Week in Review
Math 152

Week 08

Common Exam 2

Preparation

Part II

Common Exam II Prep. Part II

1. After long division, $\frac{x^4+5x^2+1}{x^2+1} = A + \frac{B}{x^2+1}$. What are A and B?

   (a) $A = x^2$ and $B = 4x^2 - 3$

   (b) $A = x^2 + 4$ and $B = -3$

   (c) $A = 1$ and $B = 5x + 1$

   (d) $A = 0$ and $B = 1$

   (e) $A = x^2 + 5x$ and $B = x + 1$

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2. What would be an appropriate substitution in order to evaluate $\int \frac{1}{\sqrt{x^2+10x}}\,dx$?

   (a) $x = 5\sec\theta - 5$

   (b) $x = 5\sin\theta + 5$

   (c) $x = 5\tan\theta$

   (d) $x = 25\sec\theta$

   (e) $x = 25\sin\theta - 5$

3. Evaluate the improper integral $\int_5^\infty \frac{1}{x(\ln x)^4}\, dx$.

(a) $-\frac{3}{(\ln 5)^3}$

(b) The integral diverges.

(c) $\frac{1}{3(\ln 5)^3}$

(d) $\frac{1}{375}$

(e) $\frac{3}{125}$

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4. The following recursive sequence is bounded and increasing. Find the limit of the sequence, if it exists.

   $a_1 = 4, \quad a_{n+1} = 8 - \dfrac{15}{a_n}.$

   (a) 8

   (b) 4

   (c) 3

   (d) 5

   (e) The sequence diverges.

5. When applying the comparison test for improper integrals, which of the following statements is true regarding

$$\int_2^\infty \frac{\cos(x)+6}{x^3}\,dx?$$

(a) $\displaystyle\int_2^\infty \frac{\cos(x)+6}{x^3}\,dx$ diverges because $\displaystyle\frac{\cos(x)+6}{x^3} < \frac{5}{x^3}$ and $\displaystyle\int_2^\infty \frac{5}{x^3}\,dx$ converges.

(b) $\displaystyle\int_2^\infty \frac{\cos(x)+6}{x^3}\,dx$ diverges because $\displaystyle\frac{\cos(x)+6}{x^3} > \frac{5}{x^3}$ and $\displaystyle\int_2^\infty \frac{5}{x^3}\,dx$ converges.

(c) $\displaystyle\int_2^\infty \frac{\cos(x)+6}{x^3}\,dx$ converges because $\displaystyle\frac{\cos(x)+6}{x^3} < \frac{5}{x^3}$ and $\displaystyle\int_2^\infty \frac{5}{x^3}\,dx$ diverges.

(d) $\displaystyle\int_2^\infty \frac{\cos(x)+6}{x^3}\,dx$ converges because $\displaystyle\frac{\cos(x)+6}{x^3} < \frac{7}{x^3}$ and $\displaystyle\int_2^\infty \frac{7}{x^3}\,dx$ converges.

(e) $\displaystyle\int_2^\infty \frac{\cos(x)+6}{x^3}\,dx$ converges because $\displaystyle\frac{\cos(x)+6}{x^3} > \frac{7}{x^3}$ and $\displaystyle\int_2^\infty \frac{7}{x^3}\,dx$ converges.

6. Find the sum of the geometric series $\sum_{n=1}^{\infty} \frac{3^{n+1}}{4^n}$.

(a) 3

(b) The series diverges.

(c) 9

(d) 4

(e) 1

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7. Which of the following series fails the test for divergence?

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{3n^2}{5n^2+1}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n}$

(c) $\displaystyle\sum_{n=1}^{\infty} \cos\left(\frac{1}{n}\right)$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\arctan n}$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{1-e^{-n}}$

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8. Given the partial fraction decomposition $\frac{2x^2 - x + 4}{x(x^2+4)} = \frac{A}{x} + \frac{Bx + C}{x^2+4}$, what are the values of A, B and C?

(a) A$=1$, B$=1$, C$=-1$.

(b) A$=4$, B$=-2$, C$=-1$.

(c) A$=1$, B$=1$, C$=1$.

(d) A$=4$, B$=2$, C$=1$.

(e) A$=1$, B$=2$, C$=4$.

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9. Which of the following statements is true about a series $\sum_{n=1}^{\infty} a_n$ whose nth partial sum is given by $s_n = \frac{n}{2n+1}$?

(a) The first term of the series is $a_1 = 1/3$ and the series diverges.

(b) The first term of the series is $a_1 = 0$ and the series converges to 1.

(c) The first term of the series is $a_1 = 1/3$ and the series converges to $1/2$.

(d) The first term of the series is $a_1 = 1/3$ and the series converges to 1.

(e) The first term of the series is $a_1 = 1$ and the series diverges.

10. Which of the following integrals are improper?

(I) $\int_1^e \ln(x-1)\,dx$ (II) $\int_{-5}^0 \frac{1}{5+2x}\,dx$ (III) $\int_1^\infty \frac{2}{x^3}\,dx$

(a) Only (III) is an improper integral.

(b) Only (I) and (III) are improper integrals.

(c) Only (II) and (III) are improper integrals.

(d) Only (I) is an improper integral.

(e) (I), (II) and (III) are all improper integrals.

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11. Let $s = \sum_{n=1}^{\infty} \frac{1}{n^4}$. Using The Remainder Estimate for the Integral Test, find the smallest value of n such that $R_n = s - s_n \leq \frac{1}{81}$.

(a) $n = 2$

(b) $n = 3$

(c) $n = 4$

(d) $n = 5$

(e) $n = 6$

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12. Evaluate the integral $\int_{1/2}^{\sqrt{3}/2} \frac{1}{\sqrt{1-x^2}}\,dx$.

(a) $\pi/3 - \pi/6$

(b) $\dfrac{\sqrt{3}-1}{2}$

(c) $\pi/6 - \pi/3$

(d) $\dfrac{\sqrt{3}+1}{2}$

(e) $\pi/3 - \pi/4$

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13. Which of the following sequences will converge?

(I) $a_n = \dfrac{\sin n}{n}$ (II) $a_n = \ln(5n+1) - \ln(3n+2)$ (III) $a_n = \dfrac{(-1)^n 5n}{n+5}$

(a) (I) and (III) only.

(b) (II) only.

(c) (I) and (II) only.

(d) (I), (II) and (III).

(e) (II) and (III) only.

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14. Which of the following is true for the sequence $a_n = 3 + \dfrac{(-1)^n n}{3n^2+1}$ for $n \geq 1$?

(a) The sequence is decreasing and the sequence converges to 3.

(b) The sequence is increasing and the sequence diverges.

(c) The sequence is neither increasing nor decreasing and the sequence diverges.

(d) The sequence is neither increasing nor decreasing and the sequence converges to 3.

(e) The sequence is increasing and the sequence converges to 3.

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15. What is a general formula for the sequence $a_n = \left\{\frac{1}{5}, \frac{-4}{8}, \frac{9}{11}, \frac{-16}{14}, \frac{25}{17}, \ldots\right\}$, for $n \geq 1$?

(a) $a_n = \dfrac{(-1)^{n-1}n^2}{2n+3}$

(b) $a_n = \dfrac{(-1)^{n-1}2n}{n^2+4}$

(c) $a_n = \dfrac{(-1)^{n+1}n^2}{4n+1}$

(d) $a_n = \dfrac{(-1)^{n}n^2}{n+3}$

(e) $a_n = \dfrac{(-1)^{n+1}n^2}{3n+2}$

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16. (10 points) Does the telescoping series $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty}(e^{1/n} - e^{1/(n+1)})$ converge or diverge?

If the series converges, find the sum of the series.

17. (10 points) Compute $\int \frac{\sqrt{9+x^2}}{x^4}\, dx$.

Your final answer MUST be presented in x, without any inverse trigonometric function(s).

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18. (10 points) Evaluate the integral $\displaystyle\int \frac{4x^2 - 5x + 11}{(x-1)(x^2+4)}\,dx$.

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19. (10 points) Use the Integral Test to show that the series $\sum_{n=1}^{\infty} \frac{3e^{1/n}}{n^2}$ converges or prove that it diverges.

Your answer MUST be presented as a complete coherent sentence with justification.