MATH 150 - WEEK-IN-REVIEW 12

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FINAL EXAM REVIEW

1. Determine whether the Law of Sines or the Law of Cosines is needed to solve the triangle, then (if possible), solve the triangle.
   $\angle B = 17°$, $\angle C = 150°$ and $c = 65$.

Law of sines:

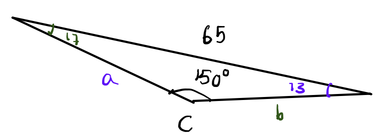
$$\frac{c}{\sin(\angle C)} = \frac{b}{\sin(\angle B)} \Rightarrow b = \frac{65\sin(17°)}{\sin(150°)} = \frac{65\sin(17°)}{\frac{1}{2}} = 130\sin(17°)$$

($\sin(150°)$: $180-30$)

now we have a pair & a side you can check: triangle exists ✓

$$\gamma = 180° - 150° - 17° = 13°$$

$$c = \frac{65\sin(13°)}{\sin(150°)} = 130\sin(13°)$$



2. Determine whether the Law of Sines or Law of Cosines is needed to solve the triangle, then (if possible), solve the triangle.
   $\angle A = 32°$, $a = 4.2$, $b = 12.4$

a pair & a side check: $a \overset{?}{\geq} h = b\sin(\angle A)$

$$h = 12.4\sin(32°) > 12.4\sin(30°) = 6.2$$

$$4.2 = a < 6.2$$

$\Rightarrow$ triangle doesn't exist

3. Determine whether the Law of Sines or Law of Cosines is needed to solve the triangle, then (if possible), solve the triangle.
   $a = 10,\ b = 12,\ c = 16$

Law of Cosines: $c^2 = a^2 + b^2 - 2ab\cos(\gamma) \Rightarrow \dfrac{-a^2 - b^2 + c^2}{-2ab} = \cos(\gamma)$

$$\gamma = \arccos\left(\frac{a^2+b^2-c^2}{2ab}\right) = \arccos\left(\frac{(10)^2+(12)^2-(16)^2}{2(10)(12)}\right) = \arccos\left(\frac{100+144-256}{2(10)(12)}\right)$$

$$= \arccos\left(\frac{-12}{2(10)(12)}\right) = \arccos\left(\frac{-1}{20}\right)$$

$$\alpha = \arccos\left(\frac{b^2+c^2-a^2}{2bc}\right) = \arccos\left(\frac{b^2+c^2-a^2}{2bc}\right) = \arccos\left(\frac{144+256-100}{2(12)(16)}\right)$$

$$= \arccos\left(\frac{300}{2(12)(16)}\right) = \arccos\left(\frac{25}{2(16)}\right) = \arccos\left(\frac{25}{32}\right)$$

$$\beta = \arccos\left(\frac{a^2+c^2-b^2}{2ac}\right) = \arccos\left(\frac{a^2+c^2-b^2}{2ac}\right) = \arccos\left(\frac{100+256-144}{2(10)(16)}\right)$$

$$= \arccos\left(\frac{212}{2(10)(16)}\right) = \arccos\left(\frac{106}{(10)(16)}\right) = \arccos\left(\frac{53}{5(16)}\right)$$

$$= \arccos\left(\frac{53}{80}\right)$$

4. Determine whether the Law of Sines or Law of Cosines is needed to solve the triangle, then (if possible), solve the triangle.
   $a = 13$, $b = 15$, $\angle A = 53°$

a pair & a side is given you can check: two triangles will exist

since $b\sin(\angle A) < a < b$

Law of Sines: $\frac{\sin\beta}{b} = \frac{\sin(\angle A)}{a}$

$$\beta = \arcsin\left(\frac{b\sin(\angle A)}{a}\right) = \arcsin\left(\frac{15\sin(53°)}{13}\right)$$

$$\gamma = 180° - 53° - \arcsin\left(\frac{15\sin(53°)}{13}\right)$$

Since we don't have calculator in exam to approximate I will keep all answers in exact form.

$$\frac{c}{\sin(\gamma)} = \frac{a}{\sin\alpha}$$

$$c = \frac{13\sin\left(\overbrace{180° - 53° - \arcsin\left(\frac{15\sin(53°)}{13}\right)}^{\gamma}\right)}{\sin(53°)}$$

you can approximate these numbers at home using a calculator, can you try finding the second triangle similar to your homework?

5. Find the component form, magnitude and directional angle of $\vec{v} = -\vec{u} - \sqrt{3}\vec{w}$, where $\vec{u} = 8\vec{i} + \sqrt{3}\vec{j}$, and $\vec{w} = -\frac{2}{\sqrt{3}}\vec{i} + \vec{j}$.

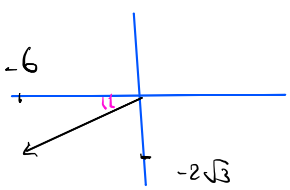
$\vec{u} = \langle 8, \sqrt{3} \rangle \qquad \vec{w} = \langle \frac{-2}{\sqrt{3}}, 1 \rangle$

$\vec{v} = -\langle 8, \sqrt{3} \rangle - \sqrt{3}\langle \frac{-2}{\sqrt{3}}, 1 \rangle = \langle -8, -\sqrt{3} \rangle + \langle 2, -\sqrt{3} \rangle$

$\boxed{\vec{v} = \langle -6, -2\sqrt{3} \rangle}$

Magnitude $\boxed{\|\vec{v}\|} = \sqrt{(-6)^2 + (-2\sqrt{3})^2} = \sqrt{36 + 4(3)} = \boxed{\sqrt{48}}$

Directional angle

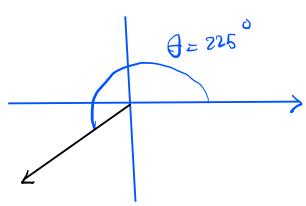


Q III & $\tan(\theta) = \frac{y}{x} = -\frac{2\sqrt{3}}{-6} = \frac{\sqrt{3}}{3}$

reference angle $\arctan\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi}{6}$

in Q III $\boxed{\theta = \pi + \frac{\pi}{6} = \frac{7\pi}{6}}$

6. Find the component form of $\vec{v}$ given its magnitude and the angle it makes with the positive x-axis.
$\|\vec{v}\| = 3$, $\theta = 225°$

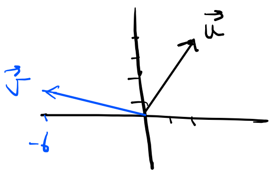


$\vec{v} = \langle \|\vec{v}\| \cos\theta, \|\vec{v}\| \sin\theta \rangle = \langle 3\cos(225°), 3\sin(225°) \rangle$ (225 = 180 + 45)

$\boxed{= \langle -\frac{3\sqrt{2}}{2}, -\frac{3\sqrt{2}}{2} \rangle}$

7. Find $\vec{u} \bullet \vec{v}$ and $(\vec{u} \bullet \vec{v})\vec{v}$ for $\vec{u} = \langle 2, 4 \rangle$ and $\vec{v} = \langle -6, 2 \rangle$.

$\vec{u} \cdot \vec{v} = \langle 2, 4 \rangle \cdot \langle -6, 2 \rangle = -12 + 8 = -4$



$(\vec{u} \cdot \vec{v})\vec{v} = -4 \cdot \langle -6, 2 \rangle = \langle 24, -8 \rangle$

8. Find the angle between $\vec{u}$ and $\vec{v}$ for $\vec{u} = \langle 2, 4 \rangle$ and $\vec{v} = \langle -6, 2 \rangle$.

$u \cdot v = \|\vec{u}\| \cdot \|\vec{v}\| \cos\theta$ where θ is the angle between two vectors

$\|\vec{u}\| = \sqrt{4 + 16} = \sqrt{20}$

$\|\vec{v}\| = \sqrt{36 + 4} = \sqrt{40}$

$-4 = \sqrt{20} \times \sqrt{40} \cos\theta$

$\frac{-4}{\sqrt{800}} = \cos\theta \Rightarrow \theta = \arccos\left(\frac{-4}{\sqrt{800}}\right)$

$= \arccos\left(\frac{-2}{5\sqrt{8}}\right)$

$= \arccos\left(\frac{-1}{5\sqrt{2}}\right)$

9. Compute the difference quotient for $f(x) = \dfrac{-x}{2x+1}$.

$$f(x+h) = \frac{-(x+h)}{2(x+h)+1} = \frac{-x-h}{2x+2h+1}$$

$$f(x+h) - f(x) = \frac{-x-h}{2x+2h+1} - \frac{-x}{2x+1} = \frac{-x-h}{2x+2h+1} + \frac{x}{2x+1}$$

Common denom

$$= \frac{(-x-h)(2x+1) + x(2x+2h+1)}{(2x+2h+1)(2x+1)} = \frac{-2x^2 - x - 2xh - h + 2x^2 + 2xh + x}{(2x+2h+1)(2x+1)}$$

Difference quotient

$$\frac{f(x+h)-f(x)}{h} = \frac{-h}{h(2x+2h+1)(2x+1)} = \frac{-1}{(2x+2h+1)(2x+1)}$$

10. Solve the following.

(a) $|9 + 2x| = 5x - 3$

$9 + 2x = 5x - 3$

$12 = 3x$

$4 = x$

$9 + 2x = -(5x - 3)$

$9 + 2x = -5x + 3$

$6 = -7x$

$x = -\frac{6}{7}$

Check: $|9 + 2(4)| \stackrel{?}{=} 5(4) - 3$

$|9 + 8| \stackrel{\checkmark}{=} 17$

$|9 + 2(-\frac{6}{7})| \stackrel{?}{=} 5(-\frac{6}{7}) - 3$

$|9 - \frac{12}{7}| \stackrel{?}{=} -\frac{30}{7} - 3$ 21

$\frac{51}{7} \neq -\frac{51}{7}$

$x = -\frac{6}{7}$ extraneous

one solution $x = 3$

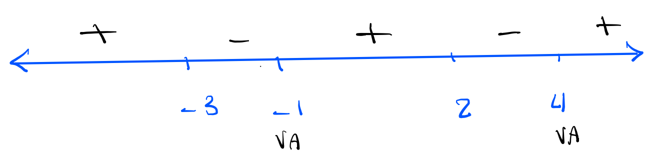
(b) $\dfrac{x^2 + x - 6}{2x^2 - 6x - 8} < 0$

$\dfrac{(x-2)(x+3)}{2(x^2 - 3x - 4)} = \dfrac{(x-2)(x+3)}{2(x-4)(x+1)} < 0$

$x^2 + x - 6 = 0 \implies x = 2, x = -3$

$2x^2 - 6x - 8 = 0 \implies x = 4, x = -1$

Sign chart:



$x \in (-3, -1) \cup (2, 4)$

11. Write the equation of the line perpendicular to $3x - 4y = 8$ and having the same y-intercept as $y = 5x - 1$.

$-4y = -3x + 8$

$y = \frac{3}{4}x - 2$ (m')

Perpendicular Slope $m = -\frac{1}{m'} = -\frac{4}{3}$

y-intercept is the same as y-intercept of $y = 5x - 1 \Rightarrow (0, -1)$

Equation of the line

$y - (-1) = -\frac{4}{3}(x - 0)$

$y + 1 = -\frac{4}{3}x \Rightarrow \boxed{y = -\frac{4}{3}x - 1}$

12. Given the equation $g(x) = -2x^2 + 4x + 9$, Identify the vertex, axis of symmetry, write the equation in vertex form and find the x-intercepts.

$h = \frac{-b}{2a} = \frac{-4}{-4} = 1$

$k = f\left(\frac{-b}{2a}\right) = f(1) = -2 + 4 + 9 = 11$

$\boxed{\text{vertex } (1, 11)}$

axis of sym. $\boxed{x = 1}$

vertex form $y = a(x-h)^2 + k$

$\boxed{y = -2(x-1)^2 + 11}$

x-intercept: $\left(1 + \frac{\sqrt{11}}{\sqrt{2}}, 0\right)$ and $\left(1 - \frac{\sqrt{11}}{\sqrt{2}}, 0\right)$

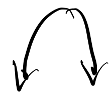
$-2(x-1)^2 + 11 = 0$

$-2(x-1)^2 = -11$

$(x-1)^2 = \frac{11}{2} \Rightarrow x - 1 = \pm\sqrt{\frac{11}{2}}$

$x = 1 \pm \frac{11}{2}$

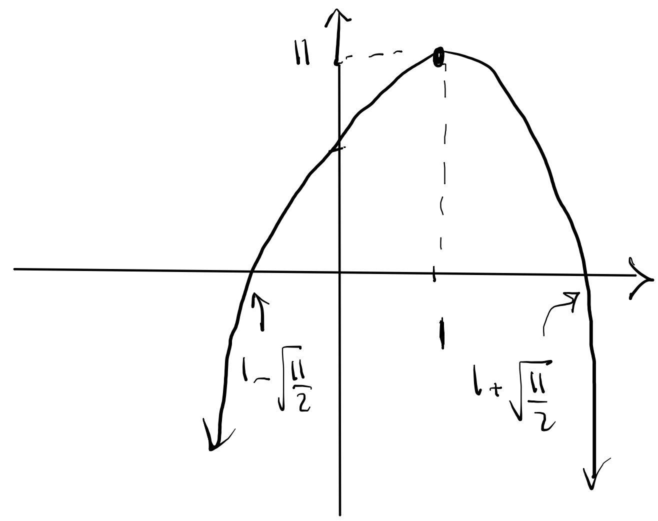
$g(x) = -2x^2 + 4x + 9$

leading term : $-2x^2$ $\Rightarrow$  $\Rightarrow$ vertex would be max

leading Coefficient: -2

So max value is 11

Constant : $+9$



y-intercept ? $(0, 9)$

interval of increase : $(-\infty, 1)$

interval of decrease: $(1, +\infty)$

13. Consider the function $g(x) = -(2x-4)^3 + 5$. Describe the transformations from $f(x) = x^3$ to $g(x)$.

or $= -(2(x-2))+5$

$f(x) = x^3$

① (Horiz. shift) Right 4 units $y_1(x) = f(x-4) = (x-4)^3$

② Horizontal shrink factor of 2 $y_2(x) = y_1(2x) = (2x-4)^3$

① Horizontal shrink factor of 2 $y_1(x) = y(2x) = (2x)^3$

② Right 2 units $y_2(x) = y_1(x-2) = (2(x-2))^3$

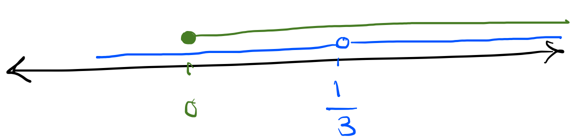
③ Reflect about x-axis $y_3(x) = -y_2(x) = -(2x-4)^3$

④ (vertical shift) up 5 units $y_4(x) = y_3(x) + 5 = -(2x-4)^3 + 5 = g(x)$

14. Find domain of the function $f(x) = e^{\frac{x^2-4}{\sqrt{3x}-1}}$

restrictions: denom. $\neq 0$ $\sqrt{3x} - 1 \neq 0 \Rightarrow \sqrt{3x} \neq 1 \Rightarrow 3x \neq 1$ $x \neq \frac{1}{3}$

even root $\sqrt{3x}$: $3x \geq 0 \Rightarrow x \geq 0$



$x \in [0, \frac{1}{3}) \cup (\frac{1}{3}, +\infty)$

15. Solve the following equation $\log_6(x-12) - \log_6(x) = \log_6(x-6)$.

$\log_6\left(\frac{x-12}{x}\right) = \log_6(x-6)$

Common base ⟹ Property

$\frac{x-12}{x} = x-6$

$x - 12 = x^2 - 6x$

$x^2 - 7x + 12 = 0$

$(x-3)(x-4) = 0$

$x = 3$ and $x = 4$

both not indomain since

$\log_6(3-12)$ & $\log_6(4-12)$

-9, -8

No solutions

16. Solve the equation $7^{2x+5} = 4^{1-x}$.

$\ln(7)^{2x+5} = \ln(4)^{1-x}$

$(2x+5)\ln(7) = (1-x)\ln 4$

$2x\ln(7) + 5\ln(7) = \ln 4 - x\ln(4)$

$2x\ln(7) + x\ln(4) = \ln 4 - 5\ln(7)$

$x(2\ln(7) + \ln(4)) = \ln(4) - 5\ln(7) \implies x = \frac{\ln(4) - 5\ln(7)}{2\ln(7) + \ln(4)}$

17. Expand the logarithmic expression $\log_8\left(\dfrac{(x^2+1)^4}{64(x^3-x)}\right)$

$$\log_8\left(\frac{(x^2+1)^4}{64(x^3-x)}\right) = \log_8 (x^2+1)^4 - \log_8 (64(x^3-x))$$

$x(x^2-1) = x(x-1)(x+1)$

$$= 4\log_8(x^2+1) - \left[\log_8 64 + \log_8(x^3-x)\right]$$

$$= 4\log_8(x^2+1) - \left[\log_8(8)^2 + \log_8(x) + \log_8(x-1) + \log_8(x+1)\right]$$

$$= 4\log_8(x^2+1) - 2 - \log_8(x-1) - \log_8(x+1)$$

18. Determine the quotient and remainder of the $(14x^3 - 2x^2 - \frac{1}{2}) \div (2x+1)$.

Quotient: $7x^2 - \frac{9}{2}x + \frac{9}{4}$ ← quotient

$2x+1 \,\overline{)\, 14x^3 - 2x^2 - \frac{1}{2}}$

$-(14x^3 + 7x^2)$

$-9x^2 - \frac{1}{2}$

$-(-9x^2 - \frac{9}{2}x)$

$\frac{9}{2}x - \frac{1}{2}$

$-(\frac{9}{2}x + \frac{9}{4})$

$-\frac{1}{2} - \frac{9}{4} = \frac{-2-9}{4} = \frac{-11}{4}$ ← remainder

$$\frac{14x^3 - 2x^2 - \frac{1}{2}}{2x+1} = 7x^2 - \frac{9}{2}x + \frac{9}{4} + \frac{-\frac{11}{4}}{2x+1}$$

$$= 7x^2 - \frac{9}{2}x + \frac{9}{4} - \frac{11}{4(2x+1)}$$

19. Determine the domain, vertical asymptote(s), horizontal asymptote(s), hole(s) and intercepts of the equation $g(x) = \dfrac{8x^2 - 12x + 4}{(x-1)(x+3)}$. State the end behavior, then sketch the graph.

$x \neq 1$, $x \neq -3$

Domain: $(-\infty, -3) \cup (-3, 1) \cup (1, +\infty)$

$$g(x) = \frac{8x^2 - 12x + 4}{(x-1)(x+3)} = \frac{4(2x^2 - 3x + 1)}{(x-1)(x+3)} = \frac{4(2x-1)(x-1)}{(x-1)(x+3)} = \frac{4(2x-1)}{x+3}$$

Hole at $x = 1$ location $\left(1, \frac{4(2-1)}{1+3}\right) = \left(1, \frac{4}{4}\right) = (1, 1)$

Vertical asy: $x = -3$

x-intercept: $\left(\frac{1}{2}, 0\right)$ $f(x) = 0 \Rightarrow \frac{4(2x-1)}{x+3} = 0 \Rightarrow x = \frac{1}{2}$

y-intercept: $\left(0, -\frac{4}{3}\right)$ $f(0) = \frac{4(0-1)}{0+3} = -\frac{4}{3}$

Horizontal asy.

$$g(x) = \frac{8x^2 - 12x + 4}{(x-1)(x+3)} = \frac{8x^2 - 12x + 4}{x^2 + 2x - 3}$$

Divide all terms by highest power of x in denominator

$$\frac{\frac{8x^2}{x^2} - \frac{12x}{x^2} + \frac{4}{x^2}}{\frac{x^2}{x^2} + \frac{2x}{x^2} - \frac{3}{x^2}} = \frac{8 - \frac{12}{x} + \frac{4}{x^2}}{1 + \frac{2}{x} - \frac{3}{x^2}}$$

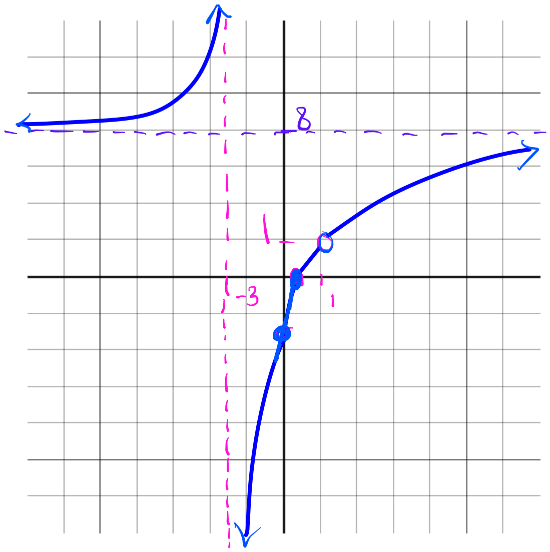
End behavior

as $x \to +\infty$ then

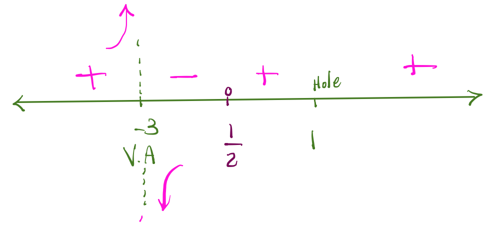
as $x \to -\infty$ then

$$\frac{8 - \frac{12}{x} + \frac{4}{x^2}}{1 + \frac{2}{x} - \frac{3}{x^2}} \to \frac{8}{1} = 8$$

$y = 8$ Horiz. asy.



Sign Chart



as $x \to -3^+$ then $g(x) \to -\infty$

as $x \to -3^-$ then $g(x) \to +\infty$

20. Given $f(x) = \dfrac{2x}{x+3}$, $g(x) = \dfrac{1}{x}$, find $(f \circ g)(x)$ its domains.

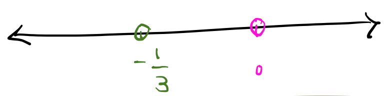
Dom(f): $x \in (-\infty, -3) \cup (-3, +\infty)$ Dom(g): $x \in (-\infty, 0) \cup (0, \infty)$

$$f \circ g\,(x) = f(g(x)) = \frac{2\left(\frac{1}{x}\right)}{\frac{1}{x} + 3 \cdot \frac{x}{x}} = \frac{\frac{2}{x}}{\frac{1+3x}{x}} = \frac{2}{1+3x}$$

Note: $x \neq 0$

$1 + 3x \neq 0$ $x \neq -\frac{1}{3}$

domain of $f \circ g$:



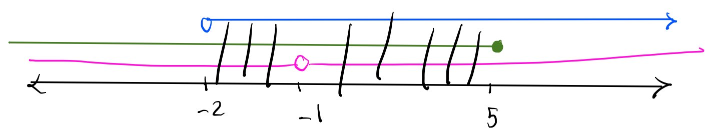
$x \in \left(-\infty, -\frac{1}{3}\right) \cup \left(-\frac{1}{3}, 0\right) \cup \left(0, +\infty\right)$

21. Given three equations $f(x) = \sqrt{5-x}+1$, $g(x) = e^{3x}-1$ and $h(x) = \log_3(x+2)$. Find domain of the function $S(x) = \dfrac{f(x)+g(x)}{h(x)}$. Then sketch f, g and h separately.

$S(x) = \dfrac{\sqrt{5-x} + e^x}{\log_3(x+2)}$

Restrictions

1. even root $\sqrt{5-x}$: $5 - x \geq 0 \Rightarrow 5 \geq x$
2. log $\log_3(x+2)$: $x+2 > 0 \Rightarrow x > -2$
3. denominator $\neq 0$: $\log_3(x+2) \neq 0 \Rightarrow x+2 \neq 3^0 = 1$ $\quad x \neq 1-2 = -1$



domain of S(x): $x \in (-2,-1) \cup (-1, 5]$

Graph of $f(x) = \sqrt{5-x} + 1$

domain of f: $x \in (-\infty, 5]$

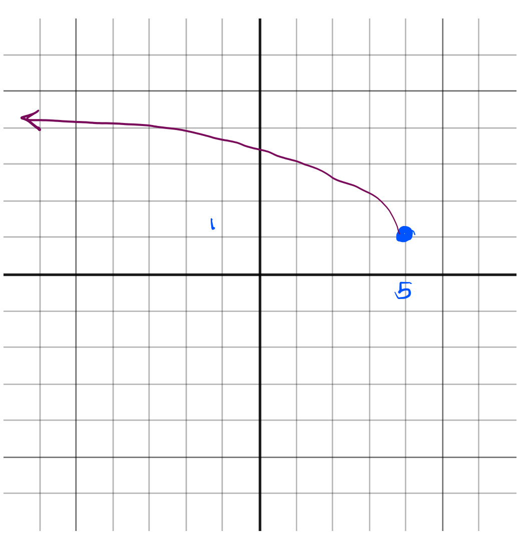
end point $(5, 1)$

End behavior as $x \to +\infty$ None!

as $x \to -\infty$ $\quad \sqrt{5-x} \to +\infty$

No horizontal asy. No vertical asymptotes

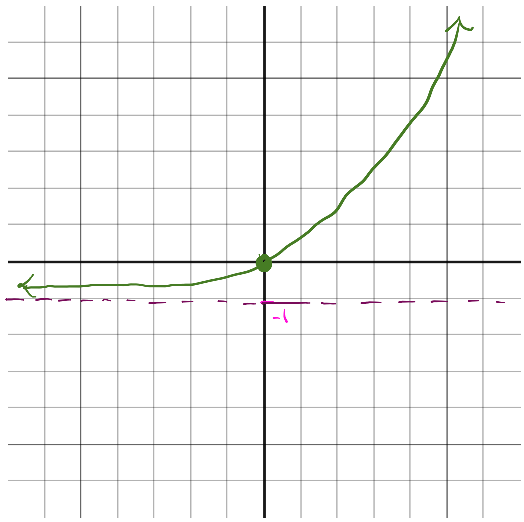
f(x)



y-intercept : $(0, \sqrt{5}+1)$

x-intercept: None

$\sqrt{5-x}+1 = 0 \quad \sqrt{5-x} = -1 \Rightarrow$ No Sol.



Graph of $g(x) = e^{3x} - 1$

domain of g: $(-\infty, +\infty)$

End behavior:

as $x \to +\infty$ $e^{3x} - 1 \to +\infty$

as $x \to -\infty$ $e^{3x} - 1 \to 0 - 1 = -1$

Horizontal asy. $y = -1$

No vertical asy.

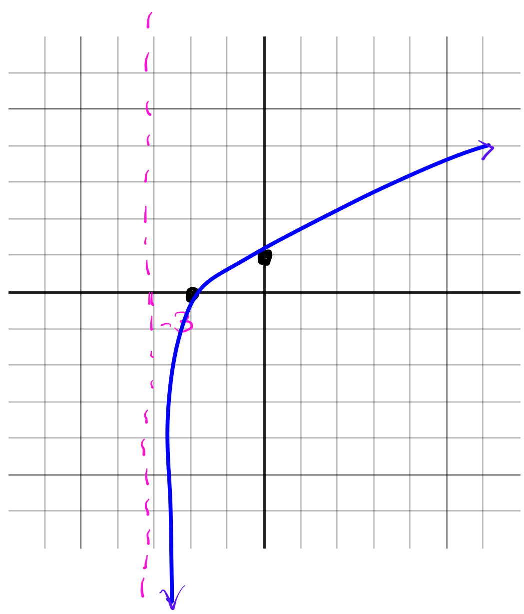
x-intercept: $(0, 0)$

$e^{3x} = 1$

$3x = \ln(1) = 0$

$x = 0$

& y-intercept $(0, 0)$



Graph of $h(x) = \log_3(x+3)$

domain $(-3, +\infty)$

No Horiz. asy.

Vertical asy. at $x = -3$

End behavior

as $x \to -3^+$ (from right) $\log_3(x+3) \to -\infty$

as $x \to +\infty$ $\log_3(x+3) \to +\infty$

x-intercept $(-2, 0)$

$\log_3(x+3) = 0$

$x + 3 = 3^0 = 1$

$x = -2$

y-intercept $(0, \log_3 3) = (0, 1)$

22. Solve the following system of equations. $\begin{cases} x^2 - y^2 = -4 \\ 2\sqrt{x} - y = 0 \end{cases}$ $\Rightarrow$ $y = 2\sqrt{x}$

Substitution Method:

$x^2 - (2\sqrt{x})^2 = -4$

$x^2 - 4x = -4$

$x^2 - 4x + 4 = 0$

$(x-2)^2 = 0 \longrightarrow$ $\boxed{x=2}$ $\boxed{y = 2\sqrt{2}}$

check: let $(x, y) = (2, 2\sqrt{2})$

in first equation $(2)^2 - (2\sqrt{2})^2 \stackrel{?}{=} -4$

$4 - 4(2) \stackrel{\checkmark}{=} -4$

in second equation

$2\sqrt{2} - 2\sqrt{2} \stackrel{\checkmark}{=} 0$

23. Simplify the expression $\dfrac{4\sin(x)\cos(x)}{2\cos(x)\cos(2x) - 2\sin(x)\sin(2x)}$. $= \dfrac{2\sin(2x)}{2\cos(x+2x)}$

$= \boxed{\dfrac{\sin(2x)}{\cos(3x)}}$

24. Complete the identity

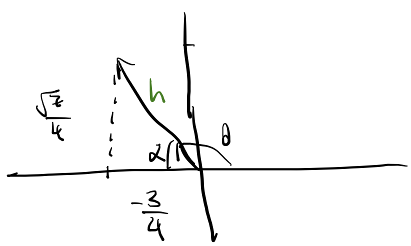
$$\frac{2}{\sec(x)+1} - \frac{2}{\sec(x)-1}$$

$$\frac{\sec(x)-1}{\sec(x)-1} \times \qquad \times \frac{\sec(x)+1}{\sec(x)+1}$$

$$= \frac{2(\sec(x)-1) - 2(\sec(x)+1)}{(\sec(x)-1)(\sec(x)+1)} = \frac{2\sec x - 2 - 2\sec x - 2}{\sec^2 x - 1}$$

$$= \frac{-4}{\sec^2 x - 1} = \frac{-4}{\tan^2 x} = -4\cot^2(x)$$

25. Given t corresponds to the point $\left(-\frac{3}{4}, \frac{\sqrt{7}}{4}\right)$ on a circle, find the value of $\sin(t) - \sec(t)$.



Q II $\quad h^2 = \left(-\frac{3}{4}\right)^2 + \left(\frac{\sqrt{7}}{4}\right)^2 = \frac{9}{16} + \frac{7}{16} = \frac{16}{16} = 1$

$h = 1$ unit circle

$$\sin(t) = \frac{\sqrt{7}}{4}$$

$$\sec(t) = \frac{1}{\cos(t)} = \frac{\text{hyp.}}{\text{adj.}} = \frac{1}{-\frac{3}{4}} = -\frac{4}{3}$$

$$\Rightarrow \sin(t) + \sec(t) = \frac{\sqrt{7}}{4} + \left(-\frac{4}{3}\right) = \frac{3\sqrt{7} - 16}{12}$$

26. Find all solutions to $\csc(3x) - \sin(3x) = 0$ then list the answers on the interval $[0, 2\pi)$.

$$\frac{1}{\sin(3x)} - \sin(3x) = 0 \rightarrow \frac{1-\sin^2(3x)}{\sin(3x)} = 0$$

$$\frac{\cos^2(3x)}{\sin(3x)} = 0$$

Domain restriction (denom)

Note: $\sin(3x) \neq 0$

$3x \neq k\pi$

$x \neq \frac{k\pi}{3}$

$\cos(3x) = 0$

$3x = \frac{\pi}{2} + k\pi$

$x = \frac{\pi}{6} + \frac{k\pi}{3}$

On $[0, 2\pi)$: let $k=0$: $\frac{\pi}{6}$, $k=1$: $\frac{\pi}{6} + \frac{\pi}{3} = \frac{3\pi}{6} = \frac{\pi}{3}$ (not in domain), $k=2$: $\frac{5\pi}{6}$, $k=3$: $\frac{7\pi}{6}$, $k=4$: $\frac{9\pi}{6} = \frac{3\pi}{2}$, $k=5$: $\frac{11\pi}{6}$

27. Find solutions to $\sin(2x) + \frac{1}{13}\sin(x) = 0$ on the interval $[0, 2\pi)$.

$2\sin(x)\cos(x) + \frac{1}{13}\sin x = 0$

$\sin(x)\left(2\cos x + \frac{1}{13}\right) = 0$

$\sin x = 0$ & $\cos(x) = \frac{1}{26}$

$x = k\pi$

$x = \arccos\left(\frac{1}{26}\right) + 2\pi k$

$x = 2\pi - \arccos\left(\frac{1}{26}\right) + 2\pi k$

on $[0, 2\pi)$: $0,\ \arccos\left(\frac{1}{26}\right),\ 2\pi - \arccos\left(\frac{1}{26}\right),\ \pi$

let $k=0$ let $k=1$

28. Evaluate each of the following:

(a) $\sin\left(\arccos\left(-\frac{1}{2}\right)\right)$ $= \sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2}$

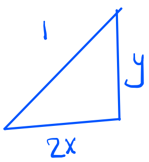
(b) $4\arctan\left(\cot\left(-\frac{\pi}{3}\right)\right)$ $= 4\arctan\left(-\frac{\sqrt{3}}{3}\right) = 4\left(-\frac{\pi}{6}\right) = \frac{-2\pi}{3}$

(c) $\tan\left(\arcsin\left(\frac{-\sqrt{2}}{2}\right)\right) + 2$ $= \tan\left(-\frac{\pi}{4}\right) + 2 = -1 + 2 = +1$

29. Simplify the following composition, then state its domain.

$$\tan\left(\arccos\left(2x\right)\right) = \tan(\theta) = \frac{\sqrt{1-4x^2}}{2x}$$

$\theta = \arccos(2x) \iff \cos(\theta) = \frac{2x}{1}$ $\quad \frac{\text{adj}}{\text{hyp}}$



find opposite: $y^2 + 4x^2 = 1$

$y^2 = 1 - 4x^2 \qquad y = \sqrt{1-4x^2}$

you can find the domain either from left-hand expression or from $\frac{\sqrt{1-4x^2}}{2x}$

domain: from $\arccos(2x) \Rightarrow -1 \leq 2x \leq 1 \Rightarrow -\frac{1}{2} \leq x \leq \frac{1}{2}$

& from $\tan(\theta) \Rightarrow \theta \neq \frac{\pi}{2} + k\pi$

$\arccos(2x) \neq \frac{\pi}{2} + k\pi$

$2x \neq \cos\left(\frac{\pi}{2} + k\pi\right) = 0 \Rightarrow x \neq 0$

$x \in \left[-\frac{1}{2}, 0\right) \cup \left(0, \frac{1}{2}\right]$

30. Find all vertical asymptotes of $y = 2\tan\left(\frac{x}{4} + \frac{\pi}{6}\right) - 5$ on the interval $\left[0, \frac{\pi}{2}\right)$

$$\frac{x}{4} + \frac{\pi}{6} = \frac{\pi}{2} + k\pi$$

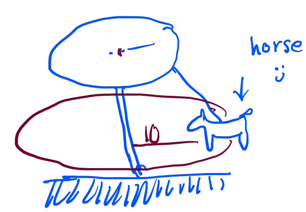
$$\frac{x}{4} = \frac{\pi}{2} - \frac{\pi}{6} + k\pi$$

$$\frac{x}{4} = \frac{2\pi}{6} + k\pi \implies \boxed{x = \frac{4\pi}{3} + 4k\pi}$$

on $\left[0, \frac{\pi}{2}\right)$: $\boxed{\frac{4\pi}{3}}$

31. Emmy chooses a horse that is 10 feet from the center of a merry-go-round. The merry-go-round makes $\frac{9}{2}$ rotations per minute. Determine Emmy's angular and linear velocity in radians per second.

1 revolution (or rotation) $\iff 2\pi$ radians



angular velocity per second

$$\omega = \frac{\theta}{t} = \frac{\frac{9}{2}\,\frac{\text{rotation}}{\text{min}} \times 2\pi\,\frac{\text{radians}}{\text{rotation}}}{60\ \text{seconds}/\text{min}}$$

$$= 9\pi\,\frac{\text{rad}}{\text{min}} \times \frac{1}{60}\,\frac{\text{min}}{\text{sec}} = \frac{9\pi}{60}\ \text{rad}/\text{sec.}$$

$$= \boxed{\frac{3\pi}{20}\ \text{rad}/\text{sec}}$$

$$v = r\cdot\omega = 10 \times \frac{3\pi}{20} = \boxed{\frac{3\pi}{2}\ \text{ft}/\text{sec}}$$

32. If you deposit \$2000 in an account with an annual interest rate of 3%, compounded continuously. Find the time it takes for the investment of \$2000 to grow to \$2500.

P $r=0.03$ $t=?$ $A(t)$

$$A(t) = Pe^{rt}$$

$$2500 = 2000e^{0.03t}$$

$$e^{0.03t} = \frac{25}{20} = \frac{5}{4}$$

$$0.03t = \ln\left(\frac{5}{4}\right)$$

$$t = \frac{\ln(5/4)}{0.03} \text{ or } = 100\left(\frac{\ln(5) - \ln(4)}{3}\right)$$

33. Find domain and range of $5\csc(2x - \frac{\pi}{3}) + 3$.

$$5\frac{1}{\sin(2x - \frac{\pi}{3})} + 3$$

Domain: $2x - \frac{\pi}{3} \neq k\pi \Rightarrow 2x \neq \frac{\pi}{3} + k\pi \Rightarrow x \neq \frac{\pi}{6} + \frac{k\pi}{2}$

$$\left\{x \mid x \neq \frac{\pi}{6} + \frac{k\pi}{2}\right\}$$

we knew: $\csc(\theta) \geq 1$ or $\csc(\theta) \leq -1$

$5\csc(\theta) + 3 \geq 5(1) + 3 = 8$ or $\csc(\theta) \leq 5(-1) + 3 = -2$

Range: $(-\infty, -2] \cup [8, +\infty)$

34. Given $y = 5\sin\left(\frac{1}{3}x - \frac{2\pi}{5}\right) - 4$, state the period and give an interval including the fundamental cycle of your function. Sketch the graph.

$A = 5$

$\frac{1}{3}x - \frac{2\pi}{5} = 0 \qquad \frac{x}{3} = \frac{2\pi}{5} \qquad x = \frac{6\pi}{5}$ (phase shift $-\frac{C}{B}$)

Period $\frac{2\pi}{\frac{1}{3}} = 6\pi$

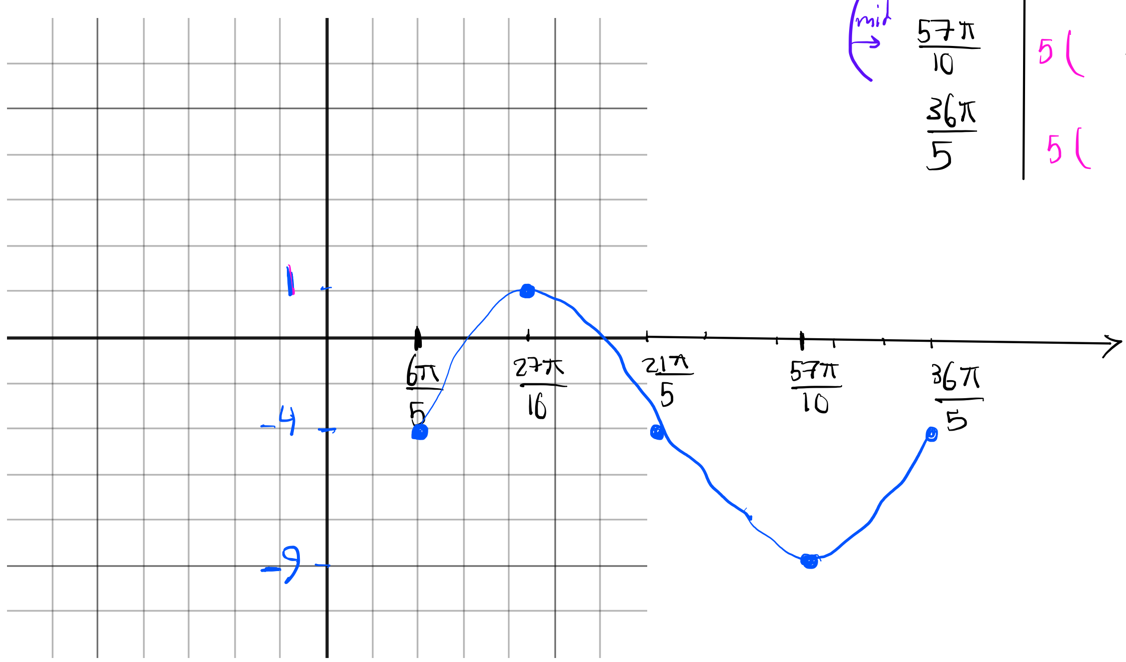
$D = -4$ base line (vertical shift)

start: $0 + \frac{6\pi}{5} = \frac{6\pi}{5}$

end: $6\pi + \frac{6\pi}{5} = \frac{36\pi}{5}$

key points:

x	$5\sin\left(\frac{1}{3}x - \frac{2\pi}{5}\right) - 4$
$\frac{6\pi}{5}$	$5(0) - 4 = -4$
$\frac{27\pi}{10}$ (middle)	$5(1) - 4 = 1$
$\frac{21\pi}{5}$	$5(0) - 4 = -4$
$\frac{57\pi}{10}$ (mid)	$5(-1) - 4 = -9$
$\frac{36\pi}{5}$	$5(0) - 4 = -4$



35. For $z_1 = 2 + 3i$ and $z_2 = 4 - i$, find $z_1 + z_2, z_1 - z_2, z_1 \cdot z_2$.

$z_1 = 2 + 3i$

$z_2 = 4 - i$

$z_1 + z_2 = 6 + 2i$

$z_1 - z_2 = -2 + 4i$

$z_1 \cdot z_2 = (2+3i)(4-i)$

$= 8 - 2i + 12i - 3i^2$

$= 8 + 10i + 3 = 11 + 10i$

Note: These problems are just a sample and should not be your sole source of studying. For additional practice, refer to your lecture notes, homework and previous Week-in-Reviews. You can also find other problems in your textbook.