MATH 150 - WEEK-IN-REVIEW 10

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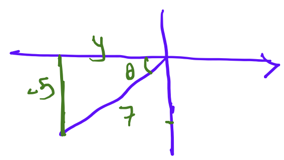
PROBLEM STATEMENTS

1. Given $\sin(\theta) = -\frac{5}{7}$ and $\tan(\theta) > 0$, find $\tan(\theta)$ and $\sec(\theta)$.

$\sin\theta = \frac{\text{opp.}}{\text{hyp.}} = -\frac{5}{7} < 0$

$\tan\theta = \frac{\sin\theta}{\cos\theta} > 0$ $\quad \frac{+}{+}$ & $\frac{-}{-}$ $\quad$ Quadrant I or III



$\tan\theta = \frac{\sin\theta}{\cos\theta}$ or $\frac{\text{opp}}{\text{adj}}$ $\Rightarrow \tan\theta = \frac{-5}{-\sqrt{24}} = \frac{5}{\sqrt{24}}$

$\sec\theta = \frac{1}{\cos\theta} = \frac{\text{hyp.}}{\text{adj}} \Rightarrow \sec\theta = \frac{7}{-\sqrt{24}} = -\frac{7}{\sqrt{24}}$

$(-5)^2 + y^2 = 49$

$25 + y^2 = 49$

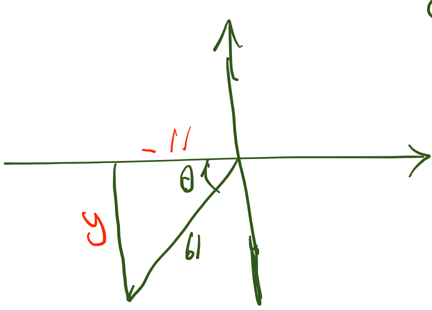
$y^2 = 24$ $\quad y = \pm\sqrt{24} \rightarrow -\sqrt{24}$

{ if interested $\sqrt{24} = 2\sqrt{6}$ }

2. Use the function value to find the indicated trigonometric value in the specified quadrant.

Function Value: $\sec\theta = -\frac{61}{11}$ **Quadrant:** III **Trigonometric Function:** $\cot\theta$

$\sec\theta = \frac{1}{\cos\theta} = \frac{\text{hyp.}}{\text{adj}} = -\frac{61}{11}$



$\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{\text{adj.}}{\text{opp.}} = \frac{-11}{y}$

$\cot\theta = \frac{-11}{-60} = \frac{11}{60}$

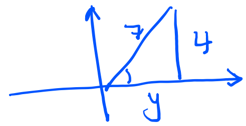
$y^2 + (-11)^2 = (61)^2$

$y^2 + 121 = 3721$

$y^2 = 3600 \rightarrow y = \pm 60 \rightarrow y = -60$

3. Given $\sin\theta = \dfrac{4}{7}$ and θ in Quadrant I, use the trigonometric identities to find the exact value of each:

a. $\cos(\theta) =$ $\dfrac{\text{adj.}}{\text{hyp}} = \dfrac{\sqrt{33}}{7}$



$y^2 + 16 = 49$

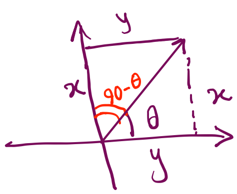
$y^2 = 33$ $\quad y = +\sqrt{33}$

or we could use $\sin^2\theta + \cos^2\theta = 1$

b. $\cot(\theta) =$ $\dfrac{\cos\theta}{\sin\theta} = \dfrac{\sqrt{33}}{4}$

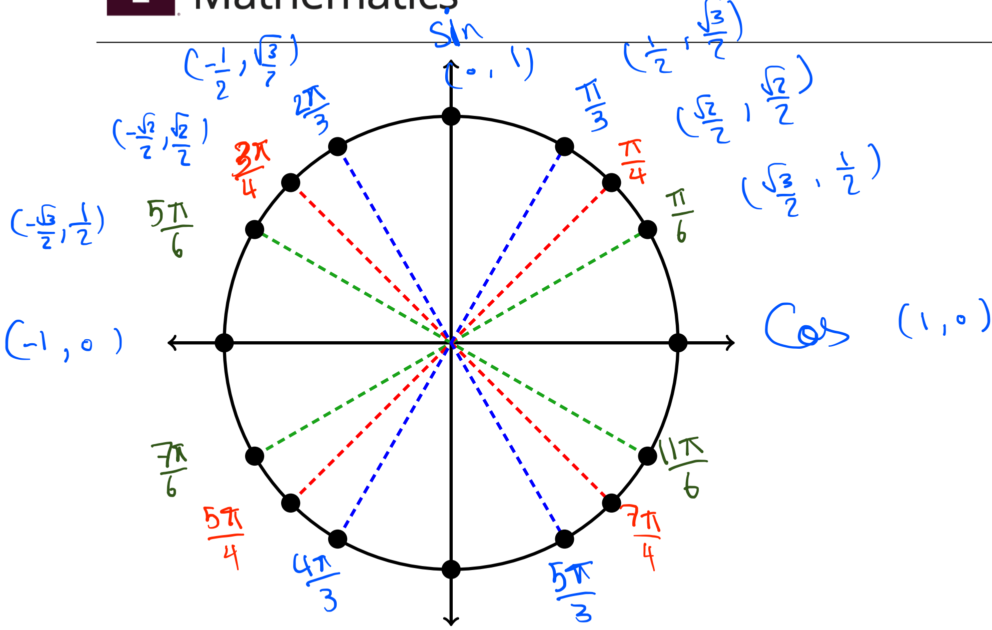
c. $\csc(\theta) =$ $\dfrac{1}{\sin\theta} = \dfrac{7}{4}$

d. $\tan(90° - \theta) =$ $\dfrac{\sin(90° - \theta)}{\cos(90° - \theta)} = \dfrac{\cos(\theta)}{\sin(\theta)} = \cot\theta = \dfrac{\sqrt{33}}{4}$



$\sin\theta = \dfrac{x}{\text{hyp.}} \rightarrow \sin(90° - \theta) = \dfrac{y}{\text{hyp.}} = \cos\theta$

$\cos\theta = \dfrac{y}{\text{hyp}} \rightarrow \cos(90° - \theta) = \dfrac{x}{\text{hyp.}} = \sin\theta$



4. Evaluate the following:

a) $\sin\frac{4\pi}{3} = -\sin\left(\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$

b) $\cos\frac{4\pi}{3} = -\frac{1}{2}$

c) $\tan\frac{4\pi}{3} = \dfrac{\sin\frac{4\pi}{3}}{\cos\frac{4\pi}{3}} = \dfrac{-\sqrt{3}/2}{-\frac{1}{2}} = \sqrt{3}$

d) $\cot\frac{4\pi}{3} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

e) $\sec\frac{4\pi}{3} = \dfrac{1}{\cos\left(\frac{4\pi}{3}\right)} = -2$

f) $\csc\frac{4\pi}{3} = \dfrac{1}{\sin\left(\frac{4\pi}{3}\right)} = \dfrac{1}{-\frac{\sqrt{3}}{2}} = -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$

a) $\sin 315° = \sin(360 - 45) = -\frac{\sqrt{2}}{2}$

b) $\cos 315° = \frac{\sqrt{2}}{2}$

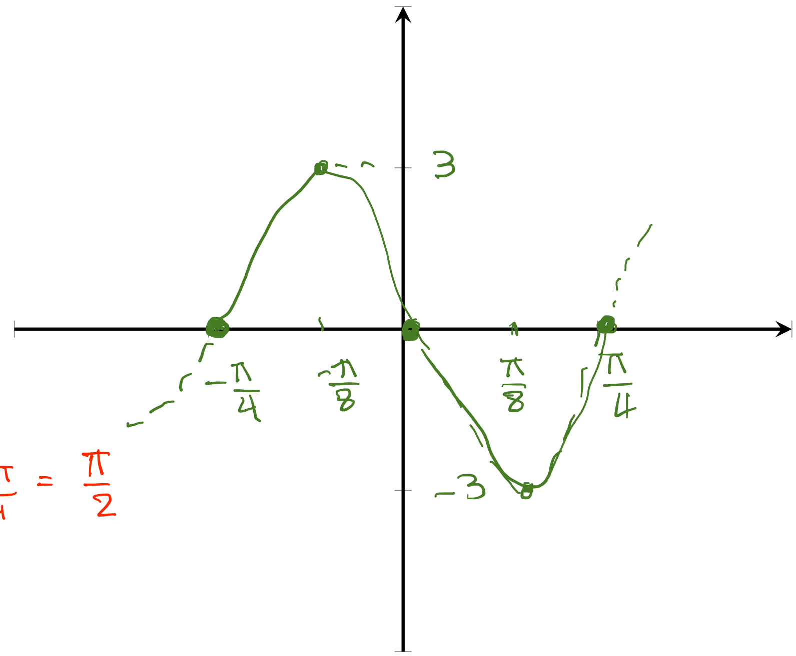
c) $\tan 315° = -1$

d) $\cot 315° = -1$

e) $\sec 315° = \sqrt{2}$

f) $\csc 315° = -\sqrt{2}$

5. Given $y = 3\sin(4x + \pi)$, describe the period, amplitude, and phase shift of the graph. Then graph the function.



Amplitude: 3

Period Endpoints

Start:

$4x + \pi = 0 \rightarrow x = -\frac{\pi}{4}$

$[-\frac{\pi}{4}, \frac{\pi}{4}]$

End:

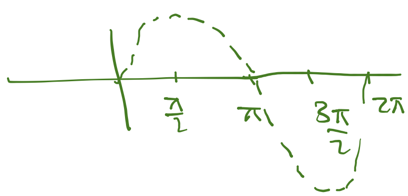
$4x + \pi = 2\pi \rightarrow x = \frac{\pi}{4}$

Period: $\frac{\pi}{4} - (-\frac{\pi}{4}) = \frac{2\pi}{4} = \frac{\pi}{2}$

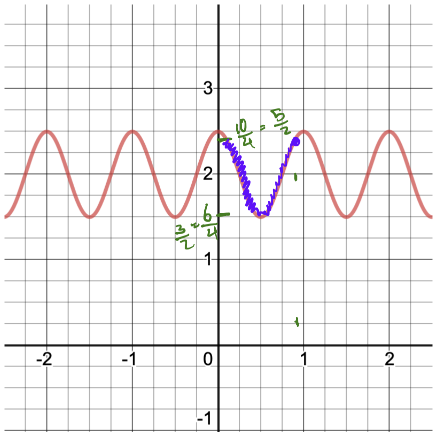
Phase Shift:

$\frac{\pi}{4}$ left

Sinx

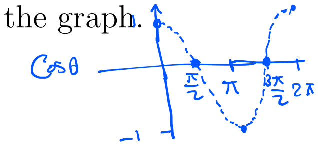


6. Given the graph, write the equation of the cosine function which matches the graph.



$y = A\cos(Bx + C) + d$

Amplitude



$[0, 2\pi]$

$A = \dfrac{\frac{5}{2} - \frac{3}{2}}{2} = \frac{1}{2}$ $\left(\text{or } 2 - \frac{3}{2} = \frac{1}{2} \text{ or } \frac{5}{2} - 2\right)$

Phase $[0, 1]$ Period: 1

$C = 0$ since no phase shift

$Bx + C = 0 \quad x = \frac{-C}{B} = 0 \rightarrow C = 0$

$Bx + C = 2\pi \rightarrow x = \frac{2\pi - C}{B} = 1 \rightarrow B = 2\pi$

$A = \frac{1}{2}\cos(2\pi x) + d$

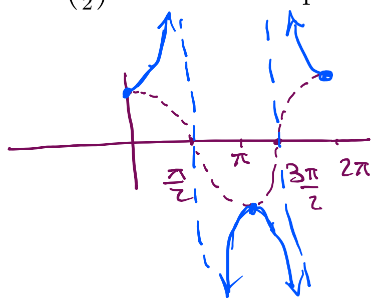
$d = 2$ vertical shift up

$$\boxed{A = \frac{1}{2}\cos(2\pi x) + 2}$$

7. Graph one cycle of the function $y = 1 + \sec\left(\frac{t}{2}\right)$ and state its period.

$\sec t = \frac{1}{\cos t}$

$[0, 2\pi]$



starts: $\frac{t}{2} = 0 \longrightarrow t = 0$

Ends: $\frac{t}{2} = 2\pi \longrightarrow t = 4\pi$

$[0, 4\pi] \Rightarrow$ Period: 4π

(or $\frac{2\pi}{B} = \frac{2\pi}{\frac{1}{2}} = 4\pi$)

Asymptotes: $\cos\left(\frac{t}{2}\right) = 0$ when $\frac{t}{2} = \frac{\pi}{2}$ & $\frac{t}{2} = \frac{3\pi}{2}$

$\hookrightarrow t = \pi$ $\hookrightarrow t = 3\pi$

as $t \to \pi^- \Rightarrow \sec(t/2) \to +\infty$

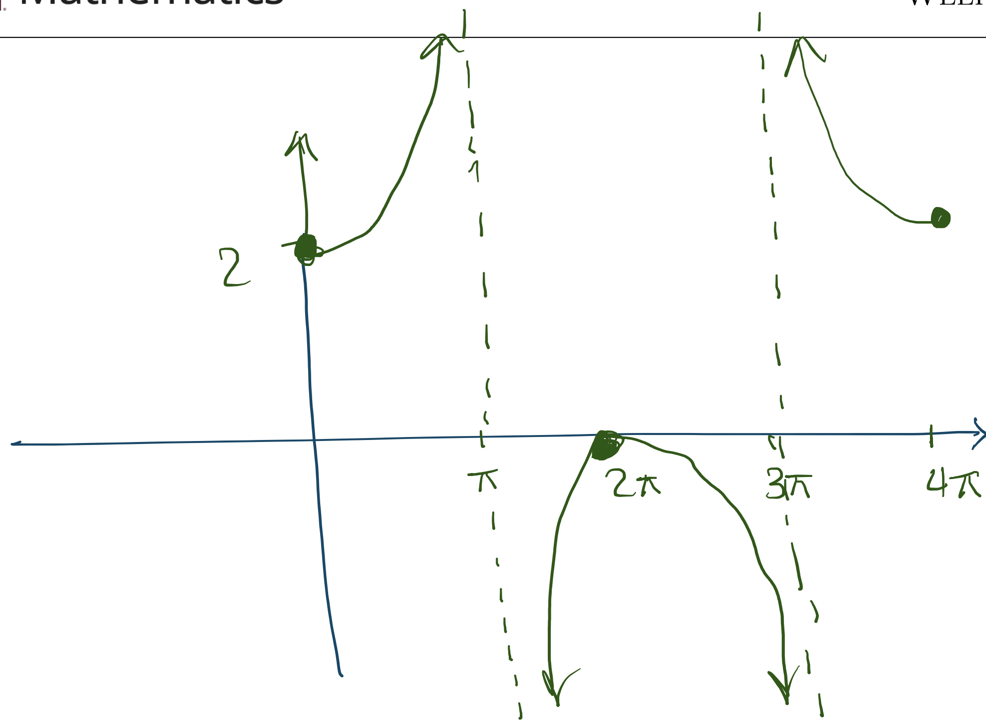
$t \to \pi^+ \Rightarrow \sec(t/2) \to +\infty$

$t \to 3\pi^+ \Rightarrow \sec(t/2) \to +\infty$

$t \to 3\pi^- \Rightarrow \sec(t/2) \to -\infty$

Key points:

t	$y = 1 + \sec(t/2)$
0	$1+1 = 2$
2π	$1-1 = 0$
4π	$1+1 = 2$



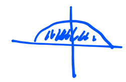
8. Evaluate each:

$\frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}}$ or $\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}$

a. $\arcsin\left(\frac{\sqrt{2}}{2}\right) =$ $\frac{\pi}{4}$ b. $\arcsin(3) =$ DNE c. $\arcsin\left(-\frac{\sqrt{3}}{2}\right) =$ $-\frac{\pi}{3}$ (not $\frac{5\pi}{3}$) d. $\arcsin(-1) =$ $-\frac{\pi}{2}$	a. $\arccos\left(-\frac{\sqrt{2}}{2}\right) =$ $\frac{3\pi}{4}$ b. $\arccos\left(\frac{1}{2}\right) =$ $\frac{\pi}{3}$ c. $\arccos(1) =$ 0 d. $\arccos(-2) =$ DNE
a. $\arctan(-\sqrt{3}) =$ $-\frac{\pi}{3}$ b. $\arctan(3) =$ Can't calculate by hand c. $\arctan(-1) =$ $-\frac{\pi}{4}$ d. $\arctan\left(\frac{5}{2}\right) =$ Can't do by hand $\tan\theta = \frac{5}{2}$	**Reminders:** • Domain $\arcsin x = [-1, 1]$, Range $\arcsin x = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ • Domain $\arccos x = [-1, 1]$, Range $\arccos x = [0, \pi]$ • Domain $\arctan x = (-\infty, \infty)$, Range $\arctan x = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

9. Evaluate each of the following:

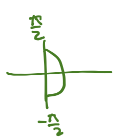
(a) $\arccos\left(\underbrace{\sin\left(\frac{11\pi}{6}\right)}_{-\frac{1}{2}}\right) = \arccos\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$



(b) $\arccos\left(\sin\left(\frac{3\pi}{4}\right)\right) = \arccos\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$

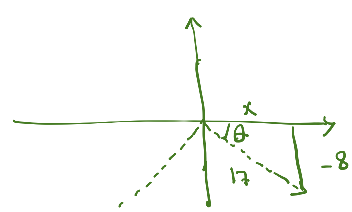
(c) $\sin\left(\underbrace{\arctan\left(-\frac{\sqrt{3}}{3}\right)}\right) = \sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-\sqrt{3}}{3} = \frac{-1}{\sqrt{3}} = \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}}$ or $\frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}}$



(d) $\cos\left(\underbrace{\arcsin\left(-\frac{8}{17}\right)}\right) = \cos(\theta) = \frac{15}{17}$

$\sin\theta = \frac{-8}{17} = \frac{opp}{hyp.}$



$x^2 + (-8)^2 = (17)^2$

$x^2 + 64 = 289$

$x^2 = 225$

$x = +15$

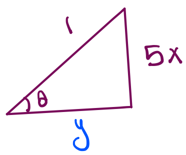
$\cos\theta = \frac{adj}{hyp.} = \frac{15}{17}$

(e) $\cos(\underbrace{\arccos(2)}_{>1})$ DNE

10. Write an algebraic expression that is equivalent to:

a. $\tan(\underbrace{\arcsin(5x)}_{\Updownarrow}) = \tan\theta = \frac{\text{opp}}{\text{adj.}} = \frac{5x}{\sqrt{1-25x^2}}$

$\sin\theta = \frac{5x}{1} = \frac{\text{opp.}}{\text{hyp}}$



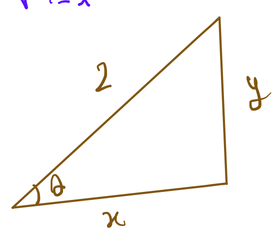
$y^2 + 25x^2 = 1$

$y^2 = 1 - 25x^2$

$y = \sqrt{1-25x^2}$

b. $\csc\left(\arccos\left(\frac{x}{2}\right)\right)$. $= \csc(\theta) = \frac{1}{\sin\theta} = \frac{\text{hyp.}}{\text{opp.}} = \frac{2}{\sqrt{4-x^2}}$

$\arccos\left(\frac{x}{2}\right) = \theta \iff \cos\theta = \frac{x}{2} = \frac{\text{adj.}}{\text{hyp.}}$



$y^2 + x^2 = 4$

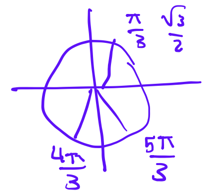
$y = \sqrt{4-x^2}$ (all y's are positive since range of arcco

11. Find all solutions to $\sqrt{3}\csc(x) + 3 = 1$

$\sqrt{3}\csc(x) = -2 \longrightarrow \csc(x) = \frac{-2}{\sqrt{3}}$

$\frac{1}{\sin(x)} = -\frac{2}{\sqrt{3}}$

$\sin(x) = -\frac{\sqrt{3}}{2}$



$\Rightarrow x = \frac{4\pi}{3} + 2k\pi$

&

$x = \frac{5\pi}{3} + 2k\pi$

Solutions between $[0, 2\pi)$: $\frac{4\pi}{3}$, $\cancel{\frac{4\pi}{3} + 2\pi = \frac{10\pi}{3}}$, $\frac{5\pi}{3}$, $\cancel{\frac{5\pi}{3} + 2\pi}$

12. Find all solutions to the equation $2\sin^2(3x) - 1 = 0$.

$2\sin^2(3x) = 1 \rightarrow \sin^2(3x) = \frac{1}{2} \rightarrow \sin(3x) = \frac{\sqrt{2}}{2}$ & $\sin(3x) = -\frac{\sqrt{2}}{2}$

let $u = 3x$ $\quad \sin(u) = \frac{\sqrt{2}}{2}$ & $\sin(u) = -\frac{\sqrt{2}}{2}$

$u = \frac{\pi}{4} + 2k\pi \qquad u = \frac{5\pi}{4} + 2k\pi$

$u = \frac{3\pi}{4} + 2k\pi \qquad u = \frac{7\pi}{4} + 2k\pi$

$[0, \pi) = [0, \frac{12\pi}{12})$

$x = \frac{\pi}{12} + \frac{2k\pi}{3} \qquad x = \frac{5\pi}{12} + \frac{2k\pi}{3}$

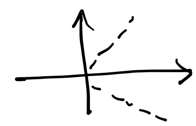
$x = \frac{3\pi}{12} + \frac{2k\pi}{3} \qquad x = \frac{7\pi}{12} + \frac{2k\pi}{3}$

$\frac{\pi}{12}, \frac{3\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{\pi}{12} + \frac{2\pi}{3} = \frac{9\pi}{12}, \frac{3\pi}{12} + \frac{2\pi}{3} = \frac{11\pi}{12}$

13. Find all solutions to $80\cos\left(\frac{\pi}{3}x+\frac{\pi}{4}\right)-40\sqrt{2}=0$. then state the solution $[0,5)$

$$\cos\left(\frac{\pi}{3}x+\frac{\pi}{4}\right)=\frac{40\sqrt{2}}{80}$$

$$\cos\left(\frac{\pi}{3}x+\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}$$



$$\Rightarrow \frac{\pi}{3}x+\frac{\pi}{4}=\frac{\pi}{4}+2k\pi \quad , \quad \frac{\pi}{3}x+\frac{\pi}{4}=\frac{7\pi}{4}+2k\pi$$

$$\frac{\pi}{3}x=2k\pi \qquad \frac{\pi}{3}x=\frac{6\pi}{4}+2k\pi$$

$$\boxed{x=6k} \qquad x=\frac{3}{\pi}\left(\frac{3\pi}{2}+k\pi\right)$$

$$\boxed{x=\frac{9}{2}+6k}$$

Solutions between $[0, 5)$ $\quad 5=\frac{10}{2}$

$$\underbrace{0,\ \frac{9}{2}}\ ,\ \cancel{\frac{9}{2}+6=\frac{21}{2}}\ ,\ \cancel{6}$$