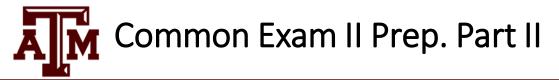


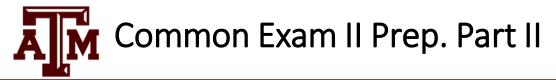
Week in Review Math 152

Week 08 Common Exam 2 Preparation Part II

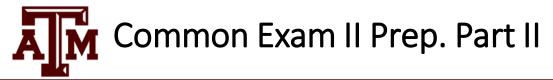
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1. After long division, $\frac{x^4 + 5x^2 + 1}{x^2 + 1} = A + \frac{B}{x^2 + 1}$. What are A and B? (a) $A = x^2$ and $B = 4x^2 - 3$ (b) $A = x^2 + 4$ and B = -3(c) A = 1 and B = 5x + 1(d) A = 0 and B = 1(e) $A = x^2 + 5x$ and B = x + 1



- 2. What would be an appropriate substitution in order to evaluate $\int \frac{1}{\sqrt{x^2 + 10x}} dx$?
 - (a) $x = 5 \sec \theta 5$
 - (b) $x = 5\sin\theta + 5$
 - (c) $x = 5 \tan \theta$
 - (d) $x = 25 \sec \theta$
 - (e) $x = 25 \sin \theta 5$



3. Evaluate the improper integral $\int_5^\infty \frac{1}{x(\ln x)^4} \ dx$.

(a)
$$-\frac{3}{(\ln 5)^3}$$

(b) The integral diverges.

(c)
$$\frac{1}{3(\ln 5)^3}$$

(d) $\frac{1}{375}$
(e) $\frac{3}{125}$



4. The following recursive sequence is bounded and increasing. Find the limit of the sequence, if it exists.

 $a_1 = 4, \quad a_{n+1} = 8 - \frac{15}{a_n}.$ (a) 8 (b) 4

- (c) 3
- (d) 5
- (e) The sequence diverges.

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5. When applying the comparison test for improper integrals, which of the following statements is true regarding



6. Find the sum of the geometric series $\sum_{n=1}^{\infty} \frac{3^{n+1}}{4^n}$.

- (a) 3
- (b) The series diverges.
- (c) 9
- (d) 4
- (e) 1



7. Which of the following series fails the test for divergence?

(a)
$$\sum_{n=1}^{\infty} \frac{3n^2}{5n^2 + 1}$$

(b)
$$\sum_{n=1}^{\infty} \frac{\ln n}{n}$$

(c)
$$\sum_{n=1}^{\infty} \cos\left(\frac{1}{n}\right)$$

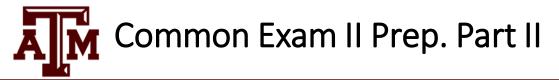
(d)
$$\sum_{n=1}^{\infty} \frac{1}{\arctan n}$$

(e)
$$\sum_{n=1}^{\infty} \frac{1}{1 - e^{-n}}$$

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8. Given the partial fraction decomposition $\frac{2x^2 - x + 4}{x(x^2 + 4)} = \frac{A}{x} + \frac{Bx + C}{x^2 + 4}$, what are the values of A, B and C?

- (a) A=1, B=1, C=-1.
- (b) A = 4, B = -2, C = -1.
- (c) A = 1, B = 1, C = 1.
- (d) A = 4, B = 2, C = 1.
- (e) A = 1, B = 2, C = 4.



9. Which of the following statements is true about a series $\sum_{n=1}^{\infty} a_n$ whose *n*th partial sum is given by $s_n = \frac{n}{2n+1}$?

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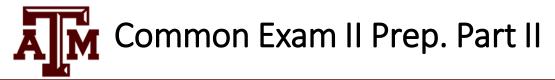
- (a) The first term of the series is $a_1 = 1/3$ and the series diverges.
- (b) The first term of the series is $a_1 = 0$ and the series converges to 1.
- (c) The first term of the series is $a_1 = 1/3$ and the series converges to 1/2.
- (d) The first term of the series is $a_1 = 1/3$ and the series converges to 1.
- (e) The first term of the series is $a_1 = 1$ and the series diverges.



10. Which of the following integrals are improper?

(I)
$$\int_{1}^{e} \ln(x-1) dx$$
 (II) $\int_{-5}^{0} \frac{1}{5+2x} dx$ (III) $\int_{1}^{\infty} \frac{2}{x^{3}} dx$

- (a) Only (III) is an improper integral.
- (b) Only (I) and (III) are improper integrals.
- (c) Only (II) and (III) are improper integrals.
- (d) Only (I) is an improper integral.
- (e) (I), (II) and (III) are all improper integrals.



11. Let s = ∑_{n=1}[∞] 1/n⁴. Using The Remainder Estimate for the Integral Test, find the smallest value of n such that R_n = s - s_n ≤ 1/81.
(a) n = 2

- (b) n = 3
- (c) n = 4
- (d) n = 5
- (e) n = 6



12. Evaluate the integral
$$\int_{1/2}^{\sqrt{3}/2} \frac{1}{\sqrt{1-x^2}} \, dx.$$

(a) $\pi/3 - \pi/6$
(b) $\frac{\sqrt{3}-1}{2}$
(c) $\pi/6 - \pi/3$
(d) $\frac{\sqrt{3}+1}{2}$
(e) $\pi/3 - \pi/4$



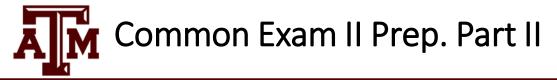
13. Which of the following sequences will converge?

(I)
$$a_n = \frac{\sin n}{n}$$
 (II) $a_n = \ln(5n+1) - \ln(3n+2)$ (III) $a_n = \frac{(-1)^n 5n}{n+5}$

- (a) (I) and (III) only.
- (b) (II) only.
- (c) (I) and (II) only.
- (d) (I), (II) and (III).
- (e) (II) and (III) only.

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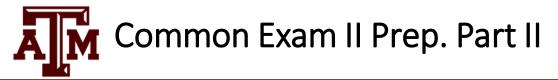
- 14. Which of the following is true for the sequence $a_n = 3 + \frac{(-1)^n n}{3n^2 + 1}$ for $n \ge 1$?
 - (a) The sequence is decreasing and the sequence converges to 3.
 - (b) The sequence is increasing and the sequence diverges.
 - (c) The sequence is neither increasing nor decreasing and the sequence diverges.
 - (d) The sequence is neither increasing nor decreasing and the sequence converges to 3.
 - (e) The sequence is increasing and the sequence converges to 3.



15. What is a general formula for the sequence
$$a_n = \left\{\frac{1}{5}, \frac{-4}{8}, \frac{9}{11}, \frac{-16}{14}, \frac{25}{17}, \dots\right\}$$
, for $n \ge 1$?

(a)
$$a_n = \frac{(-1)^{n-1}n^2}{2n+3}$$

(b) $a_n = \frac{(-1)^{n-1}2n}{n^2+4}$
(c) $a_n = \frac{(-1)^{n+1}n^2}{4n+1}$
(d) $a_n = \frac{(-1)^n n^2}{n+3}$
(e) $a_n = \frac{(-1)^{n+1}n^2}{3n+2}$



16. (10 points) Does the telescoping series $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} (e^{1/n} - e^{1/(n+1)})$ converge or diverge?

If the series converges, find the sum of the series.

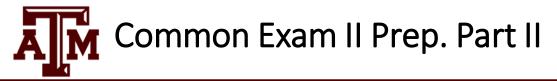


17. (10 points) Compute $\int \frac{\sqrt{9+x^2}}{x^4} dx.$

Your final answer MUST be presented in x, without any inverse trigonometric function(s).



18. (10 points) Evaluate the integral $\int \frac{4x^2 - 5x + 11}{(x-1)(x^2+4)} dx$.



19. (10 points) Use the Integral Test to show that the series $\sum_{n=1}^{\infty} \frac{3e^{1/n}}{n^2}$ converges or prove that it diverges.

Your answer MUST be presented as a complete coherent sentence with justification.