Note ♯10: Exam 04 review

Problem 1. Find all critical points and classify them as local maximum, local minimum, or saddle point. Find the local maximum and local minimum values. $f(x, y) = x^2 + y^4 + 2xy$

Problem 2. Find the extreme values(absolute maximum and minimum values) of f on the region described. $f(x, y) = x + y - xy, \quad D$ is the closed triangular region with vertices $(0, 0), (0, 2)$, and $(4, 0)$

Problem 3. Find $\int_C x ds$, where C is the right half of the circle $x^2 + y^2 = 4$, oriented counterclockwise.

Problem 4. Evaluate $\int_C z dx + (xy)dy$, where C is the line segment from $(-1, 1, 0)$ to $(1, 2, 0)$.

Problem 5. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x, y, z) = \langle y + z, xy, 2z\rangle$ and C is the curve given by $\mathbf{r}(t) = \langle 3, t, t^2\rangle$, $1 \le t \le 2$.

Problem 6. Let $\mathbf{F}(x, y) = \langle 2xy - y + 4, x^2 - x\rangle$. If $\mathbf{F}$ is conservative, find $f(1, 2)$, where f is the potential function for $\mathbf{F}$ with constant 0.

Problem 7. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x, y, z) = \langle 2x + 2y, 2x + 2y + 2z, 2y + 2z\rangle$ and C is the curve given by $\mathbf{r}(t) = \langle t^2, t, t^3 + 1\rangle$, $0 \le t \le 1$.

Problem 8. Find $\int_C \left(3y + 7e^{\sqrt{x}}\right) dx + (8x + 9\cos(y^2))\, dy$, where C is the boundary of the region enclosed by $y = x^2$ and $x = y^2$.

Problem 9. Find the surface area of the part of the paraboloid $x = y^2 + z^2$ that lies inside the cylinder $y^2 + z^2 = 9$.

Problem 10. Set up without evaluation $\iint_S (x^2 + y^2)\, dS$, where S is the hemisphere given by $x^2 + y^2 + z^2 = 4, z \ge 0$.

Problem 11. Find the flux of $\mathbf{F} = \langle x, y, 4 - z\rangle$ across S, where S is the part of the paraboloid $z = 4 - x^2 - y^2$ that is above the xy-plane. Use the positive (outward) orientation.

Problem 12. Use Stokes' Theorem to find $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F} = \langle 2xz, 4x^2, 5y^2\rangle$ and C is curve of intersection of the plane $z = x + 4$ and the cylinder $x^2 + y^2 = 4$, oriented counterclockwise when viewed from above.

Problem 13. Use the divergence theorem to evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F} = \left\langle x, e^{\sin(2z)}, (5 + 3y^{200})^7 \right\rangle$ and S is the surface bounded by $x^2 + y^2 = 4, z = 0, z = 5$.