Math 150 - Week-In-Review 2

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Exam 1 review - Chapters 1 and 2

1. Perform the indicated operation on the functions $f(x) = \dfrac{x}{x-2}$ and $g(x) = \sqrt{3x-1}$.

a. $(fg)(z) = f(z)\cdot g(z) = \left(\dfrac{z}{z-2}\right)\left(\sqrt{3z-1}\right)$

Domain of g: $3x - 1 \geq 0$, $x \geq \frac{1}{3}$, $x \in [\frac{1}{3}, +\infty)$

Domain of f: $x - 2 \neq 0 \rightarrow x \neq 2$, $x \in (-\infty, 2) \cup (2, +\infty)$

Note domain of $(fg)(x) = (\text{domain of } f(x)) \cap (\text{domain of } g(x))$

$\frac{1}{3}$ 2

$x \in (2, +\infty)$

b. $\left(\dfrac{f}{g}\right)(\frac{1}{3}) = \dfrac{f(\frac{1}{3})}{g(\frac{1}{3})}$ undefined

Note: $g(\frac{1}{3}) = 0$

c. $(f \circ g)(x) = f(g(x)) = \dfrac{g(x)}{g(x) - 2} = \dfrac{\sqrt{3x-1}}{\sqrt{3x-1} - 2}$

$\left(\text{domain of } \left(\dfrac{\sqrt{3x-1}}{\sqrt{3x-1}-2}\right)\right)$:

$\sqrt{3x-1} - 2 \neq 0 \rightarrow \sqrt{3x-1} \neq 2$
$3x - 1 \neq 4 \rightarrow 3x \neq 5 \rightarrow x \neq 5/3$

& $3x - 1 \geq 0 \rightarrow x \geq \frac{1}{3}$

$\frac{1}{3}$ $\frac{5}{3}$

domain of $f(g(x))$: $(\text{domain } g(x)) \cap \left(\text{domain of } \left(\dfrac{\sqrt{3x-1}}{\sqrt{3x-1}-2}\right)\right)$

$\left([\frac{1}{3}, +\infty)\right) \cap \left([\frac{1}{3}, \frac{5}{3}) \cup (\frac{5}{3}, +\infty)\right)$

$= [\frac{1}{3}, \frac{5}{3}) \cup (\frac{5}{3}, +\infty)$

d. $(g \circ f)(x) = g(f(x)) = \sqrt{3(f(x)) - 1} = \sqrt{3\left(\frac{x}{x-2}\right) - 1} = \sqrt{\dfrac{3x}{x-2} - 1} = \sqrt{\dfrac{3x}{x-2} - \dfrac{x-2}{x-2}}$

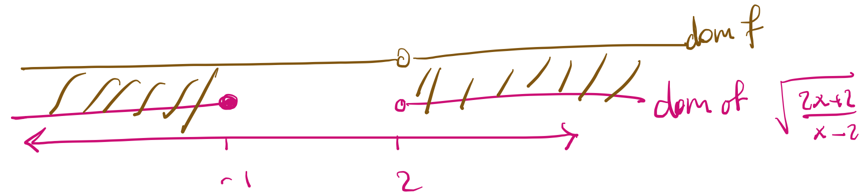
$= \sqrt{\dfrac{3x - (x-2)}{x-2}} = \sqrt{\dfrac{3x - x + 2}{x-2}} = \sqrt{\dfrac{2x+2}{x-2}}$

Need $\dfrac{\overset{2(x+1)}{2x+2}}{x-2} \geq 0$

$+ \quad - \quad +$

$-1 \quad 2$

$(-\infty, -1] \cup (2, +\infty)$


dom f
dom of $\sqrt{\frac{2x+2}{x-2}}$
-1
2

2. Let $f(x) = x^3$. Determine the formula of the function $g(x)$ whose graph is the result of the graph of $f(x)$ undergoing the following sequence of transformations.

(a) Horizontal shrink by a factor of 3. g_1
(b) Horizontal shift 5 units right. g_2
(c) Vertical shift down 2 units. g_3
(d) reflect about the y-axis. g_4
(e) Vertical shrink by factor of 2. g_5

$g_1(x) = (3x)^3$

$g_2(x) = g_1(x-5) = (3(x-5))^3$

$g_3(x) = (3(x-5))^3 - 2 = (3x-15)^3 - 2$

$g_4(x) = g_3(-x) = (-3x-15)^3 - 2$

$g_5(x) = \frac{1}{2}(g_4(x)) = \frac{1}{2}((-3x-15)^3 - 2)$

$g(x) = \frac{1}{2}((-3x-15)^3 - 2)$

3. Write the function $h(x) = \dfrac{1}{3}x^2 - 4x + 3$ in vertex form. Then determine the vertex, whether the vertex is a maximum or minimum, and the axis of symmetry.

vertex $\begin{cases} x = \frac{-b}{2a} = \frac{4}{2(\frac{1}{3})} = \frac{4}{\frac{2}{3}} = \frac{12}{2} = 6 \\ y = h(6) = \frac{1}{3}(6)^2 - 4(6) + 3 = \frac{36}{3} - 24 + 3 = -9 \end{cases}$

$\Rightarrow$ $x = 6$ is axis of symmetry

vertex $(6, -9)$

$\Rightarrow \boxed{h(x) = \frac{1}{3}(x-6)^2 - 9}$

Since $a = \frac{1}{3} > 0 \Rightarrow$ vertex is a minimum

4. Find the quadratic with axis of symmetry $x = 3$, a zero at (4,0), and a y-intercept of (0,16).

$h = 3$ $\quad f(4) = 0$ $\quad f(0) = 16$

vertex (h, k)

$f(x) = a(x-h)^2 + k$

$f(x) = a(x-3)^2 + k$

$0 = f(4) = a(4-3)^2 + k$

$0 = a + k$

$16 = f(0) = a(0-3)^2 + k$

$16 = 9a + k$

$\begin{cases} 9a + k = 16 \\ a + k = 0 \end{cases}$

$8a = 16 \rightarrow \boxed{a = 2}$ & $\boxed{k = -2}$

$\boxed{f(x) = 2(x-3)^2 - 2}$

5. Consider the function $g(x) = -\frac{5}{2} + 2(2-x)^2$. $= 2(-x+2)^2 - \frac{5}{2}$

a) Identify the parent function f.

$f(x) = x^2$

b) Describe the sequence of transformations from f to g.

1. Horiz left 2 units $g_1(x) = f(x+2) = (x+2)^2$
2. reflex w.r.t y-axis $g_2(x) = g_1(-x) = f(-x+2) = (-x+2)^2$
3. vert. stretch 2 $g_3(x) = 2g_2(x) = 2f(-x+2) = 2(-x+2)^2$
4. Vertical shift down $\frac{5}{2}$ $g_4(x) = g_3(x) - 5/2 = 2f(-x+2) - 5/2 = 2(-x+2)^2 - 5/2$

c) Use function notation to write g in terms of f.

$g(x) = 2f(-x+2) - 5/2$

d) Evaluate intercepts, vertex and axis of symmetry of $g(x)$.

X-int. $\Rightarrow \left(2-\frac{\sqrt{5}}{2}, 0\right)$ & $\left(2+\frac{\sqrt{5}}{2}, 0\right)$

$2(-x+2)^2 = 5/2 \longrightarrow (-x+2)^2 = \frac{5}{4} \longrightarrow -x+2 = \pm\frac{\sqrt{5}}{2}$

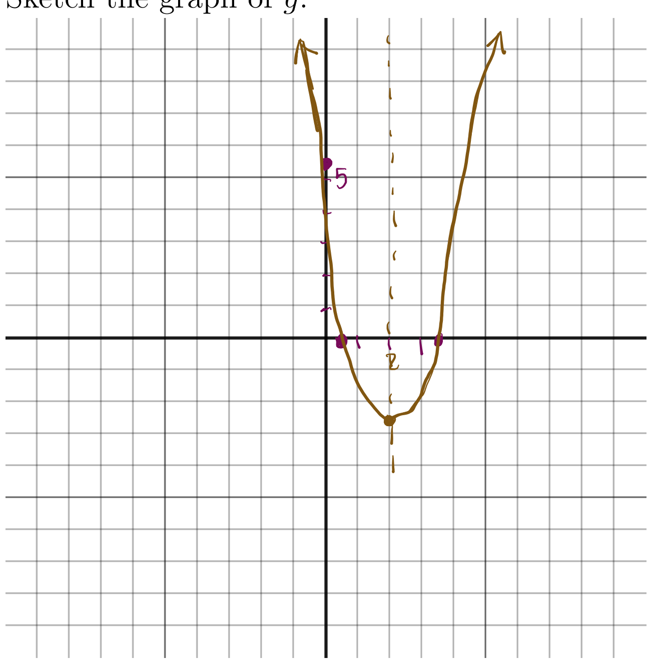
$-x = -2 + \frac{\sqrt{5}}{2}$ & $-x = -2 - \frac{\sqrt{5}}{2}$

$x = 2 - \frac{\sqrt{5}}{2} < 1$ & $x = 2 + \frac{\sqrt{5}}{2} > 3$

y-int: $\left(\frac{11}{2}, 0\right)$

$2(2)^2 - 5/2 = 8 - \frac{5}{2} = \frac{16-5}{2} = \frac{11}{2}$

e) Sketch the graph of g.



Vertex:

$3(-x+2)^2 - \frac{5}{2} = 3\left(-(x-2)\right)^2 - \frac{5}{2}$

$= 3(x-2)^2 - 5/2$

$(2, -5/2)$

$x = 2$ axis of sym.

6. Consider the function $g(x) = -3|x+3| - 4$.

a) Identify the parent function f.

$f(x) = |x|$

b) Describe the sequence of transformations from f to g.

① Horiz. left 3 units $\quad g_1(x) = |x+3|$

② vertical stretch factor of 3 $\quad g_2(x) = 3|x+3|$

③ reflection about x-axis $\quad g_3(x) = -g_2(x) = -3|x+3|$

④ vertical shift 4 down $\quad g_4(x) = g_3(x) - 4 = -3|x+3| - 4$

c) Use function notation to write g in terms of f.

$g(x) = -3f(x+3) - 4$

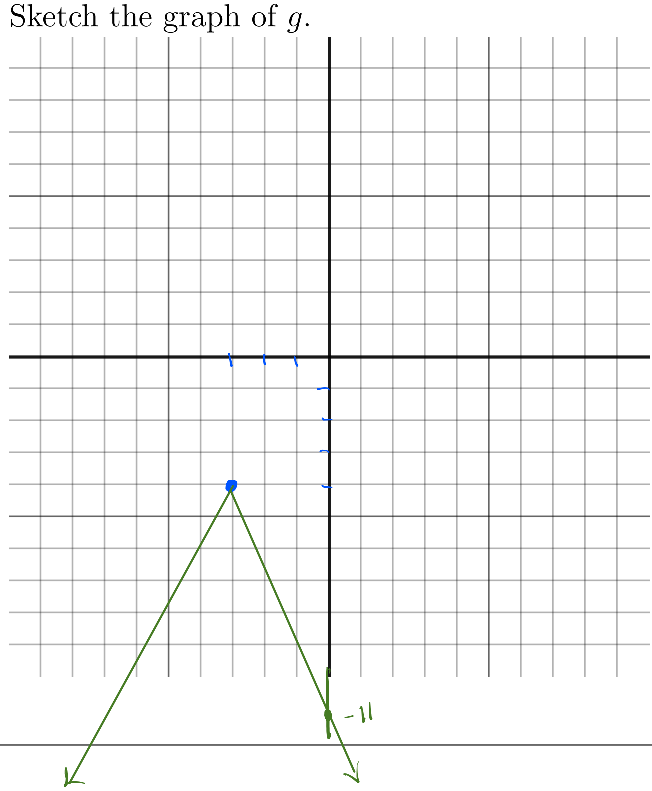
d) Evaluate intercepts, vertex and axis of symmetry of $g(x)$. format $a|x-h|+k$

$h = -3$ & $k = -4$ $\quad (-3,-4)$ vertex

$x = -3$ axis of symmetry

Y int.: $g(0) = -3|0+3| - 4 = -3(3) - 4 = -9-4 = -13$ $\quad (0,-13)$

e) Sketch the graph of g.



X-int:

$-3|x+3| - 4 = 0$

$|x+3| = -\frac{4}{3}$ No solution!

$\rightsquigarrow$ No x-intercepts

7. Most cars get their best gas mileage when traveling at a relatively modest speed. The gas mileage M for a certain new car is modeled by the function $M(s) = \frac{-1}{28}s^2 + 3s - 31$ where s is the speed in mi/h and M is measured in mi/gal. What is the car's best gas mileage and at what speed is it attained?

best gas Mil. $\Rightarrow$ Abs. Max

$a = -\frac{1}{28}$ & $b = 3$ $\Rightarrow$ Max value of $M(s)$ occurs at vertex.

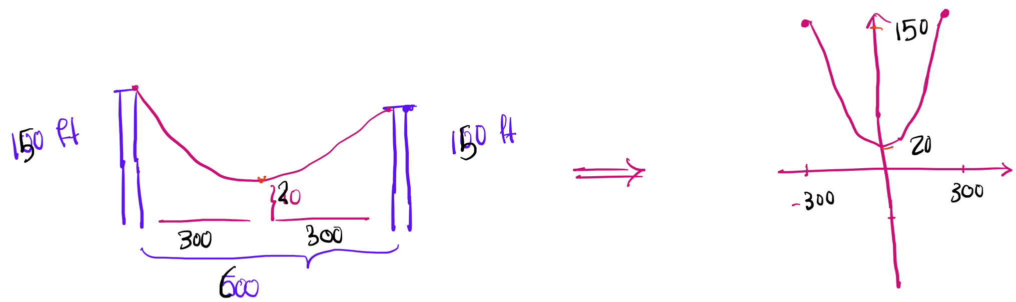
$s = \frac{-b}{2a} = \frac{-3}{2(-\frac{1}{28})} = \frac{-3}{-\frac{1}{14}} = \frac{3}{\frac{1}{14}} = 42$ mil/hr

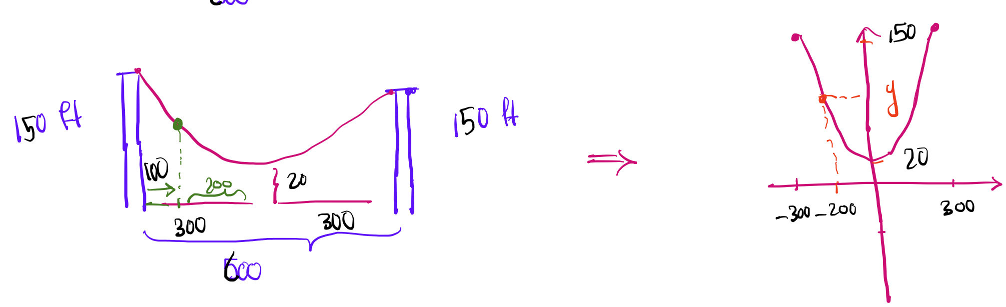
So Max value is $M(42) = -\frac{1}{28}(42)^2 + 3(42) - 31 = -\frac{1}{28}(42)(42) + 126 - 31$

$= -63 + \overbrace{126 - 31}^{95} = 32$ mil/gal

So the Cars best gas mileage is 32 mil/gal when the Car is traveling at 42 mil/h

8. The two towers of a suspension bridge are 600 feet apart. The parabolic cable attached to the tops of the towers is 20 feet above the point on the bridge deck that is midway between the towers. If the towers are 150 feet tall, find the height of the cable directly above a point of the bridge deck that is 100 feet to the right of the left-hand tower.





$$y = a(x-h)^2 + k \qquad \text{Vertex } (0, 20)$$

$$h = 0 \ \& \ k = 20$$

$$y = a(x)^2 + 10 \qquad \text{we know: } 150 = a(300)^2 + 20$$

$$130 = a(300)^2 \longrightarrow a = \frac{130}{90000} = \frac{13}{9000}$$

$$\Rightarrow y = \frac{13}{9000}(x)^2 + 20$$

$$y = \frac{13}{9000}(-200)^2 + 20 = \frac{40\cancel{0}\cancel{0}0 \times 13}{90\cancel{0}\cancel{0}} + 20$$

$$= \frac{520}{9} + 20$$

$$= \frac{520 + 180}{9} = \frac{700}{9} \quad \#$$

9. Solve the equation by using the quadratic formula $2x^2 = 3 - 2x$.

$2x^2 + 2x - 3 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2(a)}$

$x = \frac{-2 \pm \sqrt{4 - 4(2)(-3)}}{2(2)}$

$= \frac{-2 \pm \sqrt{4 + 24}}{2(2)} = \frac{-2 \pm \sqrt{28}}{2(2)}$

or

$= \frac{-1}{2} \pm \frac{\sqrt{7}}{2}$

10. Solve the equation $5x^2 + 2x - 1 = 0$ by completing the square.

$5\left(x^2 + \frac{2}{5}x - \frac{1}{5}\right) = 5\left[x^2 + \frac{2}{5}x + \left(\frac{2}{10}\right)^2 - \left(\frac{2}{10}\right)^2 - \frac{1}{5}\right]$

$= 5\left[\left(x + \frac{1}{5}\right)^2 - \frac{1}{25} - \frac{1}{5}\right] = 5\left((x+\frac{1}{5})^2 - \frac{6}{25}\right)$

$= 5\left(x + \frac{1}{5}\right)^2 - \frac{6}{5}$

Now to solve for $5x^2 + 2x - 1 = 0 \longrightarrow 5\left(x + \frac{1}{5}\right)^2 - \frac{6}{5} = 0$

$\longrightarrow \left(x + \frac{1}{5}\right)^2 = \frac{6}{25}$

$x + \frac{1}{5} = \pm\frac{\sqrt{6}}{5} \longrightarrow x = -\frac{1}{5} \pm \frac{\sqrt{6}}{5}$

11. For the given polynomial functions, determine the leading term, leading coefficient, degree, constant end behavior of the graph.

a) $g(x) = -2x^7 + 5x^3 + 4x - 16$

leading term: $-2x^7$

leading Coefficient: -2

degree: 7

Constant: -16

as $x \to +\infty$ then $f(x) \to -\infty$

as $x \to -\infty$ then $f(x) \to +\infty$

b) $g(x) = -4x^6 - 3x^5 + 8$

leading term: $-9x^6$

leading Coefficient: -4

degree 6

Constant 8

end behavior

as $x \to +\infty$ then $f(x) \to -\infty$

as $x \to -\infty$ then $f(x) \to -\infty$

12. Find the zeros and their multiplicities for the following functions, then determine the end behavior and maximum number of turning points. Roughly sketch the graph.

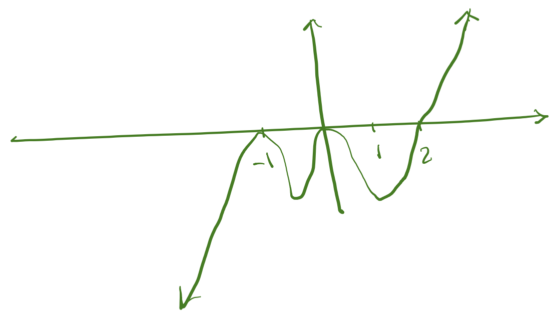
a) $k(x) = x^4(x-2)^3(x+1)^2$

leading term $x^4 (x)^3 (x^2) = x^9$ degree 9 $\to$ max # of turning points $9-1=8$

$k(x) = 0$ $\quad x^4 = 0 \to x = 0$ even mult.

$(x-2)^3 = 0 \to x = 2$ odd mult.

$(x+1)^2 = 0 \to x = -1$ even mult.



b) $f(x) = (2-x)(-x+3)x^2$

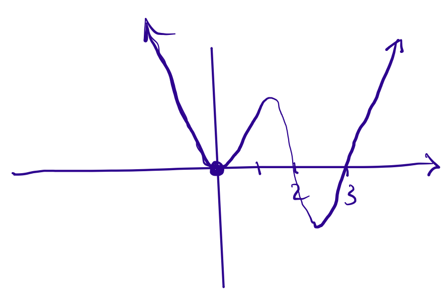
leading term: $(-x)(-x)x^2 = x^4$

degree 4 $\to$ max # of turning points $4-1=3$

$f(x) = 0 \to$ $\quad 2 - x = 0 \to x = 2$ odd mult.

$-x + 3 = 0 \to x = 3$ odd mult.

$x^2 = 0 \to x = 0$ even mult



13. Determine the quotient with fractional remainder (if necessary) of the function $(7x^5 - 46x^3 - 14x + 3) \div (x + 3)$.

$$
\begin{array}{r}
7x^4 - 21x^3 + 17x^2 - 51x + 139 \quad \leftarrow \text{Quotient!} \\
x+3 \,\big)\, 7x^5 - 46x^3 - 14x + 3 \\
-(7x^5 + 21x^4) \\
\hline
-21x^4 - 46x^3 - 14x + 3 \\
-(-21x^4 - 63x^3) \\
\hline
17x^3 - 14x + 3 \\
-(17x^3 + 51x^2) \\
\hline
-51x^2 - 14x + 3 \\
-(-51x^2 - 153x) \\
\hline
139x + 3 \\
-(139x + 417) \\
\hline
-414 \quad \leftarrow \text{remainder}
\end{array}
$$

$$= 7x^4 - 21x^3 + 17x^2 - 51x + 139 - \frac{414}{x+3}$$

14. Find a polynomial of degree four that has zeros -3, 0, 1 and the coefficient of x^3 is -6.

meaning

$P(x)$

if Polynomial was of degree 3:

$P(-3) = 0 \quad x = -3 \rightarrow x + 3 = 0$

$P(0) = 0 \quad x = 0 \rightarrow x - 0 = 0$

$P(1) = 0 \quad x = 1 \rightarrow x - 1 = 0$

$\Rightarrow P(x) = -6\,x(x+3)(x-1)$

Polynomial of degree 4 $\rightarrow$ only 3 zeros are given $\rightarrow$ So one of the zeros have been repeated let's say $x = 0$ was repeated twice

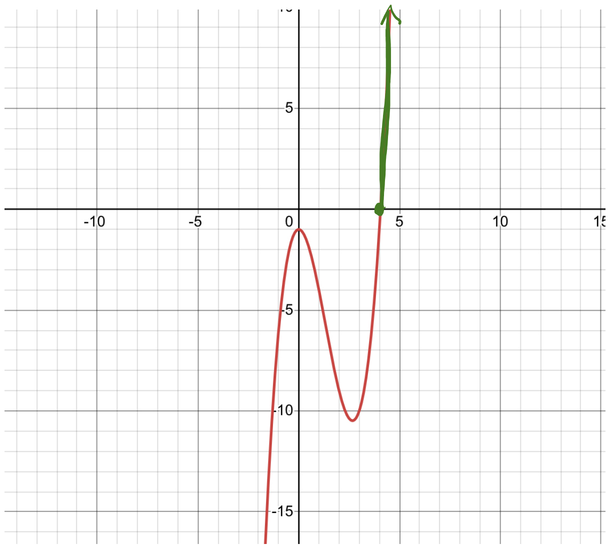
$$P(x) = ax^2(x+3)(x-1) = a(x^3 + 3x^2)(x-1) = a\left[x^4 - x^3 + 3x^3 - 3x^2\right] = a(x^4 + 2x^3 - 3x^2)$$

$$= ax^4 + 2ax^3 - 3x^2$$

we want coefficient of x^3 to be -6 So $2a = -6 \Rightarrow a = -3$

$$\Rightarrow \boxed{P(x) = -3x^2(x+3)(x-1)}$$

15. Graph of the function $f(x) = x^3 - 4x^2 - 25$ is given. Solve for $x^3 - 4x^2 \geq 25$

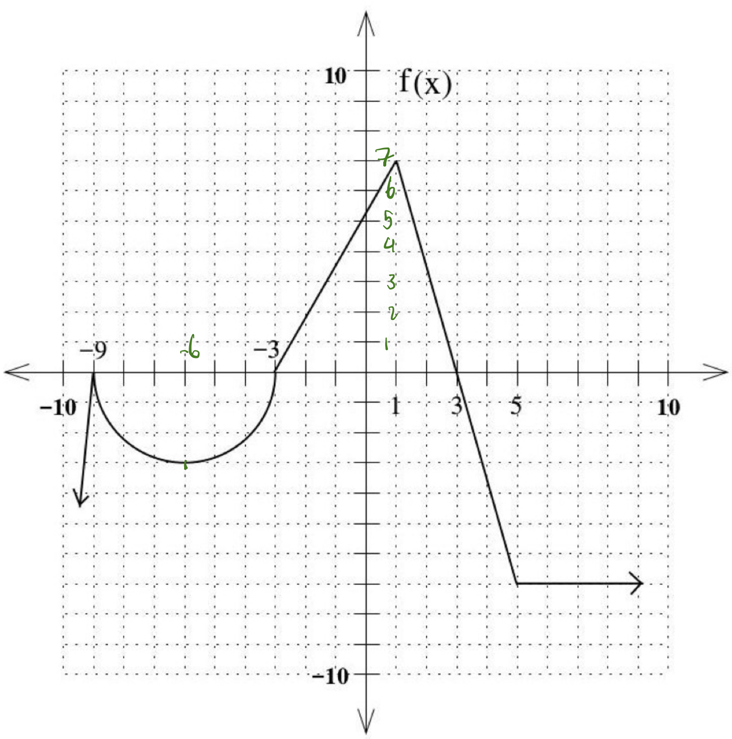


$\Updownarrow$

$x^3 - 4x^2 - 25 \geq 0$

$x \in [5, +\infty)$

16. In the following graph, state domain, range, interval of increase, interval of decrease and absolute extrema.



domain $(-\infty, +\infty)$

Range: $(-\infty, 7]$

increase on $(-\infty, -9), (-6, 1)$

decrease on $(-9, -6), (1, 5)$

Abs Min: DNE

Abs Max value 7

17. Test the equation $y = x^3 - 9|x|$ for symmetry.

Sym. about x-axis? change y with $-y$:

$-y = x^3 - 9|x|$

$y = -x^3 + 9|x|$ X not true

Sym. about y-axis? change x with $-x$

$y = (-x)^3 - 9|-x| = -x^3 - 9|x|$ X

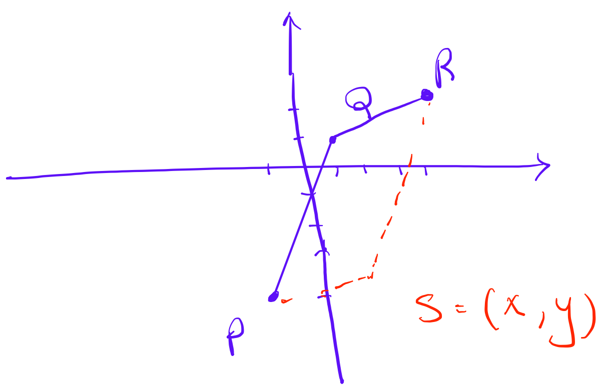
Sym. about the origin? Change x with $-x$ & y with $-y$

$-y = (-x)^3 - 9|-x|$

$-y = -x^3 - 9|x|$

$y = x^3 + 9|x|$ X No Symmetry!

18. Plot the points $P = (-1, -4)$, $Q = (1, 1)$, and $R = (4, 2)$ on a coordinate plane. Where should the point S be located so that the figure PQRS is a parallelogram?



$d_{PQ} = d_{SR}$

& $d_{QR} = d_{PS}$

$d_{PQ} = \sqrt{(2)^2 + (5)^2} = \sqrt{29}$

$d_{SR} = \sqrt{(x-4)^2 + (y-2)^2}$

$\longrightarrow (x-4)^2 + (y-2)^2 = 29$

$d_{QR} = \sqrt{(3)^2 + (1)^2} = \sqrt{10}$

$d_{PS} = \sqrt{(x+1)^2 + (y+4)^2}$

$\longrightarrow (x+1)^2 + (y-4)^2 = 10$

Not helpful?

Midpoint of diagonals should be the same

$$M_{PR} = M_{QS}$$

$$M_{PR} = \left(\frac{4-1}{2} , \frac{2+(-4)}{2} \right) = \left(\frac{3}{2} , -1 \right)$$

$$M_{QS} = \left(\frac{x+1}{2} , \frac{1+y}{2} \right)$$

$x+1=3 \rightarrow x=2$

$1+y=-2 \rightarrow y=-3$

$$\boxed{S=(2,-3)}$$

19. Find average rate of change of the function $r(t) = 3 - \frac{1}{3}t$ from $t = 1$ to $t = 5$.

$$\text{Ave R.O.C} = \frac{r(5) - r(1)}{5-1} = \frac{4/3 - 8/3}{4} = \frac{-\frac{4}{3}}{4} = -\frac{1}{3}$$

$$r(5) = 3 - \frac{5}{3} = \frac{9-5}{3} = \frac{4}{3}$$

$$r(1) = 3 - \frac{1}{3} = \frac{8}{3}$$

20. Solve the inequality $|3x + 2| \geq 4x^2 + 1$.

$$|3x+2| - 4x^2 - 1 \geq 0$$

$$y = |3x+2| - 4x^2 - 1 = 0$$

recall: $|3x+2| = \begin{cases} 3x+2 & \text{if } 3x+2 \geq 0 \\ -(3x+2) & \text{if } 3x+2 < 0 \end{cases}$

$$= \begin{cases} 3x+2 & \text{if } x \geq -2/3 \\ -3x-2 & x < -2/3 \end{cases}$$

Case 1) if $x \geq -\frac{2}{3}$

3, -1, $+4$, -4

$$y = 3x+2 - 4x^2 - 1 = -4x^2 + 3x + 1 = 0 \rightarrow \underbrace{-4x^2 + 4x}\, - x + 1 = 0$$

$$4x(-x+1) + (-x+1) = (-x+1)(4x+1) = 0$$

$\rightarrow$ $\boxed{x = -\frac{1}{4}}$

$\boxed{x = 1}$

$$-\frac{3}{4} = -\frac{1}{4} > -\frac{2}{3} = -\frac{8}{12}$$

Case 2) if $x < -\frac{2}{3}$

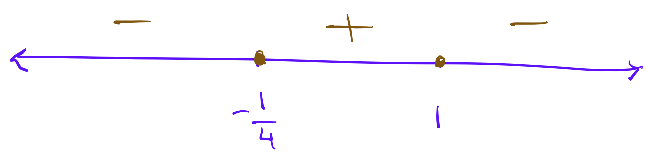
$$y = -(3x+2) - 4x^2 - 1 = -3x - 2 - 4x^2 - 1 = -4x^2 - 3x - 3$$

$$= -(4x^2 + 3x + 3) = 0$$

$$x = \frac{-3 \pm \sqrt{9 - 4(4)(3)}}{2(4)}$$ ← negative!

No solution!

$$|3x+2| - 4x^2 - 1 \geq 0$$



$$x \in \left[-\frac{1}{4}, 1\right]$$

21. $z_1 = 1 + \sqrt{-27}$ $\qquad z_2 = 2 - \sqrt{-12}$

a)

$$z_1 = 1 + \sqrt{27 \times -1} = 1 + \sqrt{27} \cdot \sqrt{-1} = 1 + \sqrt{27}\, i = 1 + 3\sqrt{3}\, i$$

$$z_2 = 2 - \sqrt{12}\, i = 2 - 2\sqrt{3}\, i$$

b)

$$z_1 + z_2 = 1 + 3\sqrt{3}\, i + 2 - 2\sqrt{3}\, i = 3 + (\sqrt{3})\, i$$

$$z_1 - z_2 = (1 + 3\sqrt{3}\, i) - (2 - 2\sqrt{3}\, i) = -1 + (5\sqrt{3})\, i$$

$$z_1 \cdot z_2 = (1 + 3\sqrt{3}\, i)(2 - 2\sqrt{3}\, i) = 2 - 2\sqrt{3}\, i + 6\sqrt{3}\, i - 6 \times 3\, i^2$$

$$= 2 + 18 + 4\sqrt{3}\, i$$

$$= 20 + 4\sqrt{3}\, i$$

c & d not needed ^^