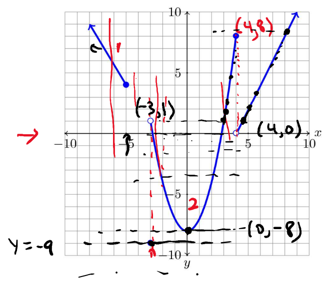
Exam 3 Review over Chapter 5

Pr 1. State the domain and range of the function given in the graph below, using interval notation.



domain: $(-\infty, -5] \cup [-3, 4] \cup (4, \infty)$

$= \boxed{(-\infty, -5] \cup [-3, \infty)}$

range: $[-9, -9] \cup [-8, \infty)$

horizontal lines or $\{-9\} \cup [-8, \infty)$

Pr 2. Determine if the given function is a polynomial function. If the answer is yes, state the degree, leading coefficient, and constant term.

$$f(x) = 2^5 - 17x^7 + 12 - 42x^2.$$

yes

No if $f(x) = x^{3.1}$, x^{-1}, $x^{1/2}$

$2^5 = 32$

degree $= 7$

leading coef. $= -17$

constant term $= 12 + 2^5$
$= 12 + 32$
$= 44$

$f(x) = -17x^7 - 42x^2 + 44$

Pr 3. Let $f(x) = -3x^2 + 18x - 15$. Find the domain, range, x-intercepts, and y-intercept.

$-3<0$

domain: $(-\infty, \infty)$

range: $(-\infty, 12]$

vertex $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

k

$[k, \infty)$ or $(-\infty, k]$

$a = -3 < 0$ $\quad b = 18$ $\quad c = -15$

$h = \frac{-b}{2a} = \frac{-18}{2\cdot(-3)} = \frac{-18}{-6} = 3$

$k = f\left(\frac{-b}{2a}\right) = f(3) = -3\cdot 3^2 + 18\cdot 3 - 15$

$= 12$

x-intercept

$-3x^2 + 18x - 15 = 0$

$x = \dfrac{-18 \pm \sqrt{18^2 - 4(-3)(-15)}}{2(-3)}$

$x = \dfrac{-18 \pm \sqrt{144}}{-6}$

$= \dfrac{-18 \pm 12}{-} = \begin{cases} \frac{-6}{-6} = \frac{1}{1} \\ \frac{-30}{-6} = 5 \end{cases}$

$\boxed{\text{x-intercepts: } (1,0),\ (5,0)}$

y-intercept: $(0, f(0)) = (0, -15)$

constant term.

Pr 4. The price-demand function (in dollars) for a particular item is given by $p(x) = -0.06x + 56$, where x is the number of items. The company who produces these items has a production cost of \$5 per item and fixed costs of \$150. Determine the maximum profit for the company from the sales of this item.

maximize $P(x) = R(x) - C(x)$

$R(x) = p(x)x = (-.06x + 56)x$

$= -.06x^2 + 56x$

$C(x) = 5x + 150$

$P(x) = -.06x^2 + 56x - (5x + 150)$

$= -.06x^2 + 51x - 150$

(h,k)

Maximum

$h =$ # of items we need to sell to max profit

$k =$ maximum profit.

$h = \frac{-b}{2a} = \frac{-51}{2(-.06)} = \frac{5100}{2\times 6} = \frac{5100}{12} = \frac{1700}{4} = 425.$

425 items

$P(425) = -.06\times 425^2 + 51\times 425 - 150$

$\boxed{= \$\ 10{,}687.50}$

Pr 5. State the domain of the following rational function. Then classify each each domain restriction as the location of a hole or vertical asymptote on the graph of the function. $f(x) = \dfrac{(3x-2)(2x-5)(x-5)}{(x-5)(2x+5)(x+1)}$

Domain $= \{x : \text{denominator} \neq 0\}$

den. $= (x-5)(2x+5)(x+1) = 0$

$x - 5 = 0$ or $2x + 5 = 0$ or $x + 1 = 0$

$x = 5$ $\quad \frac{2x}{2} = \frac{-5}{2}$ $\quad x = -1$

$x = -\frac{5}{2}$

$-5/2 \quad -1 \quad 0 \quad 5$

Domain: $(-\infty, -5/2) \cup (-5/2, -1) \cup (-1, 5) \cup (5, \infty)$

hole at $x = 5$

vertical asymptote at $x = -5/2$, $x = -1$

Pr 6. Compute and simplify the difference quotient of $g(x) = \dfrac{2x}{3x-1}$.

$\dfrac{g(x+h) - g(x)}{h}$ ← rise ← run

Pr 6. Compute and simplify the difference quotient of $g(x) = \frac{2x}{3x-1}$.

$$\frac{g(x+h) - g(x)}{h} \begin{matrix} \leftarrow \text{rise} \\ \leftarrow \text{run} \end{matrix}$$

$$\frac{g(x+h) - g(x)}{h} = \frac{1}{h}\left(\frac{2(x+h)}{3(x+h)-1} - \left(\frac{2x}{3x-1}\right)\right) = \frac{1}{h}\left(\frac{2x+2h}{3x+3h-1} + \frac{-2x}{3x-1}\right)$$

$$= \frac{1}{h}\left(\frac{(2x+2h)}{(3x+3h-1)}\frac{(3x-1)}{(3x-1)} + \frac{(-2x)}{(3x-1)}\frac{(3x+3h-1)}{(3x+3h-1)}\right)$$

$$= \frac{1}{h}\left(\frac{(2x)(3x) + 2x(-1) + 2h(3x) + 2h(-1) - (2x)(3x) - 2x(3h) - 2x(-1)}{(3x+3h-1)(3x-1)}\right)$$

$$= \frac{1}{\cancel{h}}\left(\frac{-2\cancel{h}}{(3x+3h-1)(3x-1)}\right)$$

$$= \boxed{\frac{-2}{(3x+3h-1)(3x-1)}}$$

more practice:

$f(x) = \sqrt{ax+6}$

$\sqrt{2x-3}$

"conjugate"

Pr 7. State the domain of $f(x) = \frac{(3x-2)\sqrt{1-2x}}{(x+5)^{4/7}}$ using interval notation.

rules: $1-2x \geq 0$

$(x+5)^{4/7} \to$ no restriction

7 is odd

$1-2x \geq 0$ Then $-2x \geq -1$

$\frac{-2x}{-2} \geq \frac{-1}{-2}$

$x \leq \frac{1}{2}$

$(-\infty, \frac{1}{2}]$

Denominator:

$(x+5)^{4/7} \neq 0$

iff $(x+5) \neq 0$

$x \neq -5$

$$\boxed{\text{domain: } (-\infty, -5) \cup (-5, \tfrac{1}{2}]}$$

Pr 8. Rationalize $f(x) = \frac{(\sqrt{x+6}-\sqrt{x})}{(6)} \frac{(\sqrt{x+6}+\sqrt{x})}{(\sqrt{x+6}+\sqrt{x})}$

$$= \frac{(\sqrt{x+6})^2 + \sqrt{x+6}\sqrt{x} - \sqrt{x+6}\sqrt{x} + (-\sqrt{x})(\sqrt{x})}{6(\sqrt{x+6}+\sqrt{x})}$$

$$= \frac{(\sqrt{x+6})^2 - (\sqrt{x})^2}{6(\sqrt{x+6}+\sqrt{x})}$$

$$= \frac{x+6-x}{6(\sqrt{x+6}+\sqrt{x})}$$

$$= \frac{\cancel{6}}{\cancel{6}(\sqrt{x+6}+\sqrt{x})}$$

$$= \frac{1}{\sqrt{x+6}+\sqrt{x}}$$

wrong approaches:

$(\sqrt{x+6}-\sqrt{x})^2 \neq x+6-x$?

$(a+b)^2 \neq a^2+b^2$

$a^2+2ab+b^2$

Pr 9. State the domain of $f(x) = \begin{cases} \frac{1}{(x+5)(x-3)} & x < -3 \\ \ln(12-2x) & x \geq 3 \end{cases}$ using interval notation.

$(-\infty, -3)$

$[3, \infty)$

$(x+5)(x-3) = 0$

$x+5=0 \quad x-3=0$

$x=-5 \quad x=3$

in $(-\infty,-3)$

not in $(-\infty,-3)$

domain of $\frac{1}{(x+5)(x-3)} = (-\infty,-5) \cup (-5,3) \cup \ldots$

domain of $\ln(12-2x)$

$12-2x > 0$

$\frac{-2x}{-2} > \frac{-12}{-2}$

$x < 6 \leftarrow (-\infty, 6) \cap [3,\infty)$

3

6

$\boxed{\text{domain: } (-\infty,-5) \cup (-5,-3) \cup [3,6)}$

Pr 10. State the domain of $h(x) = 2^{\sqrt{3-4x}}$ using interval notation

exponent: $\sqrt{3-4x}$

domain of $h(x)$ = domain of $\sqrt{3-4x}$

$3-4x \geq 0$

$\frac{-4x}{-4} \geq \frac{-3}{-4}$

$x \leq 3/4$

$\boxed{\text{domain: } (-\infty, 3/4]}$

for $f(x) = \sqrt[2k]{g(x)}$

answer $(-\infty, a]$

or $[a, \infty)$

$f(x) = \ln \ldots$

$(-\infty, a)$

(a, ∞)

$$27 \cdot 9^{(2x-1)} = 81$$

Pr 11. Algebraically solve: $27 \cdot 9^{2x-1} = 81$.

Base 3

$27 = 3^3$

$9 = 3^2$

$81 = 3^4$

$(a^b)^c = a^{b \cdot c}$

$$3^3 \cdot (3^2)^{(2x-1)} = 3^4 \Rightarrow 3^3 \cdot 3^{2\cdot(2x-1)} = 3^4$$

$$\Rightarrow 3^{2(2x-1)+3} = 3^4$$

Tricky

$3^{\text{something}} = 1 \ (= 3^0)$

something $= 0$

$$\Rightarrow 2(2x-1) + 3 = 4$$

$4x - 2 + 3 = 4$

$4x + 1 = 4$

$4x = 3$

$\boxed{x = 3/4}$

Pr 12. You would like to save \$2000 by making an initial deposit in a savings account earning annual interest at a rate of 0.35% and leave it there for 4 years. How much should be place in the account initially, if no other deposits are made during that time and the account is compounded continuously?

$\rightarrow P(t) = Ae^{rt}$

$r =$ rate

know: $P(4) = 2000$

$r = .35\% = .0035$

$2000 = A e^{4 \times .0035} = A e^{.014}$

Type I: find A.

$A = \dfrac{2000}{e^{.014}} \approx \$1972.19$

Type II: given r, A, find t.

b) when will there be \$2100 in the account?

$2100 = 1972.19\, e^{.0035t}$

$\ln(e^{.0035t}) = \ln\left(\dfrac{2100}{1972.19}\right)$

$\ln(e^x) = x$

$.0035t = \ln\left(\dfrac{2100}{1972.19}\right)$

$t = \dfrac{1}{.0035} \ln\left(\dfrac{2100}{1972.19}\right)$

$= \ln\left(\dfrac{2100}{1972.19}\right) / .0035$

$\approx 17.9\ldots$ years