Section 1.1: Basic Matrix Operations

- Size (Dimensions): rows × columns
- Entries: Labeled based on row and column position, a_{ij}
- Addition/Subtraction:
  - Matrices must be the same size for the operation to be performed
  - Combined corresponding entries based on operation given
- Scalar Product: multiplying a matrix by a constant results in a matrix of the same size
- Transpose of a matrix A: A^T
- Matrix Equality: two matrices are equal if they are the same size AND corresponding entries are equal
- Operations of matrices which contain variables must be done by hand

Pr 1. Use the given matrices A, B, C, D, and E below, to Determine the dimensions of the resulting matrices, if possible. If the given operation is not possible, explain why.

A is a 1×2, B is a 1×2, C is a 2×3, D is a 2×3, E is a 3×2

a. $\frac{1}{2}B$. is a 1×2 matrix.

b. $(B + C)^T$. ← not possible, since B has 1 row and C has 2 rows.

c. $E^T - D + 2C$. → 2×3 ⟶ $E^T - D + 2C$ is a 2×3 matrix.

2×3

E^T has size 2×3

Pr 2. Use the given matrices A, B, C, D, E, and F below, to compute each operation, if possible.

$A = \begin{bmatrix} 5 & 2 & 6 \\ 6 & \frac{1}{5} & 0 \end{bmatrix}$ $\quad B = \begin{bmatrix} -9 & x & 3 \end{bmatrix}$ $\quad C = \begin{bmatrix} -3 & w \\ -y & 0 \\ 5 & (x+1) \end{bmatrix}$

$D = \begin{bmatrix} 1.6 & 3 \\ 5 & p \end{bmatrix}$ $\quad E = \begin{bmatrix} v & 9 \\ 6m & -1 \end{bmatrix}$ $\quad F = \begin{bmatrix} -3r \\ 6z \end{bmatrix}$

a. State the dimensions of each matrix.

A is 2×3, B is 1×3, C is 3×2,

D is 2×2, E is 2×2, F is 2×1

east

SEC

South

b. State the value of c_{32}.

$c_{32} = x+1$

c. State the value of b_{21}.

b_{21} does not exist

d. Given $M = B^T$, state the value of m_{21}.

long way - write down B^T

$B^T = \begin{bmatrix} -9 \\ x \\ 3 \end{bmatrix}$

$m_{21} = b_{12}$

$= x$

$$A = \begin{bmatrix} 5 & 2 & 6 \\ 6 & \frac{1}{5} & 0 \end{bmatrix} \qquad B = \begin{bmatrix} -9 & x & 3 \end{bmatrix} \qquad C = \begin{bmatrix} -3 & w \\ -y & 0 \\ 5 & (x+1) \end{bmatrix}$$

$$D = \begin{bmatrix} 1.6 & 3 \\ 5 & p \end{bmatrix} \qquad E = \begin{bmatrix} v & 9 \\ 6m & -1 \end{bmatrix} \qquad F = \begin{bmatrix} -3r \\ 6z \end{bmatrix}$$

e. Compute $D + E$. → is 2×2

$$\begin{bmatrix} 1.6 & 3 \\ 5 & p \end{bmatrix} + \begin{bmatrix} v & 9 \\ 6m & -1 \end{bmatrix} = \begin{bmatrix} 1.6+v & 3+9 \\ 5+6m & p-1 \end{bmatrix}$$

$$= \begin{bmatrix} v+1.6 & 12 \\ 6m+5 & p-1 \end{bmatrix}$$

f. Compute $2D - 3E$.

$$2\begin{bmatrix} 1.6 & 3 \\ 5 & p \end{bmatrix} - 3\begin{bmatrix} v & 9 \\ 6m & -1 \end{bmatrix} = \begin{bmatrix} 2\cdot 1.6 & 2\cdot 3 \\ 2\cdot 5 & 2p \end{bmatrix} - \begin{bmatrix} 3v & 3\times 9 \\ 3\times(6m) & 3(-1) \end{bmatrix}$$

$$= \begin{bmatrix} 2\times 1.6 - 3v & 2\cdot 3 - 3\times 9 \\ 2\cdot 5 - 3\cdot 6m & 2p - 3(-1) \end{bmatrix} = \begin{bmatrix} 3.2-3v & -21 \\ 10-18m & 3+2p \end{bmatrix}$$

g. Compute $C^T - 6A$.

$$\begin{bmatrix} -3 & w \\ -y & 0 \\ 5 & x+1 \end{bmatrix}^T - 6\begin{bmatrix} 5 & 2 & 6 \\ 6 & \frac{1}{5} & 0 \end{bmatrix} = \begin{bmatrix} -3 & -y & 5 \\ w & 0 & x+1 \end{bmatrix} - \begin{bmatrix} 6\times 5 & 6\times 2 & 6\times 6 \\ 6\times 6 & 6\times\frac{1}{5} & 6\times 0 \end{bmatrix}$$

$$= \begin{bmatrix} -3-6\times 5 & -y-6\times 2 & 5-6\times 6 \\ w-6\times 6 & 0-6\times\frac{1}{5} & x+1-6\times 0 \end{bmatrix}$$

$$= \begin{bmatrix} -33 & -y-12 & -31 \\ w-36 & -\frac{6}{5} & x+1 \end{bmatrix}$$

$C - 6A$ is undefined..

$$A = \begin{bmatrix} 5 & 2 & 6 \\ 6 & \frac{1}{5} & 0 \end{bmatrix} \qquad B = \begin{bmatrix} -9 & x & 3 \end{bmatrix} \qquad C = \begin{bmatrix} -3 & w \\ -y & 0 \\ 5 & (x+1) \end{bmatrix}$$

$$D = \begin{bmatrix} 1.6 & 3 \\ 5 & p \end{bmatrix} \qquad E = \begin{bmatrix} v & 9 \\ 6m & -1 \end{bmatrix} \qquad F = \begin{bmatrix} -3r \\ 6z \end{bmatrix}$$

h. If $3D = E$, solve for m, v and p.

$$3\begin{bmatrix} 1.6 & 3 \\ 5 & p \end{bmatrix} = \begin{bmatrix} v & 9 \\ 6m & -1 \end{bmatrix} \qquad \begin{bmatrix} 3\cdot 1.6 & 3\cdot 3 \\ 3\cdot 5 & 3\cdot p \end{bmatrix} = \begin{bmatrix} v & 9 \\ 6m & -1 \end{bmatrix}$$

$3 \times 1.6 = v$ $\qquad 3 \times 3 = 9$ ✓

$3 \cdot 5 = 6m$ $\qquad 3 \cdot p = -1$

$$\boxed{\begin{aligned} v &= 4.8 \\ m &= 5/2 = 2.5 \\ p &= -1/3 \end{aligned}}$$

$4.8 = v \rightarrow$

$\frac{15}{6} = \frac{\cancel{6}m}{\cancel{6}} \rightarrow \quad m = \frac{15}{6} = \frac{5}{2}$

$\frac{3p}{3} = \frac{-1}{3}$

Pr 3. Solve the matrix equation for the matrix X.

$$3X - \begin{bmatrix} 260 & 165 \\ 130 & 60 \end{bmatrix} = \begin{bmatrix} 120 & 165 \\ 320 & -30 \end{bmatrix} - 3X$$

$\rightarrow 2\times 2$

X is 2×2

$$X = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$3\begin{bmatrix} a & b \\ c & d \end{bmatrix} - \begin{bmatrix} 260 & 165 \\ 130 & 60 \end{bmatrix} \rightarrow \text{solve for it} = A$$

$$\begin{bmatrix} 120 & 165 \\ 320 & -30 \end{bmatrix} - 3X = B \qquad A = B$$

$$3X - \begin{bmatrix} 260 & 165 \\ 130 & 60 \end{bmatrix} = \begin{bmatrix} 120 & 165 \\ 320 & -30 \end{bmatrix} - 3X$$

$+3X \qquad\qquad +3X$

$$6X - \begin{bmatrix} 260 & 165 \\ 130 & 60 \end{bmatrix} + \begin{bmatrix} 260 & 165 \\ 130 & 60 \end{bmatrix} = \begin{bmatrix} 120 & 165 \\ 320 & -30 \end{bmatrix} + \begin{bmatrix} 260 & 165 \\ 130 & 60 \end{bmatrix}$$

$$\frac{1}{6} 6X = \frac{1}{6}\begin{bmatrix} 120+260 & 165+165 \\ 320+130 & -30+60 \end{bmatrix}$$

$$X = \frac{1}{6}\begin{bmatrix} 390 & 330 \\ \end{bmatrix} \quad \begin{bmatrix} 65 & 55 \\ \end{bmatrix}$$

$\frac{}{6}$ $\frac{}{\cancel{6}}$ $\frac{}{6} = \frac{}{2}$

$\frac{3p}{3} = \frac{-1}{3}$

Pr 3. Solve the matrix equation for the matrix X.

$$3X - \begin{bmatrix} 260 & 165 \\ 130 & 60 \end{bmatrix} = \begin{bmatrix} 120 & 165 \\ 320 & -30 \end{bmatrix} - 3X$$

2x2

X is 2x2

$$X = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$3\begin{bmatrix} a & b \\ c & d \end{bmatrix} - \begin{bmatrix} 260 & 165 \\ 130 & 60 \end{bmatrix} \rightarrow \text{solve for it} = A$$

$$\begin{bmatrix} 120 & 165 \\ 320 & -30 \end{bmatrix} - 3X = B$$

$A = B$

$$3X - \begin{bmatrix} 260 & 165 \\ 130 & 60 \end{bmatrix} = \begin{bmatrix} 120 & 165 \\ 320 & -30 \end{bmatrix} - 3X$$

$+3X$ $\qquad +3X$

$$6X - \begin{bmatrix} 260 & 165 \\ 130 & 60 \end{bmatrix} = \begin{bmatrix} 120 & 165 \\ 320 & -30 \end{bmatrix}.$$

$$+\begin{bmatrix} 260 & 165 \\ 130 & 60 \end{bmatrix} \qquad + \begin{bmatrix} 260 & 165 \\ 130 & 60 \end{bmatrix}.$$

$$\frac{1}{6}6X = \frac{1}{6}\begin{bmatrix} 120+260 & 165+165 \\ 320+130 & -30+60 \end{bmatrix}$$

$$X = \frac{1}{6}\begin{bmatrix} 390 & 330 \\ 450 & 30 \end{bmatrix} = \begin{bmatrix} 65 & 55 \\ 75 & 5 \end{bmatrix}$$

Section 1.2: Matrix Multiplication

- For the matrix product AB to exist the number of columns of matrix A must be the same as the number of rows of matrix B.
- Matrix multiplication is not commutative.

$a \times \cancel{b} \quad \cancel{b} \times c \qquad a \times c$

Pr 1. Use the given matrices A, B, C, D, E, and F below, to compute each matrix product, if possible.

$$A = \begin{bmatrix} 5 & 2 & 6 \\ 6 & \frac{1}{5} & 0 \end{bmatrix} \qquad B = \begin{bmatrix} -9 & x & 3 \end{bmatrix} \qquad C = \begin{bmatrix} -3 & w \\ -y & 0 \\ 5 & (x+1) \end{bmatrix}$$

$$D = \begin{bmatrix} 1.6 & 3 \\ 5 & p \end{bmatrix} \qquad E = \begin{bmatrix} v & 9 \\ 6m & -1 \end{bmatrix} \qquad F = \begin{bmatrix} -3r \\ 6z \end{bmatrix}$$

a. DA

$$\begin{bmatrix} 1.6 & 3 \\ 5 & p \end{bmatrix} \times \begin{bmatrix} 5 & 2 & 6 \\ 6 & \frac{1}{5} & 0 \end{bmatrix} = \begin{bmatrix} 1.6\cdot 5 + 3\cdot 6 & 1.6\cdot 2 + 3\cdot\frac{1}{5} & 1.6\cdot 6 + 3\cdot 0 \\ 5\cdot 5 + p\cdot 6 & 5\cdot 2 + p\cdot\frac{1}{5} & 5\cdot 6 + p\cdot 0 \end{bmatrix}$$

$2\times\underline{2} \quad \underline{2}\times 3 \rightarrow 2\times 3$

$$= \begin{bmatrix} 26 & 3.8 & 9.6 \\ 6p+25 & \frac{p}{5}+10 & 30 \end{bmatrix}$$

$A \times B$

$2\times\underline{3} \quad \underline{1}\times 3$

not possible

b. AB^T

$$\begin{bmatrix} 5 & 2 & 6 \\ 6 & \frac{1}{5} & 0 \end{bmatrix} \times \begin{bmatrix} -9 \\ x \\ 3 \end{bmatrix} = \begin{bmatrix} 5(-9) + 2x + 6\cdot 3 \\ 6(-9) + \frac{1}{5}x + 0\cdot 3 \end{bmatrix} = \begin{bmatrix} 2x - 27 \\ \frac{1}{5}x - 54 \end{bmatrix}$$

$2\times\underline{3} \quad \underline{3}\times 1 \rightarrow 2\times 1$

c. Use the given matrices A, B, C, D, E, and F below, to compute each matrix product, if possible.

$A = \begin{bmatrix} 5 & 2 & 6 \\ 6 & \frac{1}{5} & 0 \end{bmatrix}$ $\quad B = \begin{bmatrix} -9 & x & 3 \end{bmatrix}$ $\quad C = \begin{bmatrix} -3 & w \\ -y & 0 \\ 5 & (x+1) \end{bmatrix}$

$D = \begin{bmatrix} 1.6 & 3 \\ 5 & p \end{bmatrix}$ $\quad E = \begin{bmatrix} v & 9 \\ 6m & -1 \end{bmatrix}$ $\quad F = \begin{bmatrix} -3r \\ 6z \end{bmatrix}$

d. $-6BC = (-6B)C$ or $-6(BC)$

$\rightarrow \begin{bmatrix} -9 & x & 3 \end{bmatrix} \begin{bmatrix} -3 & w \\ -y & 0 \\ 5 & x+1 \end{bmatrix} = \begin{bmatrix} -9(-3)+x(-y)+3(5) & -9\cdot w+x\cdot 0+3\cdot(x+1) \end{bmatrix}$

$1\times 3 \quad 3\times 2 \rightarrow 1\times 2$

$= \begin{bmatrix} 27 - xy + 15 & -9w + 3x + 3 \end{bmatrix}$

$(3(x+1) = 3x+3$

$-6\begin{bmatrix} 42 - xy & -9w + 3x + 3 \end{bmatrix}$

$= \begin{bmatrix} -252 + 6xy & 54w - 18x - 18 \end{bmatrix}$

e. $ACE = (AC)E$ or $A(CE)$

$2\times 3 \; 3\times 2 \; 2\times 2$ $\quad$ $2\times 3 \;\; 3\times 2$

$2\times 2 \;\; 2\times 2$

B

$\begin{bmatrix} -3 & w \\ -y & 0 \\ 5 & (x+1) \end{bmatrix}$

$A\times C = \begin{bmatrix} 3 & 2 & 6 \\ 6 & \frac{1}{5} & 0 \end{bmatrix} \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}$

A

$= \begin{bmatrix} 3(-3)+2(-y)+6\cdot 5 & 3\cdot w+2\cdot 0+6\cdot(x+1) \\ 6\cdot(-3)+\frac{1}{5}(-y)+0\cdot 5 & 6\cdot w+\frac{1}{5}\cdot 0+0\cdot(x+1) \end{bmatrix}$

$= \begin{bmatrix} -2y + 21 & 3w + 6x + 6 \\ -\frac{y}{5} - 18 & 6w \end{bmatrix}$

E

$\begin{bmatrix} v & 9 \\ 6m & -1 \end{bmatrix}$

T

$$= \begin{bmatrix} -252 + 6xy & 54w - 18x - 18 \end{bmatrix}$$

e. $ACE = \underset{2\times 3\;3\times 2\;2\times 2}{(AC)E}$ or $\underset{2\times 3\;\;3\times 2}{A(CE)}$

$2\times 2 \quad 2\times 2$

$$\begin{array}{c} B \\ \begin{bmatrix} -3 & w \\ -y & 0 \\ 5 & (x+1) \end{bmatrix} \end{array}$$

$$A \times C = \begin{bmatrix} 3 & 2 & 6 \\ 6 & \frac{1}{5} & 0 \end{bmatrix} \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}$$

A

$$= \begin{bmatrix} 3(-3)+2(-y)+6\cdot 5 & 3\cdot w + 2\cdot 0 + 6\cdot(x+1) \\ 6\cdot(-3)+\frac{1}{5}(-y)+0\cdot 5 & 6\cdot w + \frac{1}{5}\cdot 0 + 0\cdot(x+1) \end{bmatrix}$$

$$= \begin{bmatrix} -2y + 21 & 3w + 6x + 6 \\ -\frac{y}{5} - 18 & 6w \end{bmatrix}$$

$$\begin{array}{c} E \\ \begin{bmatrix} v & 9 \\ 6m & -1 \end{bmatrix} \end{array}$$

T

$$tC \quad \begin{bmatrix} -2x + 21 & 3w + 6x + 6 \\ -\frac{y}{5} - 18 & 6w \end{bmatrix} \leftarrow \begin{bmatrix} \checkmark & \cancel{2} \\ \cancel{3} & 0 \end{bmatrix}$$

$$\begin{bmatrix} (-2x+21)\cdot v + (3w+6x+6)\cdot(6m) & (-2x+21)\cdot 9 + (3w+6x+6)\cdot(-1) \\ (-\frac{y}{5} - 18)\cdot v + 6w(6m) & (-\frac{y}{5} - 18)\cdot 9 + 6w(-1) \end{bmatrix}$$

$$= \begin{bmatrix} -2vx + 21v + 18mw + 36xm + 36m & -18x + 183 - 3w - 6x \\ -\frac{vy}{5} - 18w + 36mw & -\frac{9y}{5} - 162 - 6w \end{bmatrix}$$

Pr 2. A content creator, Miss Terri, owns three channels on youtube - one with science videos, one with movie reviews, and one with cooking videos. Miss Terri records the number of total views for videos and shorts on each channel across two months as follows:

- In January, the science channel experienced 1 million total video views, and 5 million total views of shorts. The movie review channel experienced 5 million total video views, and 12 million total views of shorts. Finally, the cooking channel experienced 6 million total video views, and 17 million total views of shorts.
- In February, the science channel experienced 1.2 million total video views, and 4 million total views of shorts. The movie review channel experienced 3 million total video views, and 11 million total views of shorts. Finally, the cooking channel experienced 7 million total video views, and 21 million total views of shorts.

Suppose that Youtube pays a content creator \$2 for every 1,000 views on videos and \$.05 for every 1,000 views on a short. How much total revenue did Miss Terri make on each channel?

3 channels

3×1 matrix or 1×3

$$\begin{array}{c|c} & \begin{array}{cc} v & sh \end{array} \\ \begin{array}{c} S \\ M \\ C \end{array} & \begin{bmatrix} \quad & \quad \\ & \\ & \end{bmatrix} \end{array} \times \begin{bmatrix} 2 \\ .05 \end{bmatrix} \begin{array}{c} v \\ sh \end{array} = \begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix} \begin{array}{c} sh \\ M \\ C \end{array}$$

revenue

J = January, F = February

$J \cdot \begin{bmatrix} 2 \\ .05 \end{bmatrix}$

$$(J+F)\begin{bmatrix} 2 \\ .05 \end{bmatrix} = \text{answer}$$

1 million = 1000 thousands

$$\to 1000\,(J+F)\begin{bmatrix} 2 \\ .05 \end{bmatrix}$$

$$(c \cdot A)B = c \cdot (A \cdot B)$$

$$1000\left(\begin{array}{c|c} & \begin{array}{cc} v & sh \end{array} \\ \begin{array}{c} S \\ M \\ C \end{array} & \begin{bmatrix} 1 & 5 \\ 5 & 12 \\ 6 & 17 \end{bmatrix} \end{array} + \begin{bmatrix} 1.2 & 4 \\ 3 & 11 \\ 7 & 21 \end{bmatrix}\right)\begin{bmatrix} 2 \\ .05 \end{bmatrix}$$

$$1000\left(\begin{bmatrix} 2.2 & 9 \\ 8 & 23 \\ 13 & 38 \end{bmatrix}\begin{bmatrix} 2 \\ .05 \end{bmatrix}\right) = 1000\begin{bmatrix} 2.2 \times 2 + 9 \times .05 \\ 8 \times 2 + 23 \times .05 \\ 13 \times 2 + 38 \times .05 \end{bmatrix}$$

$$= 1000\begin{bmatrix} 4.85 \\ 16.95 \\ 27.9 \end{bmatrix} = \begin{bmatrix} 4850 \\ 16950 \\ 27900 \end{bmatrix}$$

$$\begin{bmatrix} \$4,850 \\ \$16,950 \\ \$27,900 \end{bmatrix}$$