able to collect random samples of 10 hens. The sample mean of the number of eggs is recorded, and we repeat these 1,000 times, and build a new distribution of sample means. How will the variability of this new distribution compare to the variability of the original distribution?

    Standard deviation of $(\underline{X}) = \sigma_{\underline{X}} = \frac{\sigma}{\sqrt{n}} = \frac{18.2}{\sqrt{10}}$, the variability will be larger.

Problem 4

Suppose that the mean outstanding credit card balance for all young couples in the US is $650 with a standard deviation of $420. The distribution is highly skewed to the right.

12. A simple random sample of 16 young couples is selected and their mean credit card balance is calculated. Therefore, we can use the Central Limit Theorem to estimate the probability that the sample mean is less than $500?

    No, because the population distribution is skewed and the sample size is less than 30.

13. Suppose now that one random sample of size 200 is selected from the population of young couples. The mean of this sample is $623. Based on this information, fill in the blanks:

    The shape of the distributions of this one random sample is _skewed to the right_ with mean ____$623______ and the shape of the sampling distribution of the sample mean for samples of size 200 is approximately ______Normal____ with a mean of ____$650_______.

14. What is the probability that the sample mean of a random sample of size 200 is larger than 680?

$$P(\bar{X} > 680) = P(Z > \frac{680 - 650}{420 / \sqrt{200}}) = P(Z > 1.01) = .1562$$

15. Researchers fed cockroaches a sugar solution. Ten hours later, they dissected the cockroaches and measured the amount of sugar in various tissues. Here are the amounts (in micrograms) of d-glucose in the hindguts of 5 cockroaches:

55.95 68.24 52.73 21.50 23.78

Suppose the researchers were not sure that the population of responses is Normal. Which of the following would then be a violated inference assumption?

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a. Normal sampling distribution
b. Population standard deviation σ
c. Equal variance
d. Random sample

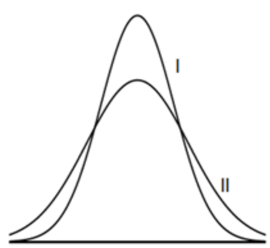
Problem 5

16. Suppose that 65% of all college women have been on a diet within the last 6 months. A survey is planned to interview a simple random sample of 100 college women if they were on a diet within the last 6 months. What is the probability that 70% or more of the women in the sample have been on a diet in the last 6 months? [4]

Thus, $P(\hat{p} > .7) = P\left(Z > \frac{.7 - .65}{\sqrt{(.65)(.35)/100}}\right) = 0.14727$

Problem 6

17. The following graph shows two sampling distributions of two sample proportions (Distribution I is taller.) The population from which we sample is the same in both cases. What can we conclude? Hint: Think in terms of the spread.



e. The sample proportion in I comes from a larger sample from that of II.
f. The sample proportion in II comes from a larger sample from that of I.
g. The sample sizes for both sample proportions are equal to each other.
h. The sample sizes are different, but it cannot be determined which is larger.
i. Something is wrong. If the population is the same in both cases, the sampling distributions should look the same.

Problem 7

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18. Historically, 51% of voters in a certain state voted for a Republican candidate as state governor. A new governor election is coming up and a survey of randomly selected 100 voters from this state will be conducted. What is the probability that more than 55% will vote for the Republican candidate? Find the closest answer.
    a) 0.83 b) 0.04 c) 0.79 d) 0.96 e) 0.21

Check CLT conditions

$P(\hat{p} > .55) = P(Z > \frac{.55 - .51}{\sqrt{(.51)(.49)/100}}) = .21181$

Problem 8

19. The state of California reported a total of 1904 cases of pertussis (whooping cough) for all of the year 2013, 12% of which were infants less than six months old. Infants this young cannot be vaccinated yet and must rely on herd immunity for protection from the virus.

If we took many random samples of 100 California residents diagnosed with pertussis in 2013 and computed the proportion in each sample who are infants, the distribution of these values would be

a. the sampling distribution of the proportion who are infants.
b. the sampling distribution of infants diagnosed with Pertussis.
c. a Normal distribution because of the law of large numbers.
d. the population distribution of infants diagnosed with Pertussis.

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