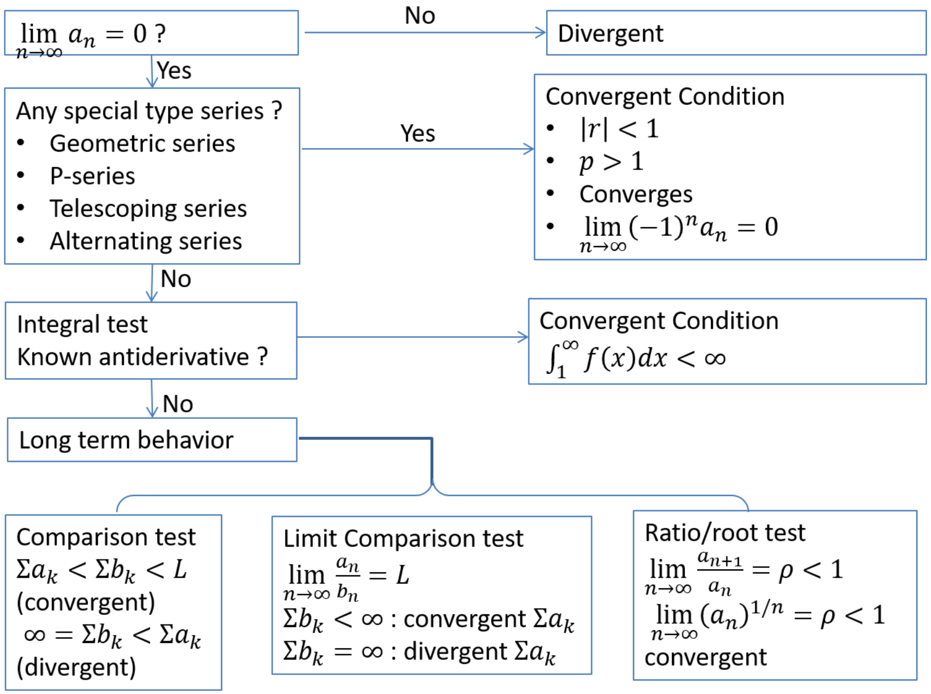
Week in Review
Math 152

Week 09

Comparison Test

Alternating Series

Review



For each of the following series, use the sequence of partial sums to determine whether the series converges or diverges.

a. $\sum_{n=1}^{\infty} \frac{n}{n+1}$

b. $\sum_{n=1}^{\infty} (-1)^n$

c. $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$

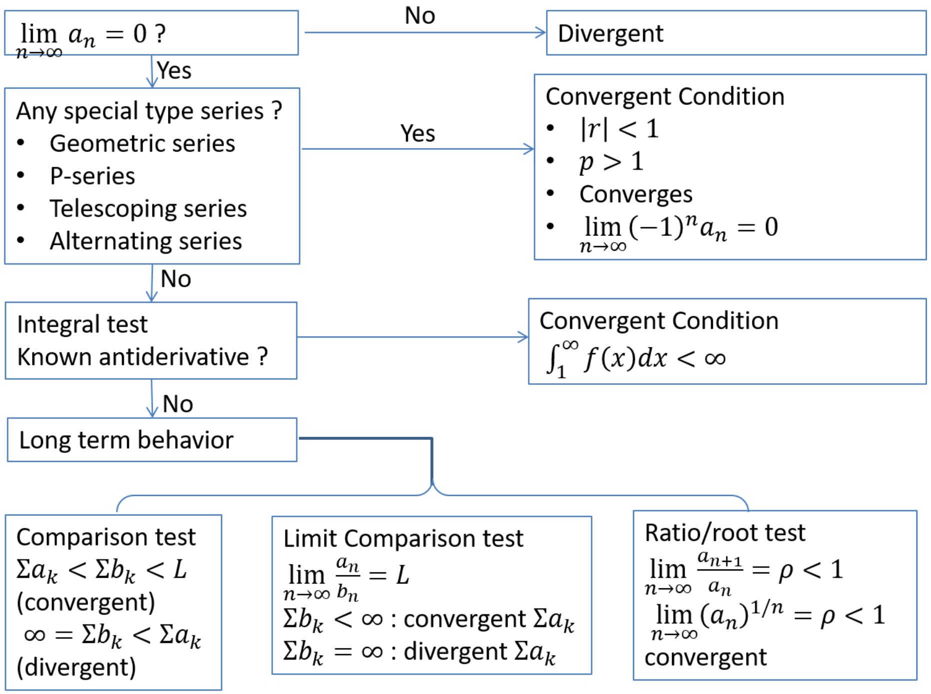
By divergence test,

a. Diverges

b. Diverges

c. divergence test fails,
but convergent (telescoping series)

Review



a.,c. Diverges

b. divergence test fails,
but convergent (p series)

For each of the following series, apply the divergence test. If the divergence test proves that the series diverges, state so. Otherwise, indicate that the divergence test is inconclusive.

a. $\sum_{n=1}^{\infty} \frac{n}{3n-1}$

b. $\sum_{n=1}^{\infty} \frac{1}{n^3}$

c. $\sum_{n=1}^{\infty} e^{1/n^2}$

Review

$\lim_{n\to\infty} a_n = 0$?

No → Divergent

Yes ↓

Any special type series ?
- Geometric series
- P-series
- Telescoping series
- Alternating series

Yes → Convergent Condition
- $|r| < 1$
- $p > 1$
- Converges
- $\lim_{n\to\infty} (-1)^n a_n = 0$

No ↓

Integral test
Known antiderivative ?

→ Convergent Condition
$\int_1^\infty f(x)dx < \infty$

No ↓

Long term behavior

Comparison test
$\Sigma a_k < \Sigma b_k < L$
(convergent)
$\infty = \Sigma b_k < \Sigma a_k$
(divergent)

Limit Comparison test
$\lim_{n\to\infty} \frac{a_n}{b_n} = L$
$\Sigma b_k < \infty$: convergent Σa_k
$\Sigma b_k = \infty$: divergent Σa_k

Ratio/root test
$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \rho < 1$
$\lim_{n\to\infty} (a_n)^{1/n} = \rho < 1$
convergent

Determine whether the series

$\sum_{n=1}^{\infty} (n+1)/n$ converges or diverges.

By divergence test,
The series diverges

Review

$\lim_{n\to\infty} a_n = 0$?

- No → Divergent
- Yes → Any special type series ?
  - Geometric series
  - P-series
  - Telescoping series
  - Alternating series

Any special type series? Yes → Convergent Condition
- $|r| < 1$
- $p > 1$
- Converges
- $\lim_{n\to\infty} (-1)^n a_n = 0$

No → Integral test: Known antiderivative ? → Convergent Condition: $\int_1^\infty f(x)dx < \infty$

No → Long term behavior

- Comparison test: $\Sigma a_k < \Sigma b_k < L$ (convergent); $\infty = \Sigma b_k < \Sigma a_k$ (divergent)
- Limit Comparison test: $\lim_{n\to\infty} \frac{a_n}{b_n} = L$; $\Sigma b_k < \infty$: convergent Σa_k; $\Sigma b_k = \infty$: divergent Σa_k
- Ratio/root test: $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \rho < 1$; $\lim_{n\to\infty} (a_n)^{1/n} = \rho < 1$; convergent

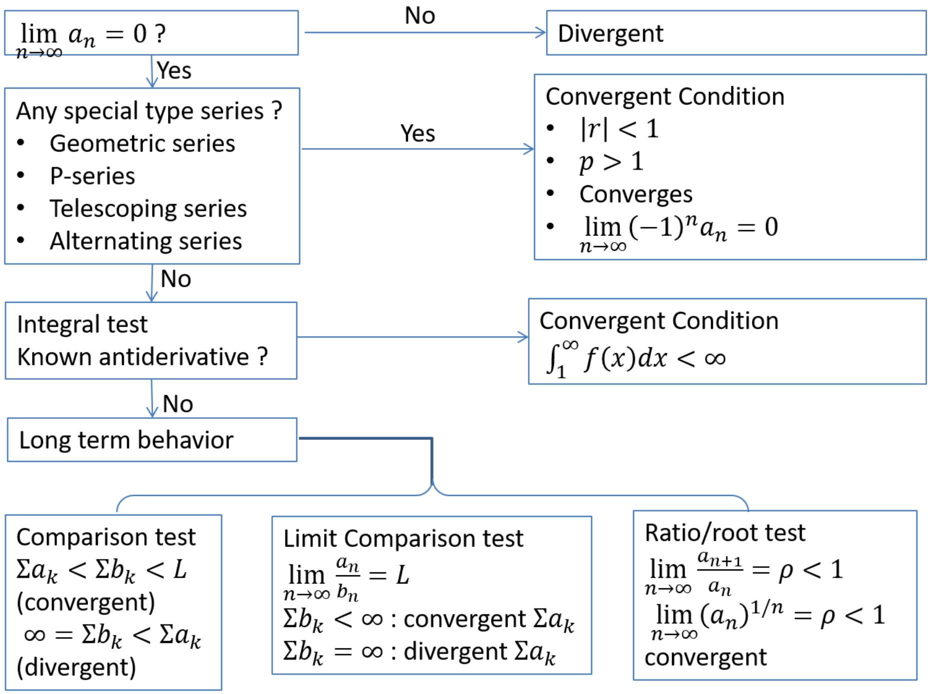
Determine whether the telescoping series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{1}{2n+1}$$

The divergence test fails.

The series diverges by comparison test

Review



Evaluate

$$\sum_{n=1}^{\infty} \frac{3}{n(n+1)}$$

Telescoping series ;

$$3 \lim_{N\to\infty} \left(1 - \frac{1}{N+1}\right) = 3$$

Review

$\lim_{n\to\infty} a_n = 0$?

- No → Divergent
- Yes → Any special type series ?
  - Geometric series
  - P-series
  - Telescoping series
  - Alternating series

Any special type series? Yes → Convergent Condition
- $|r| < 1$
- $p > 1$
- Converges
- $\lim_{n\to\infty} (-1)^n a_n = 0$

No → Integral test

Known antiderivative ? → Convergent Condition

$\int_1^\infty f(x)dx < \infty$

No → Long term behavior

Comparison test

$\Sigma a_k < \Sigma b_k < L$

(convergent)

$\infty = \Sigma b_k < \Sigma a_k$

(divergent)

Limit Comparison test

$\lim_{n\to\infty} \frac{a_n}{b_n} = L$

$\Sigma b_k < \infty$: convergent Σa_k

$\Sigma b_k = \infty$: divergent Σa_k

Ratio/root test

$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \rho < 1$

$\lim_{n\to\infty} (a_n)^{1/n} = \rho < 1$

convergent

Evaluate

$$\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{n-2}$$

$$\frac{2}{1-\frac{1}{2}} = 4$$

Evaluate $\sum_{n=1}^{\infty} \frac{5}{2^{n-1}}$.

$$\frac{5}{1-\frac{1}{2}} = 10$$

Review

$\lim_{n\to\infty} a_n = 0$?

- No → Divergent
- Yes ↓

Any special type series ?
- Geometric series
- P-series
- Telescoping series
- Alternating series

Yes → Convergent Condition
- $|r| < 1$
- $p > 1$
- Converges
- $\lim_{n\to\infty} (-1)^n a_n = 0$

No ↓

Integral test
Known antiderivative ?

→ Convergent Condition
$\int_1^\infty f(x)dx < \infty$

No ↓

Long term behavior

Comparison test
$\Sigma a_k < \Sigma b_k < L$
(convergent)
$\infty = \Sigma b_k < \Sigma a_k$
(divergent)

Limit Comparison test
$\lim_{n\to\infty} \frac{a_n}{b_n} = L$
$\Sigma b_k < \infty$: convergent Σa_k
$\Sigma b_k = \infty$: divergent Σa_k

Ratio/root test
$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \rho < 1$
$\lim_{n\to\infty} (a_n)^{1/n} = \rho < 1$
convergent

Determine whether each of the following geometric series converges or diverges, and if it converges, find its sum.

a. $\sum_{n=1}^{\infty} \frac{(-3)^{n+1}}{4^{n-1}}$

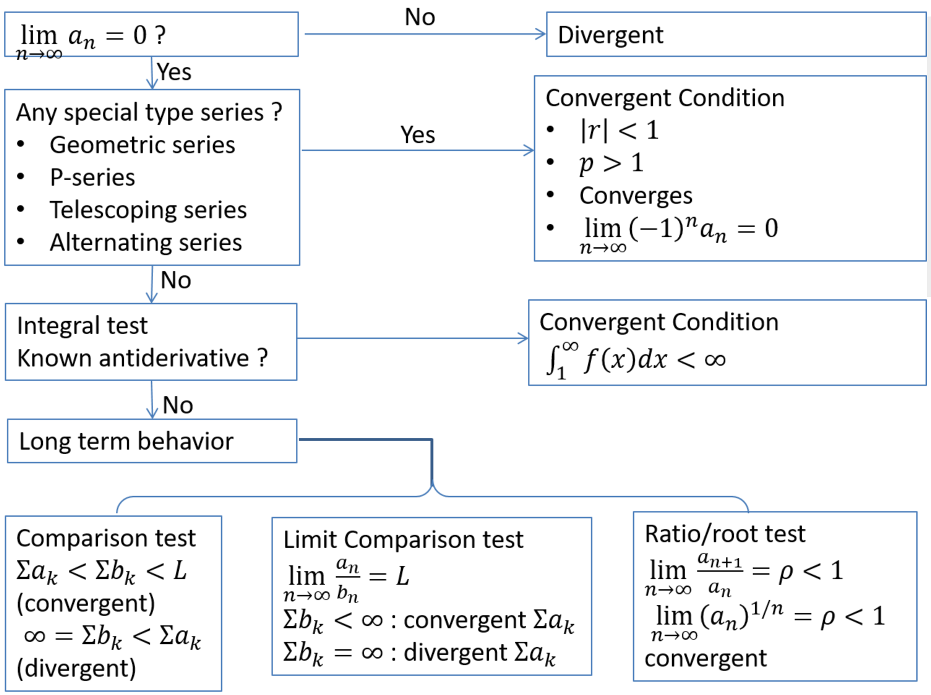
b. $\sum_{n=1}^{\infty} e^{2n}$

a. Convergent

$$\frac{9}{1+\frac{3}{4}} = \frac{36}{7}$$

b. Divergent

Review



Determine whether the telescoping series

$$\sum_{n=1}^{\infty} \left[e^{1/n} - e^{1/(n+1)}\right]$$

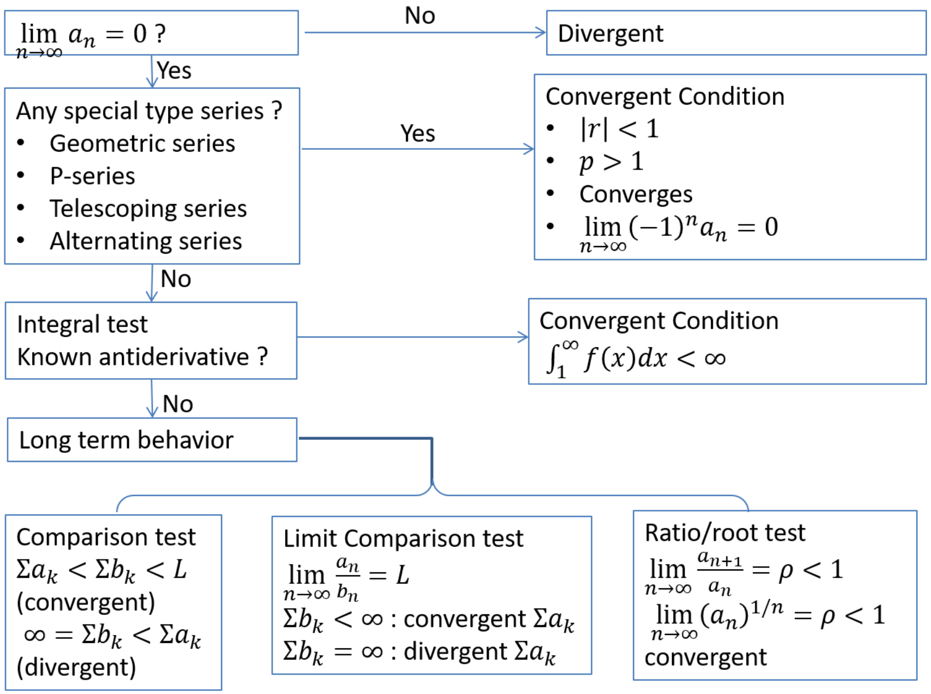
converges or diverges. If it converges, find its sum.

Telescoping series

Convergent

$$\lim_{N\to\infty} \left(e - e^{\frac{1}{N+1}}\right) = e - 1$$

Review



Determine whether the telescoping series

$$\sum_{n=1}^{\infty} \left[\cos\left(\frac{1}{n}\right) - \cos\left(\frac{1}{n+1}\right)\right]$$

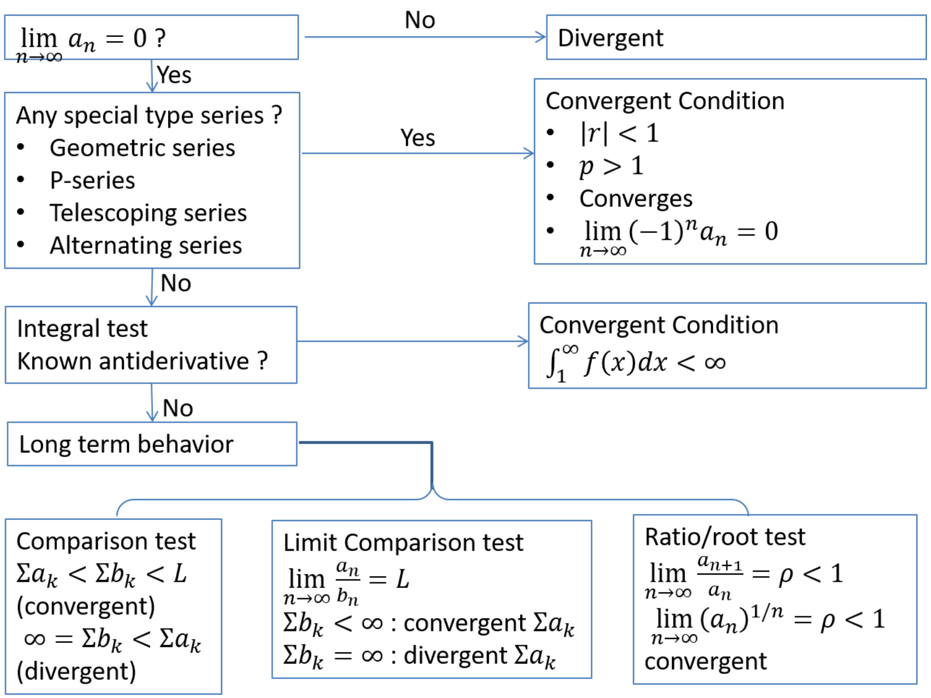
converges or diverges. If it converges, find its sum.

Telescoping series

Convergent

$$\lim_{N\to\infty} \left(\cos 1 - \cos\left(\frac{1}{N+1}\right)\right) = \cos 1 - 1$$

Review



For each of the following alternating series, determine whether the series converges or diverges.

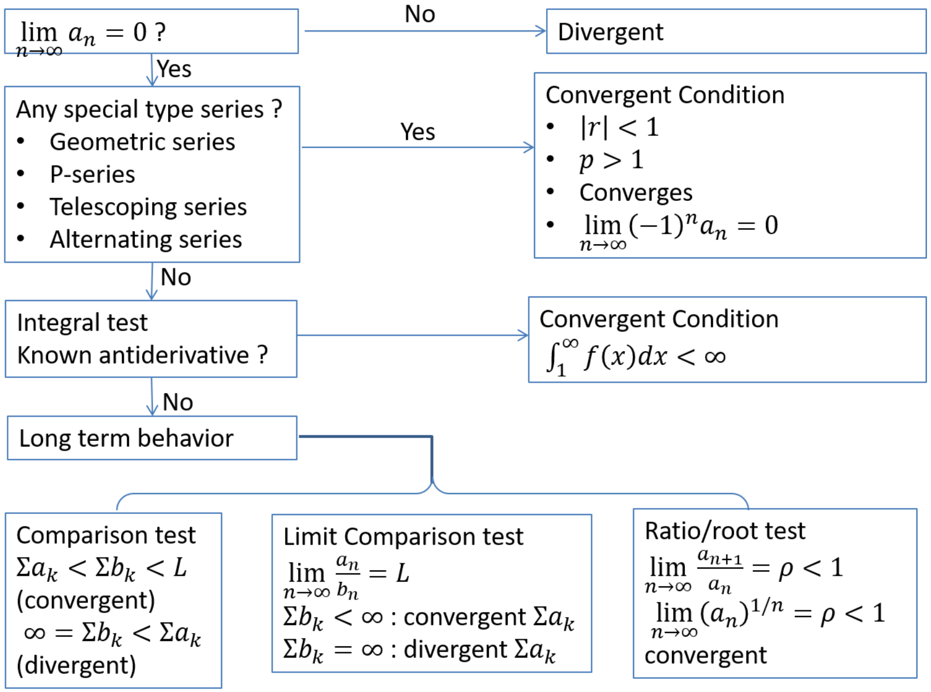
a. $\sum_{n=1}^{\infty} (-1)^{n+1}/n^2$

b. $\sum_{n=1}^{\infty} (-1)^{n+1} n/(n+1)$

c. $\sum_{n=1}^{\infty} (-1)^{n+1} n/2^n$

a. Convergent
b. Divergent
c. Convergent

Review



For each of the following series, determine whether the series converges or diverges.

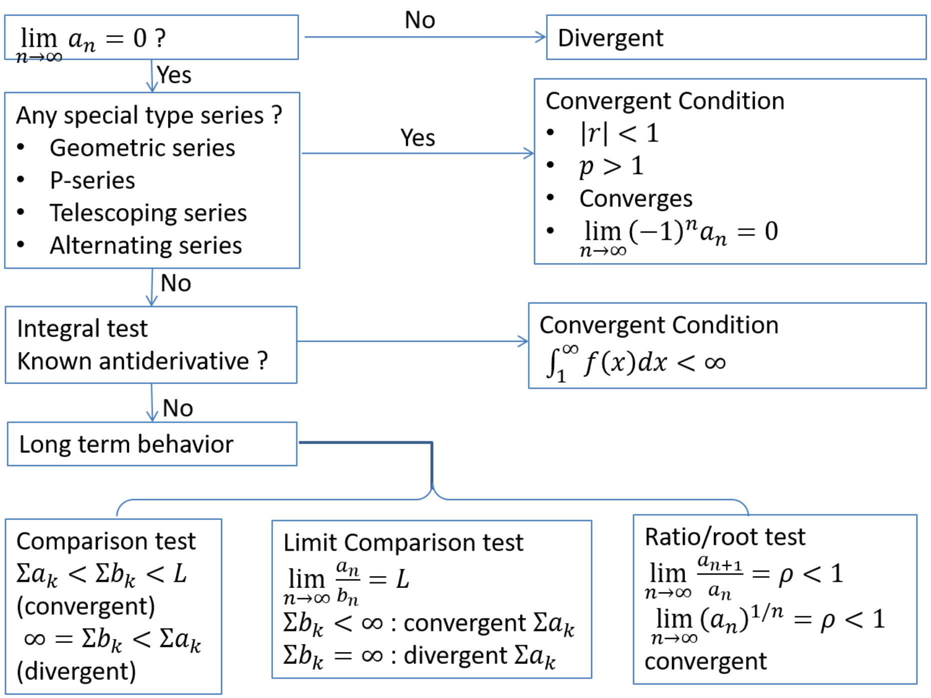
a. $\sum_{n=1}^{\infty} 1/n^3$

b. $\sum_{n=1}^{\infty} 1/\sqrt{2n-1}$

a. p −series : convergent

b. Divergent by comparison test

Review



Use the integral test to determine whether the series $\sum_{n=1}^{\infty} \frac{n}{3n^2+1}$ converges or diverges.

Divergent by (limit) comparison test

Review

$\lim_{n\to\infty} a_n = 0$?

- No → Divergent
- Yes ↓

Any special type series ?
- Geometric series
- P-series
- Telescoping series
- Alternating series

Yes → Convergent Condition
- $|r| < 1$
- $p > 1$
- Converges
- $\lim_{n\to\infty} (-1)^n a_n = 0$

No ↓

Integral test
Known antiderivative ?

→ Convergent Condition
$\int_1^\infty f(x)dx < \infty$

No ↓

Long term behavior

Comparison test
$\Sigma a_k < \Sigma b_k < L$
(convergent)
$\infty = \Sigma b_k < \Sigma a_k$
(divergent)

Limit Comparison test
$\lim_{n\to\infty} \frac{a_n}{b_n} = L$
$\Sigma b_k < \infty$: convergent Σa_k
$\Sigma b_k = \infty$: divergent Σa_k

Ratio/root test
$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \rho < 1$
$\lim_{n\to\infty} (a_n)^{1/n} = \rho < 1$
convergent

For each of the following series, determine whether it converges or diverges.

a. $\sum_{n=1}^{\infty} \frac{1}{n^4}$

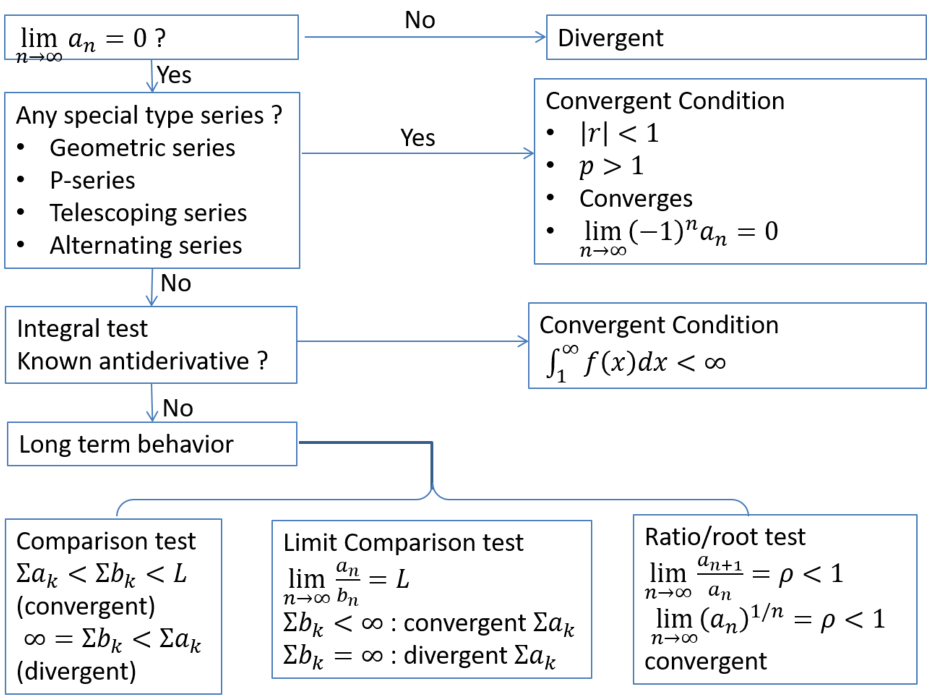
b. $\sum_{n=1}^{\infty} \frac{1}{n^{2/3}}$

P-series

a. Convergent

b. Divergent

Review



For each of the following series, use the comparison test to determine whether the series converges or diverges.

a. $\sum_{n=1}^{\infty} \frac{1}{n^3+3n+1}$

b. $\sum_{n=1}^{\infty} \frac{1}{2^n+1}$

c. $\sum_{n=2}^{\infty} \frac{1}{\ln(n)}$

a. Convergent

b. Convergent

c. Divergent

Review

$\lim_{n\to\infty} a_n = 0$?

- No → Divergent
- Yes ↓

Any special type series ?
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- Alternating series

Yes → Convergent Condition
- $|r| < 1$
- $p > 1$
- Converges
- $\lim_{n\to\infty} (-1)^n a_n = 0$

No ↓

Integral test
Known antiderivative ?

→ Convergent Condition
$\int_1^{\infty} f(x)dx < \infty$

No ↓

Long term behavior

Comparison test
$\Sigma a_k < \Sigma b_k < L$
(convergent)
$\infty = \Sigma b_k < \Sigma a_k$
(divergent)

Limit Comparison test
$\lim_{n\to\infty} \frac{a_n}{b_n} = L$
$\Sigma b_k < \infty$: convergent Σa_k
$\Sigma b_k = \infty$: divergent Σa_k

Ratio/root test
$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \rho < 1$
$\lim_{n\to\infty} (a_n)^{1/n} = \rho < 1$
convergent

For each of the following series, use the limit comparison test to determine whether the series converges or diverges.

a. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}+1}$
b. $\sum_{n=1}^{\infty} \frac{2^n+1}{3^n}$
c. $\sum_{n=1}^{\infty} \frac{\ln(n)}{n^2}$

a. Divergent
b. Convergent
c. Convergent by integral test

Review

$\lim_{n\to\infty} a_n = 0$?

- No → Divergent
- Yes ↓

Any special type series ?
- Geometric series
- P-series
- Telescoping series
- Alternating series

Yes → Convergent Condition
- $|r| < 1$
- $p > 1$
- Converges
- $\lim_{n\to\infty} (-1)^n a_n = 0$

No ↓

Integral test
Known antiderivative ?

→ Convergent Condition
$\int_1^\infty f(x)dx < \infty$

No ↓

Long term behavior

Comparison test
$\Sigma a_k < \Sigma b_k < L$
(convergent)
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Ratio/root test
$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \rho < 1$
$\lim_{n\to\infty} (a_n)^{1/n} = \rho < 1$
convergent

Determine whether the series $\sum_{n=1}^{\infty} \frac{n^3}{3^n}$ converges or diverges.

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{(n+1)^3}{3^{n+1}} \frac{3^n}{n^3}$$

$$= \lim_{n\to\infty} \frac{1}{3}\left(1+\frac{1}{n}\right)^3 = \frac{1}{3} < 1$$

convergent

For each of the following series, determine whether the series converges or diverges.

a. $\sum_{n=1}^{\infty} \frac{2^n}{n!}$

b. $\sum_{n=1}^{\infty} \frac{n^n}{n!}$

c. $\sum_{n=1}^{\infty} \frac{(-1)^n (n!)^2}{(2n)!}$

a. $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{2^{n+1}}{(n+1)!} \cdot \frac{n!}{2^n}$

$= \lim_{n\to\infty} \frac{2}{(n+1)} = 0 < 1$ (convergent)

b. $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{(n+1)^{n+1}}{(n+1)!} \cdot \frac{n!}{n^n}$

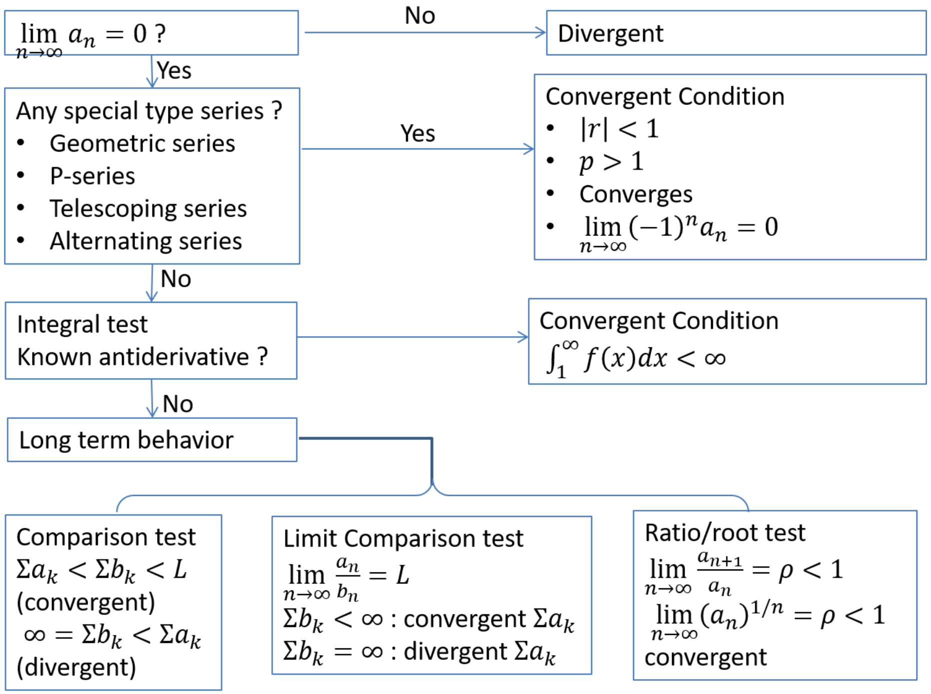
$= \lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n = e > 1$ (Divergent)

c. $\frac{n! 1\cdot 2 \cdots n}{n!(n+1)(n+2)\cdots(2n)}$

$= \left(\frac{1}{n+1}\right)\left(\frac{2}{n+2}\right)\cdots\left(\frac{n}{2n}\right) \to 0$

convergent

Review



For each of the following series, determine whether the series converges or diverges.

a. $\sum_{n=1}^{\infty} \frac{(n^2+3n)^n}{(4n^2+5)^n}$

b. $\sum_{n=2}^{\infty} \frac{n^n}{(\ln(n))^n}$

$$\lim_{n\to\infty} [a_n]^{\frac{1}{n}} = \lim_{n\to\infty} \left[\left(\frac{n^2+3n}{4n^2+5}\right)^n\right]^{\frac{1}{n}} = \frac{1}{4}$$

Convergent

$$\lim_{n\to\infty} [a_n]^{\frac{1}{n}} = \lim_{n\to\infty} \left[\left(\frac{n}{\ln n}\right)^n\right]^{\frac{1}{n}} = \infty$$

Divergent

Determine whether the series $\sum_{n=1}^{\infty} 1/n^n$ converges or diverges.

$$\lim_{n\to\infty} [a_n]^{\frac{1}{n}} = \lim_{n\to\infty} \left[\left(\frac{1}{n}\right)^n\right]^{\frac{1}{n}} = 0$$

Convergent

Review

$\lim_{n\to\infty} a_n = 0$?

- No → Divergent
- Yes ↓

Any special type series ?
- Geometric series
- P-series
- Telescoping series
- Alternating series

Yes → Convergent Condition
- $|r| < 1$
- $p > 1$
- Converges
- $\lim_{n\to\infty} (-1)^n a_n = 0$

No ↓

Integral test
Known antiderivative ?

→ Convergent Condition
$\int_1^{\infty} f(x)dx < \infty$

No ↓

Long term behavior

Comparison test
$\Sigma a_k < \Sigma b_k < L$
(convergent)
$\infty = \Sigma b_k < \Sigma a_k$
(divergent)

Limit Comparison test
$\lim_{n\to\infty} \frac{a_n}{b_n} = L$
$\Sigma b_k < \infty$: convergent Σa_k
$\Sigma b_k = \infty$: divergent Σa_k

Ratio/root test
$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \rho < 1$
$\lim_{n\to\infty} (a_n)^{1/n} = \rho < 1$
convergent

Determine whether the series converges or diverges.

a. $\sum_{n=1}^{\infty} \frac{n^2+2n}{n^3+3n^2+1}$

b. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(3n+1)}{n!}$

c. $\sum_{n=1}^{\infty} \frac{e^n}{n^3}$

d. $\sum_{n=1}^{\infty} \frac{3^n}{(n+1)^n}$

e. $\sum_{n=1}^{\infty} \frac{2^n}{3^n+n}$

Divergent (Divergence test)
Convergent (Alternating series)
Divergent (Divergence test)
Convergent (root series)
Convergent (limit comparison)