1 MATH 140 HOGU: Exam 1 Review

Problem 1. Consider the line $7x - 3y = 11$. Circle the correct choice when prompted in the sentence below, then complete the sentence.

(a) If y decreases by 7 units, x (increases/decreases) by 3 units.

•Start by finding slope: $3y = 7x - 11 \to y = \frac{7}{3}x - \frac{11}{3}$

$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{-7}{\Delta x} = \frac{7}{3} \to -21 = 7\Delta x \to \underline{\Delta x = -3}$$

(b) If x increases by 11 units, y (increases/decreases) by $\frac{77}{3}$ units.

$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{\Delta y}{11} = \frac{7}{3} \to \Delta y = 11 \cdot \frac{7}{3} = \frac{77}{3}$$

(c) If y increases by 3 units, x (increases/decreases) by $\frac{9}{7}$ units.

$$\text{slope} = \frac{3}{\Delta x} = \frac{3}{\Delta x} = \frac{7}{3} \to \frac{7}{3}\Delta x = 3 \to \Delta x = 3 \cdot \frac{3}{7} = \frac{9}{7}$$

(d) If x decreases by 9 units, y (increases/decreases) by 21 units.

$$\text{slope} = \frac{\Delta y}{-9} = \frac{\Delta y}{-9} = \frac{7}{3} \to \Delta y = -9 \cdot \frac{7}{3} = -21$$

Problem 2. The web server that `https://reveille.tamu.edu/` is hosted on originally cost \$12,000 in the year 2001. In 2005 the server reached its scrap value of \$6,000.

(a) Compute the rate of depreciation of the web server.

$$\text{slope} = \frac{12000-6000}{2001-2005} = \frac{6000}{-4} = -1500$$

$\boxed{\$1500 \text{ per year}}$ rate of depreciation is always positive!

(b) Let t be the amount of time since 2001 (in years). Find a depreciation function $V(t)$ where V is the value (in dollars) of the web server.

• Remember, in the year 2001, $t=0$! What would be the year 2005? $t=4$!

Points: $(0, 12000), (4, 6000)$

Point-slope form

$$V(t) - 6000 = -1500(t-4)$$

$$V(t) - 6000 = -1500t + 6000$$

$$\boxed{V(t) = -1500t + 12000}$$

(c) In what year was the server worth \$7500?

$$7500 = V(t) = 1500t + 12000$$

$$1500t = 4500$$

$$t = 3$$

$$2001 + 3 = \boxed{2004}$$

(d) What was the value of the server in 2006?

NOTE: Scrap value is \$6000!

$$2006 - 2001 = 5$$

$$V(5) = -1500 \cdot 5 + 12000$$

$$= -7500 + 12000 = 4500 < 6000$$

$\boxed{\text{Value is } \$6000}$

Problem 3. Last October Taylor Swift was trying to decide how much to sell the vinyl album of *The Tortured Poets Department* for, so she had her producer Jack Antonoff collect some data for her here at Texas A&M. He reported back: a \$20 vinyl album would create a demand for 24,000 copies among A&M students, but suppliers would only stock 8,000 copies at that price. However, for a \$60 vinyl album suppliers would carry 24,000 copies at their stores, but at exactly that price and above A&M students would not buy the vinyl at all.

(a) Compute the supply function $p(x)$, where x is the number of vinyl albums sold.

$(8000, 20)$ + $(24000, 60)$

slope : $\frac{60-20}{24000-8000} = \frac{40}{16000} = \frac{1}{400}$

$y - 20 = \frac{1}{400}(x - 8000) \to y - 20 = \frac{1}{400}x - 20 \to \boxed{p = \frac{1}{400}x}$

(b) Determine the demand function $p(x)$, where x is as above.

$(24000, 20)$ + $(0, 60)$

slope : $\frac{20-60}{24000-0} = \frac{-40}{24000} = \frac{-1}{600}$

$y - 60 = \frac{-1}{600}(x - 0) \to \boxed{p = \frac{-1}{600}x + 60}$

(c) At what price would both students and vinyl album suppliers be happy?

$-\frac{1}{600}x + 60 = \frac{1}{400}x$

$\to 60 = \left(\frac{1}{400} + \frac{1}{600}\right)x = \frac{3+2}{1200}x$

$60 = \frac{1}{240}x$

$x = 14400$ items

$p = \frac{1}{400} 14400 = \boxed{\$36}$

Problem 4. The demand equation for a sports watch is given by

$$p = -0.025x + 50.$$

(a) Given this model, above what price will consumers not buy the watch?

Here x is the number of people who would pay for the sports watch at a price of p dollars.

No one pays $\longleftrightarrow$ $x=0$

$p = -0.025(0) + 50 = 50$

$\boxed{\$50}$

(b) Given this model, how many people would demand this item if it were free?

free $\longleftrightarrow$ $p=0$

$0 = -0.025x + 50 \rightarrow .025x = 50$

$x = \frac{50}{.025} = \boxed{2000 \text{ people}}$

Problem 5. Texas Instruments sell each TI-84 calculator to retailers for $50. It only costs them $18 to make one of these calculators. After ~~selling~~ manufacturing 9,000 calculators to Texas A&M students this year, TI records their total ~~profit~~ cost from the ~~sales~~ is to be $216,000.

calculator production

(a) State the cost function $C(x)$ of Texas Instruments, where x is the number of TI-84 calculators ~~sold to Texas A&M students~~ manufactured this year.

cost: $18 per calculator $\longleftrightarrow$ production cost

$$C(x) = mx + F \longleftrightarrow C(x) = 18x + F$$

If $x = 9000$, total cost $= C(x) = 216000$

$$C(9000) = 18(9000) + F = 216000$$

$$162000 + F = 216000$$

$$\underline{-162000 \qquad -162000}$$

$$F = 54000$$

$$\boxed{C(x) = 18x + 54000}$$

(b) Determine the profit function $P(x)$, where x is the same as it is in part (a).

Need revenue function first.

Price per calculator is $50 $\longrightarrow$

$$R(x) = 50x$$

$$P(x) = R(x) - C(x) = 50x - (18x + 54000)$$

$$= \boxed{32x - 54000}$$

(c) Can this company truly break-even? No! $P(x) = 0 \longrightarrow$

$0 = 32x - 54000 \rightarrow x = \frac{54000}{32} = 1687.5!$ Not a whole number

Problem 6. A clumsy problem writer for MATH-140 has written two equations:

$$p(x) = -\frac{1}{2}x + 40 \qquad \text{and} \qquad p(x) = 2x + 20,$$

where x is the number of items sold or supplied. However, the writer has forgotten which equation is meant to be the supply function and which is meant to be the demand function!

(a) Which is the demand function and which is the supply function? Why?

$p(x) = -\frac{1}{2}x + 40$ is the demand function: as price increases, the number of items sold decreases.

$p(x) = 2x + 20$ is the supply function: as price increases, the number of items supplied increases.

(b) How many items will consumers demand if the items are prices at $10?

$10 = -\frac{1}{2}x + 40 \rightarrow \frac{1}{2}x + 10 = 40 \rightarrow \frac{1}{2}x = 30$

$\rightarrow \boxed{x = 60 \text{ items}}$

(c) What is the equilibrium point of this item? Give your answer as an ordered pair.

Equilibrium point: $S(x) = D(x)$

$-\frac{1}{2}x + 40 = 2x + 20$

$+\frac{1}{2}x - 20 \qquad +\frac{1}{2}x - 20$

$20 = \frac{5}{2}x \rightarrow x = 20 \cdot \frac{2}{5} = 8$ items

$S(8) = 2 \cdot 8 + 20 = 36$ dollars

$\boxed{(8, 36)}$

Problem 7. Solve the system of equations given below using the stated methods:

$$\begin{aligned} (1)\quad 2x - y &= 1 \\ (2)\quad 3x + 2y &= 12 \end{aligned}$$

(a) Use the Substitution Method to solve.

(1) Solve for y:

$2x - y = 1$

$-1 + y \quad -1 + y$

$2x - 1 = y$

(2) Substitute $y = 2x - 1$:

$3x + 2(2x - 1) = 12$

$3x + 4x - 2 = 12$

$7x = 14 \rightarrow x = 2$

(3) Substitute x=2

$2(2) - 1 = y$

$3 = y$

$\boxed{(2,3)}$

(b) Use the Addition Method to solve.

Multiply $2 \cdot (1)$:

$$\begin{array}{rrcl} & 4x - 2y &=& 2 \\ + & 3x + 2y &=& 12 \\ \hline & 7x &=& 14 \\ & x &=& 2 \end{array}$$

(1) Substitute $x = 2$:

$2(2) - y = 1$

$-1 \quad +y \quad -1 \quad +y$

$3 = 4 - 1 = y$

$\boxed{(2,3)}$

(c) Use the RREF function in your calculator to solve.

$$\left[\begin{array}{cc|c} 2 & -1 & 1 \\ 3 & 2 & 12 \end{array}\right] \xrightarrow{RREF} \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 3 \end{array}\right]$$

$x = 2$

$y = 3$

$\boxed{(2,3)}$

Problem 8. Let $B = \begin{bmatrix} -15v & -3 & -12 \\ 9w & -5r & 1 \end{bmatrix}$ and let $A = \begin{bmatrix} -1 & 18 \\ -8q & 4 \\ 12m & 2n \end{bmatrix}$.

(a) Let $E = 3A^T - 2B$. Find e_{12}.

$$3A^T = 3\begin{bmatrix} -1 & -8q & 12m \\ 18 & 4 & 2n \end{bmatrix} = \begin{bmatrix} -3 & -24q & 36m \\ 54 & 12 & 6n \end{bmatrix}$$

$$2B = \begin{bmatrix} -30v & -6 & -24 \\ 18w & -10r & 2 \end{bmatrix}$$

$$3A^T - 2B = \begin{bmatrix} -3+30v & -24q+6 & 36m+24 \\ 54-18w & 12+10r & 6n-2 \end{bmatrix}$$

(b) Calculate BA.

$$\begin{bmatrix} -15v & -3 & -12 \\ 9w & -5r & 1 \end{bmatrix}\begin{bmatrix} -1 & 18 \\ -8q & 4 \\ 12m & 2n \end{bmatrix}$$

$$= \begin{bmatrix} 15v+24q-144m & -270v-12-24n \\ -9w+40rq+12m & 162w-20r+2n \end{bmatrix}$$

Problem 9. Assume matrices A, B, C, D, and E have the following sizes:

$$A \text{ is } 3 \times 5, \quad B \text{ is } 2 \times 4, \quad C \text{ is } 4 \times 3, \quad D \text{ is } 1 \times 5, \quad E \text{ is } 2 \times 1$$

Which of the following matrix operations are possible? What are the sizes of the resulting matrices?

(a) $A^T C^T$

A^T is 5×3

C^T is 3×4

$\underbrace{\underset{5\times 3}{A^T} \quad \underset{3\times 4}{C^T}}$ matrix is $\boxed{5 \times 4}$

inner dimensions agree: matrix product exists!

(b) $2BCA$

$\underset{B}{(2\times 4)}\underset{C}{(4\times 3)}\underset{A}{(3\times 5)}$

Multiplying a matrix times 2 does not change its size!

BCA is a 2×5 matrix! $\rightarrow$ $2BCA$ is a $\boxed{2\times 5}$ matrix

(c) $ED + A$

$\underset{2\times 1}{E} \quad \underset{1\times 5}{D} \quad + \quad \underset{3\times 5}{A}$

ED is a 2×5 matrix

cannot add a 2×5 matrix plus 3×5 matrix!

Matrix $\boxed{\text{does not exist}}$

(d) $AC^T B$

C^T is 3×4

$(3\times 5)(3\times 4)(2\times 4)$

Inner dimensions do not agree!

Matrix $\boxed{\text{does not exist}}$

Problem 10. The Aggie football bobblehead manufacturer manufactures a bobblehead at a cost of \$4 a unit and sells them for \$10 a unit. If making 1,000 bobbleheads costs the manufacturer \$16,000 in total, compute their break-even point. *Interpret* what your answer means in the context of this problem.

$R(x) = 10x$

$C(x) = 4x + b \longrightarrow C(x) = 4x + 12000$

$C(1000) = 4(1000) + b = 16000$

$4000 + b = 16000$

$b = 12000$

Break-even point: $R(x) = C(x)$

$10x = 4x + 12000$

$6x = 12000$

$x = 2000$

$\boxed{(2000, 20000)}$

$R(2000) = 10(2000) = 20000$

When the bobblehead manufacturer makes 2000 bobbleheads, they will make as much revenue from selling them as it cost to make them. They would be making \$20000 in revenue.

Problem 11. Texas A&M is starting a chain of Aggie-Strong Protein Shakes! There are small and large versions of these protein shakes. A batch of small shakes takes 20 minutes to prepare and 15 minutes to blend. A batch of large shakes takes 30 minutes to prepare and 30 minutes to blend. Texas A&M has 3,900 minutes dedicated to preparing the shakes and 3,300 minutes dedicated to blending. If they want to fully utilize all of these minutes, how many batches of each version (small or large) of the protein shake should they make?

(a) Let x be the number of batches of small shakes that the chain makes. Let y be the number of batches of large shakes that the chain makes. Write a system of two equations describing this situation.

Let x be the number of minutes the chain spends preparing protein shakes, and let y be the number of minutes the chain spends blending shakes.

$$20x + 30y = 3900$$
$$15x + 30y = 3300$$

(b) How many batches of each type of shake they should produce?

Subtract these two equations:

$$5x = 600 \rightarrow x = 120$$

Substitute in for the first equation:

$$20(120) + 30y = 3900$$
$$2400 + 30y = 3900$$
$$\rightarrow 30y = 1500$$
$$y = 50$$

They should make 120 small shakes and 50 large shakes.