Math 251/221. WEEK in REVIEW 3. Fall 2024

1. (a) Find the angle between the planes $x - 2y + z = 1$ and $2x + y + z = 1$.

   (b) Find symmetric equation for the line of intersection of the planes.

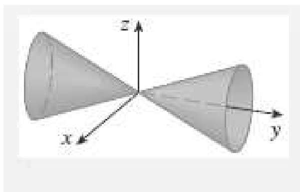
2. Find the distance from the point $(1, -2, 4)$ to the plane $4x - 6y + 2z = 3$.

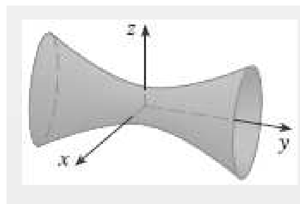
3. Match the equation with its graph. Give reasons for your choice.

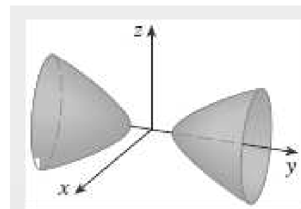
   a) $x^2 + 4y^2 + 9z^2 = 1$ b) $x^2 - y^2 + z^2 = 1$ c) $y = 2x^2 + z^2$

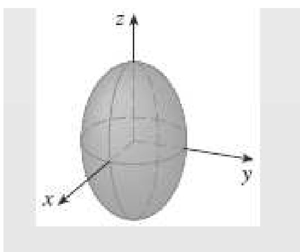
   d) $x^2 + 2z^2 = 1$ e) $9x^2 + 4y^2 + z^2 = 1$ f) $-x^2 + y^2 - z^2 = 1$

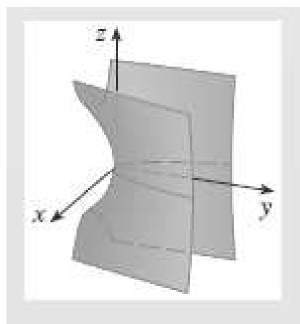
   g) $y^2 = x^2 + 2z^2$ h) $y = x^2 - z^2$

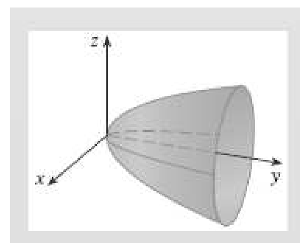


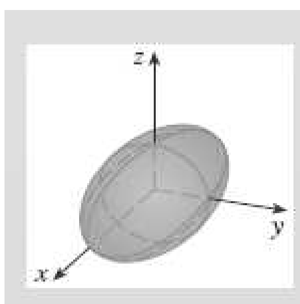


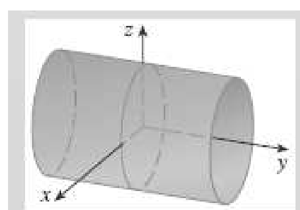












4. Reduce the equation to the standard form and classify the surface.

   (a) $z = (x-1)^2 + (y+5)^2 + 7$

   (b) $4x^2 - y^2 + (z-4)^2 = 20$

   (c) $x^2 + y^2 + z + 6x - 2y + 10 = 0$

5. Find the domain of $\mathbf{r}(t) =< \ln(4 - t^2), \sqrt{1+t}, \sin(\pi t) >$.

6. Find a vector equation for the curve of intersection of the surfaces $x = y^2$ and $z = x$ in terms of the parameter $y = t$.

7. Does the graph of the vector-function $\mathbf{r}(t) = \left\langle \frac{1-t^2}{t}, \frac{t+1}{t}, t \right\rangle$ lie in the plane $x - y + z = -1$?

8. Find the points where the curve $\mathbf{r}(t) =< 1-t, t^2, t^2 >$ intersects the plane $5x - y + 2z = -1$.

9. Find parametric equations of the line tangent to the graph of $\mathbf{r}(t) =< e^{-t}, t^3, \ln t >$ at the point $t = 1$.

10. Find symmetric equations of the line tangent to the graph of $\mathbf{r}(t) = \left\langle t^2, 4-t^2, -\frac{3}{1+t} \right\rangle$ at the point $(4, 0, 3)$.

11. Let

$$\mathbf{r}_1(t) =< \arctan t, t, -t^4 >$$

and

$$\mathbf{r}_2(t) =< t^2 - t, 2\ln t, \frac{\sin(2\pi t)}{2\pi} > .$$

   (a) Show that the graphs of the given vector-functions intersect at the origin.

   (b) Find their angle of intersection at the origin.

12. Evaluate the integral $\int_1^4 \left(\sqrt{t}\mathbf{i} + te^{-t}\mathbf{j} + \frac{1}{t^2}\mathbf{k} \right) dt$

13. A moving particle starts at an initial position $\mathbf{r}(0) =< 1, 0, 0 >$ with initial velocity $\vec{v}(0) = \mathbf{i} - \mathbf{j} + \mathbf{k}$. Its acceleration is $\vec{a}(t) = 4t\mathbf{j} + 6t\mathbf{j} + \mathbf{k}$. Find its velocity and position at time t.

14. Find the length of the curve given by the vector function $\mathbf{r}(t) = \cos^3 t\ \mathbf{i} + \sin^3 t\ \mathbf{j} + \cos(2t)\ \mathbf{k}$, $0 \le t \le \frac{\pi}{2}$.

15. For the curve given by $\mathbf{r}(t) = \langle \sin^3 t, \cos^3 t, \sin^2 t \rangle$, $0 \le t \le \frac{\pi}{2}$, find

   (a) the unit tangent vector $\mathbf{T}(t)$

   (b) the unit normal vector $\mathbf{N}(t)$

   (c) the binormal vector $\mathbf{B}(t)$

   (d) the curvature

16. A particle starts at the origin with initial velocity $\mathbf{i} - \mathbf{j} + 3\mathbf{k}$. Its acceleration is $\mathbf{a}(t) = 6t\mathbf{i} + 12t^2\mathbf{j} - 6t\mathbf{k}$. Find its position function.

Review for Exam 1.

1. Find the center and radius of the sphere given by the equation $2x^2 + 2y^2 + 2z^2 = 8x - 24z + 1$.

2. If $\mathbf{a} = 4\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$ and $\mathbf{b} = 2\mathbf{i} - 4\mathbf{k}$, find $4\mathbf{a} + 2\mathbf{b}$, $|\mathbf{a} - \mathbf{b}|$, $\mathbf{a} \cdot \mathbf{b}$, $\mathbf{a} \times \mathbf{b}$.

3. Find the vector of length 6 in the direction of $8\mathbf{i} - \mathbf{j} + 4\mathbf{k}$.

4. Find the scalar and vector projections of the vector $3\mathbf{i} - 3\mathbf{j} + \mathbf{k}$ onto the vector $2\mathbf{i} + 4\mathbf{j} - \mathbf{k}$.

5. Find the volume of the parallelepiped with adjacent edges PQ, PR and PS, if $P(-2, 1, 0)$, $Q(2, 3, 2)$, $R(1, 4, -1)$, $S(3, 6, 1)$.

6. (a) Find an equation of the plane that passes through the points $A(2, 1, 1)$, $B(-1, -1, 10)$, and $C(1, 3, -4)$.

   (b) Find **symmetric** equations for the line through B that is perpendicular to the plane in part (a).

   (c) A second plane passes through $(2, 0, 4)$ and has the normal vector $\langle 2, -4, -3 \rangle$. Find an equation of the plane.

(d) Find the angle between planes in parts (a) and (c).

(e) Find parametric equations for the line of intersection of the planes in parts (a) and (c).

7. (a) Find the point of intersection for the lines $\mathbf{r}_1(t) = \langle 1, 1, 0 \rangle + t \langle 1, -1, 2 \rangle$ and $\mathbf{r}_2(s) = \langle 2, 0, 2 \rangle + s \langle -1, 1, 0 \rangle$

(b) Find an equation of the plane that contains these lines.