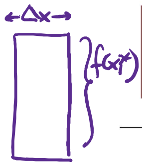
Riemann sums : $\sum_{i=1}^{n} f(x_i^*)\Delta x = f(x_1^*)\Delta x + f(x_2^*)\Delta x + \dots + f(x_n^*)\Delta x$

on $[a, b]$



$\Delta x = \frac{b-a}{n}$

MATHEMATICS
Math 142 - Spring 2024
WIR 10: Exam 3 review

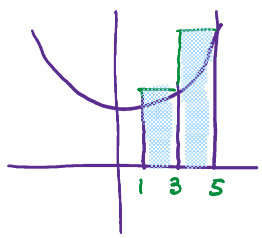
TEXAS A&M UNIVERSITY
College of Arts & Sciences

Problem 1. Estimate the area under the graph of $f(x) = x^2 + 5$ on the interval $[1, 5]$ using Reimann sums and with 2 rectangles of equal width. Sketch the graph and the rectangles and show your Riemann sum formula.

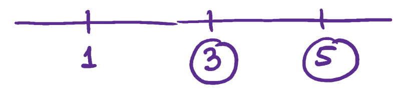
$f(x) = x^2+5$, $n = 2$

$\Delta x = \frac{b-a}{n} = \frac{5-1}{2} = \frac{4}{2} = 2$

(1) Use a right-hand Riemann sum.



$\Delta x = 2$



$R_2 = f(3)\cdot 2 + f(5)\cdot 2$

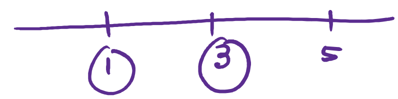
$= (14)(2) + (30)(2)$

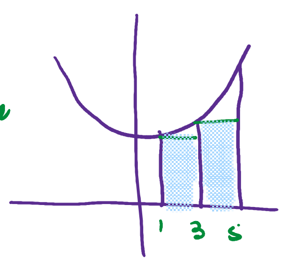
$= 88$

over estimation

(2) Use a left-hand Riemann sum.

$\Delta x = 2$





under estimate

$L_2 = f(1)\cdot 2 + f(3)\cdot 2$

$= (6)(2) + (14)(2)$

$= 40$

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$b = 4 \qquad a = -4$

$R_4 \qquad n=4$

Problem 2. Approximate $\int_{-4}^{4} (2x^2 - x - 2)\, dx$ using a right-hand Riemann sum with 4 subintervals of equal width. Show your Riemann sum formula. What does your answer represent? What does the definite integral represent?

$$\int_{-4}^{4} \underbrace{(2x^2-x-2)}\, dx$$

$$\Delta x = \frac{b-a}{n} = \frac{4-(-4)}{4} = \frac{8}{4} = 2$$

$-4 \quad -2 \quad 0 \quad 2 \quad 4$

$$\sum_{i=1}^{4} f(x_i^*)\Delta x = f(x_1^*)\cdot 2 + f(x_2^*)\cdot 2 + f(x_3^*)\cdot 2 + f(x_4^*)\cdot 2$$

$$= f(-2)\cdot 2 + f(0)\cdot 2 + f(2)\cdot 2 + f(4)\cdot 2 = R_4$$

$f(x) = 2x^2 - x - 2$

x	$f(x)$
-2	8
0	-2
2	4
4	26

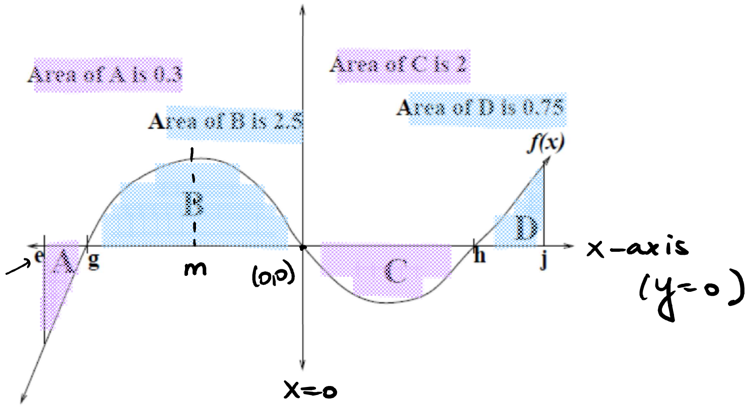
$R_4 = (8)(2) + (-2)(2) + (4)(2) + (26)(2)$

$R_4 = 72$

your answer: approximate net area

definite integral: exact net area.

Problem 3. Use the diagram below to calculate the following:



(1) $\int_g^j f(x)\,dx$ $= B + C + D = 2.5 + (-2) + (0.75) = 1.25$

$\int_e^0 f(x)dx = -\int_0^e f(x)dx$

(2) $\int_e^0 f(x)\,dx + \int_0^h 2f(x)\,dx$ $= (-0.3) + (2.5) + 2(-2) = -1.8$

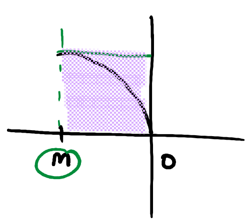
$[A+B] + [2(C)]$

(3) $\int_e^h 4f(x)\,dx - \int_h^j f(x)\,dx$ $= 4(-0.3 + 2.5 - 2) - (0.75)$

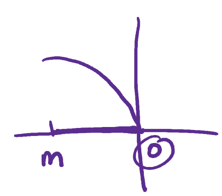
$4(A+B+C) - D$

$= 0.05$

(4) When finding a left-hand Reimann sum of $f(x)$ on the interval $[m, 0]$, will we get an over estimate or an under estimate if m is halfway between g and 0?



over estimate



$$\int_1^5 x^2 dx$$

Problem 4. Given $\int_1^4 x dx = 7.5$, $\int_1^4 x^2 dx = 21$, and $\int_4^5 x^2 dx = \frac{61}{3}$, evaluate

(1) $\int_1^4 (4x^2 - 9x) dx$

$$= \int_1^4 4x^2 dx - \int_1^4 9x\, dx$$

$$= 4 \int_1^4 x^2 dx - 9 \int_1^4 x\, dx\,.$$

$$= 4(21) - 9(7.5)$$

$$= 16.5$$

(2) $\int_1^5 (-4x^2) dx$

$$= (-4) \int_1^5 x^2 dx$$

$$= (-4) \int_1^4 x^2 dx + (-4) \int_4^5 x^2 dx$$

$$= (-4)(21) + (-4)\left(\frac{61}{3}\right)$$

$$= -165.333$$

Problem 5. Evaluate the following integrals.

(1) $\int_0^3 (3t^2(t+1) + \sqrt{t} + e^t)\ dt$ $= \int_0^3 \underbrace{3t^3} + \underbrace{3t^2} + \underbrace{t^{1/2}} + \underbrace{e^t}\ dt$

$= 3.\frac{t^4}{4} + \cancel{3}\frac{t^3}{\cancel{3}} + \frac{t^{3/2}}{3/2} + e^t \Big|_{t=0}^{t=3}$

$= \left[\frac{3}{4}\cdot 3^4 + 3^3 + \frac{2}{3}\cdot(3^{3/2}) + e^3\right] - \left[\frac{3}{4}\cdot 0 + 0 + \frac{2}{3}\cdot 0 + e^0\right]$

1

$= \frac{3^5}{4} + 3^3 + \frac{2}{3}(3^{3/2}) + e^3 - 1$

(2) $\int_a^{2a} (4x - 5)\ dx$ for $a > 0$

$\frac{d}{dx}(\ln(x)) = \frac{1}{x}$

(3) $\int_2^e \frac{5}{2x\ln(5x)}\ dx$

$= \frac{5}{2}\int_2^e \frac{1}{x\cdot\ln(5x)}\cdot dx$ $= \frac{5}{2}\int_2^e \left(\frac{1}{x}\right)\left(\frac{1}{\ln(5x)}\right)dx$ $= \frac{5}{2}\int_2^e \left(\frac{1}{\ln(5x)}\right)\left(\frac{1}{x}\right)dx$

$\boxed{u = \ln(5x)}$

$du = \left(\frac{1}{\cancel{5}x}\right)(\cancel{5}\ dx) = \frac{1}{x}dx$

$x = 2,\quad u = \ln(10)$

$x = e,\quad u = \ln(5e)$

$= \frac{5}{2}\int_{u=\ln(10)}^{u=\ln(5e)} \frac{1}{u}\cdot du$ $= \frac{5}{2}\ln|u| \Big|_{\ln(10)}^{\ln(5e)}$ $= \frac{5}{2}\ln|\ln(5e)| - \frac{5}{2}\ln|\ln(10)|$

(4) $\displaystyle\int \frac{30x+3}{10x^2+2x-4}\,dx$ $= \displaystyle\int \frac{3(10x+1)}{2(5x^2+x-2)}\,dx$

$u = 5x^2+x-2$

$\frac{du}{dx} = 10x+1$

(5) $\displaystyle\int \frac{2e^{5x}-4}{e^{5x}-10x}\,dx$ $= \displaystyle\int \frac{2(e^{5x}-2)}{e^{5x}-10x}\,dx.$

$u = e^{5x}-10x.$

$\frac{du}{dx} = 5e^{5x}-10 = 5(e^{5x}-2)$

(6) $\int (\underline{x^9+4x})e^{x^{10}+20x^2}\,dx$

$u = x^{10}+20x^2$

$\frac{du}{dx} = 10x^9 + 20(2x) = 10(x^9+4x)$

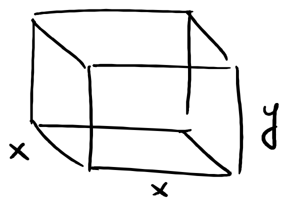
(7) $\int_1^4 \frac{x}{(x^2-9)^5}\, dx$

(8) $\int_{-1}^2 (x-5)(x^2-10x)^3\, dx$

$\ell = w = x$ and $ht = y$

Problem 6. A box with a square base and an open top must have a volume of $216m^3$. Find the dimensions of the box so that the least amount of material is used to make the box.

minima. surface area.



① want : $S = x^2 + 4xy$

know : $V = \underbrace{x^2 y = 216}\ m^3$

② $y = \frac{216}{x^2} \leftarrow$

$$S(x) = x^2 + 4x \cdot \left(\frac{216}{x^2}\right) = x^2 + \frac{4(216)}{x} = x^2 + 864x^{-1}$$

minimize $\boxed{S(x) = x^2 + 864x^{-1}}$

③ Interval : $x > 0$, $y > 0$

$\frac{216}{x^2} > 0 \Rightarrow x < 0$ or $x > 0$

$(0, \infty)$

④ Calculus : $S'(x) = 2x - 864 \cdot x^{-2}$

$= 2x - \frac{864}{x^2} = 0$

$2x = \frac{864}{x^2}$

$2x^3 = 864$

$x^3 = 432$

$x = \sqrt[3]{432}$

$x \sim 7.560$

$y = \frac{216}{x^2} \sim 3.780$

one cv on interval $(0, \infty)$

1st or 2nd derivative test

$S''(x) = 2 - 864(-2) \cdot x^{-3} = 2 + \frac{2(864)}{x^3}$

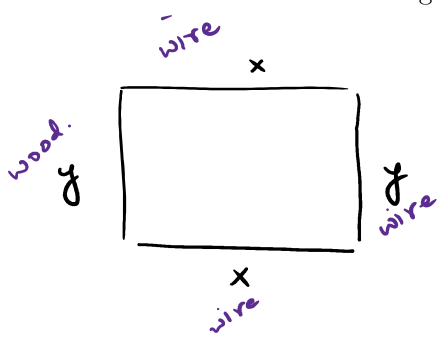
$S''(\sqrt[3]{432}) = 6 \Rightarrow$ cv is a minima.

⑤ Answer: Box must be $7.560\,m \times 7.560\,m \times 3.780\,m$

ℓ w ht

cost .

Problem 7. A rectangular garden needs to be fenced. There is \$320 available for this project. Three sides of the fence will be constructed with wire fencing at the cost of \$2 per foot. The fourth side will be constructed with wood fencing at a cost of \$6 per foot. Find the length of the sides as well as the area of the largest garden that can be fenced in this way.



① want: $A = xy$

know: $C = x(2) + y(2) + x(2) + y(6)$

$C = 4x + 8y = 320$

$4x = 320 - 8y$

② $\boxed{x = 80 - 2y}$

maximize $A(y) = (80-2y)y = 80y - 2y^2$

③ Interval: $x > 0$ $y > 0$

$80 - 2y > 0$

$(0, 40)$ $80 > 2y$

$2y < 80$

$y < 40$

④ Calculus: $A'(y) = 80 - 4y = 0 \Rightarrow y = \frac{80}{4} = 20$.

2nd derivative test $A''(y) = -4 \longrightarrow$ cv must be a maxima.

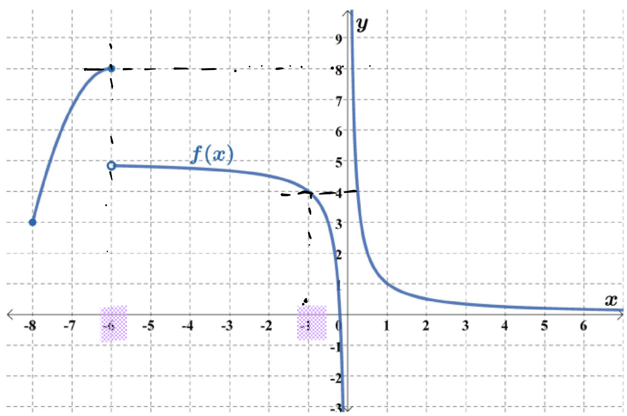
$x = 80 - 2y = 40$

⑤ Ans: Garden can be 20 feet × 40 feet

largest area = $(20)(40) = 800 \text{ ft}^2$

Problem 8. Use the graph below to find any absolute maxima or absolute minima on each of the given intervals. Is the extrema a maxima or a minima?

and/or



(1) $[-8, 6)$

Absolute maxima : no

Abs. minima : no

(2) $(0, 1)$

Abs max: no

Abs min: no

(3) $(-7, -1)$

Abs max : 8

Abs min : no

(4) $[-8, -1]$

Abs max = 8

Abs min = 3

(5) $[-6, -1]$

Abs max = 8

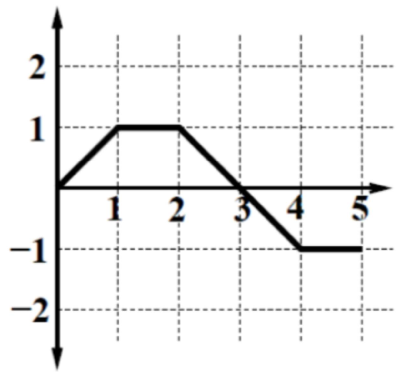
Abs min = 4

Problem 9. A lawnmower company's marginal profit is given by $M(x) = 35e^{-0.01x}$ dollars per lawnmower, where x is the number of lawnmowers produced and sold each day. The profits earned from producing and selling 120 lawnmowers each day is \$300.

(1) If the current production level is 250 lawnmowers each day and the goal is to increase production to 275 lawnmowers each day, how will this production affect profit?

(2) What is the profit earned when 200 lawnmowers are produced and sold each day?

Problem 10. Use the graph of $f(x)$ below to answer the following questions.



(1) $\int_0^3 f(x)\ dx =$

(2) $\int_0^4 f(x)\ dx =$

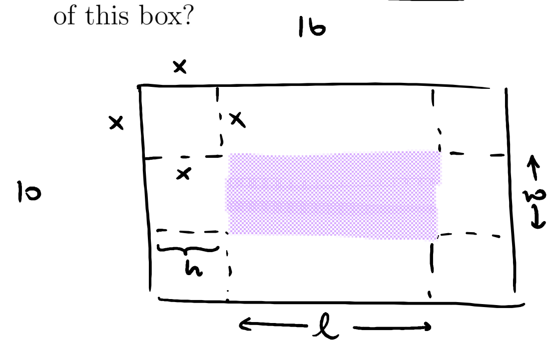
(3) $\int_0^5 f(x)\ dx =$

(4) $\int_2^5 f(x)\ dx =$

(5) $\int_2^2 f(x)\ dx =$

(6) $\int_3^0 f(x)\ dx =$

Problem 11. An open box is made from a 10-inch by 16-inch piece of cardboard by cutting out squares of equal size from the four corners and folding up the sides. Find the length and width of each square so that the volume of the box is maximized. What is the largest possible volume of this box?



① want $V = l w \cdot h$

$= (16-2x)(10-2x)\,x$

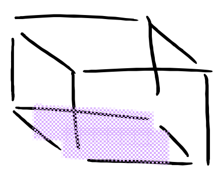
$= 160x - 52x^2 + 4x^3$

know: dont need a constraint since V is already only in terms of x

$l = 16 - 2x$

$w = 10 - 2x$

$h = x$



② maximize $V(x)$

③ interval: $x > 0$, $2x < 10$, $2x < 16$

$(0, 5)$

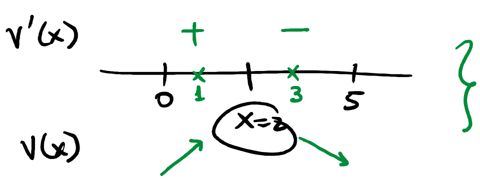
④ Calculus: $\boxed{V'(x)} = 160 - 52(2x) + 4(3x^2) = 0$.

$= 4(40 - 26x + 3x^2) = 0$.

$40 - 26x + 3x^2 = 0$ $\quad x = \dfrac{+26 \pm \sqrt{(-26)^2 - 4(3)(40)}}{2(3)}$

$\cancel{x = \frac{20}{3}}$ $\quad x = 2$

Only 1 cv in $(0,5)$ → 1st derivative test



cv is a maxima.

⑤ Answer: Square should be 2x2 inches

largest volume: $(2)(16-4)(10-4) = 2(12)(6)$

$= 144$ inches^2

Problem 12. Find the absolute maximum and the absolute minimum values for the following functions

(1) $f(x) = x^3 + 4x^2 + 4$ on the closed interval $[-4, 1]$.

(2) $f(x) = (5 - x)(x + 7)^2$ on the closed interval $[-9, -5]$.