Math 151 - Week-In-Review 1

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Topics for the week:

1.5 Inverse Trigonometric Functions
J.1 Vectors

1.5 Inverse Trigonometric Functions

1. State the domain and range of $f(x) = \arcsin(x)$, $g(x) = \arccos(x)$, and $h(x) = \arctan(x)$.

- $f(x) = \arcsin(x) = \sin^{-1}(x)$
  Domain: $[-1, 1]$
  Range: $[-\frac{\pi}{2}, \frac{\pi}{2}]$

- $g(x) = \arccos(x) = \cos^{-1}(x)$
  Domain: $[-1, 1]$
  Range: $[0, \pi]$

- $h(x) = \arctan(x) = \tan^{-1}(x)$
  Domain: $(-\infty, \infty)$
  Range: $(-\frac{\pi}{2}, \frac{\pi}{2})$

2. Compute the exact value of each expression.

If $y = \text{arc}\_\_(x)$ then for all x in the correct domain and y in the correct range. $\_\_(y) = x$

(a) $\arcsin(-1) = -\frac{\pi}{2}$

angle in $[-\pi/2, \pi/2]$

(b) $\arccos\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$

angle in $[0, \pi]$

(c) $\arctan\left(\sqrt{3}\right) = \frac{\pi}{3}$

angle in $(-\frac{\pi}{2}, \pi/2)$

(d) $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$

angle in $[-\frac{\pi}{2}, \frac{\pi}{2}]$

(e) $\cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$

angle in $[0, \pi]$

(f) $\tan^{-1}(0) = 0$

angle in $(-\frac{\pi}{2}, \frac{\pi}{2})$

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3. Simplify each expression.

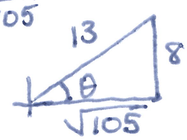
(a) $\arctan(\underbrace{\cos(\pi)}_{=-1}) = \arctan(-1)$

$= -\frac{\pi}{4}$

(b) $\sec\left(\underbrace{\sin^{-1}\left(\frac{8}{13}\right)}_{=\theta}\right) = \sec(\theta)$

$= \frac{13}{\sqrt{105}}$ from the picture

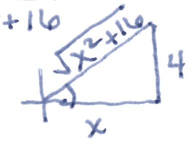
side $= \sqrt{169-64}$
$= \sqrt{105}$



(c) $\csc\left(\underbrace{\arctan\left(\frac{x}{4}\right)}_{=\theta}\right) = \csc(\theta)$

$= \frac{\sqrt{x^2+16}}{4}$ from the picture

hyp $= \sqrt{x^2+16}$

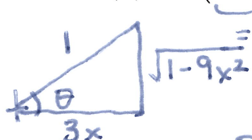


(d) $\sin\left(\underbrace{\cos^{-1}(3x)}_{=\theta}\right) = \sin(\theta)$

$= \sqrt{1-9x^2}$ from the picture

note:
$-\frac{1}{3} \le x \le \frac{1}{3}$

side $=$
$\sqrt{1-9x^2}$



Could be in QII

J.1 Vectors

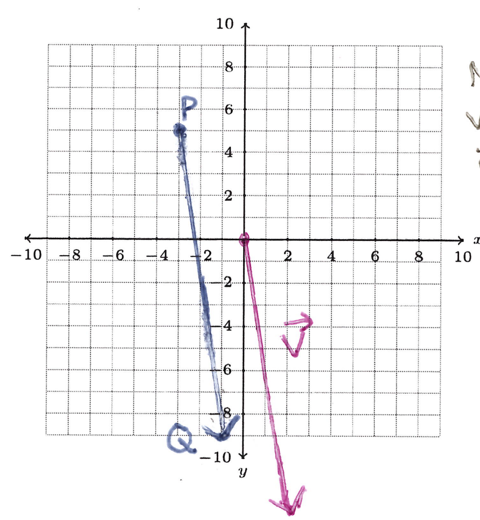
4. Compute a vector, $\vec{v}$, which is given by the directed line segment $\overrightarrow{PQ}$ with points $P = (-3, 5)$ and $Q = (-1, -9)$. Then sketch both the directed line segment $\overrightarrow{PQ}$ and vector, $\vec{v}$.

$\overrightarrow{PQ}$ says P is the initial point
Q is the terminal point

$\vec{v} = \langle x_2 - x_1, y_2 - y_1 \rangle$

$= \langle (-1) + (+3), (-9) - 5 \rangle$

$\vec{v} = \langle 2, -14 \rangle$



Note that vector $\vec{v}$ is parallel to $\overrightarrow{PQ}$

and the same length as $\overrightarrow{PQ}$

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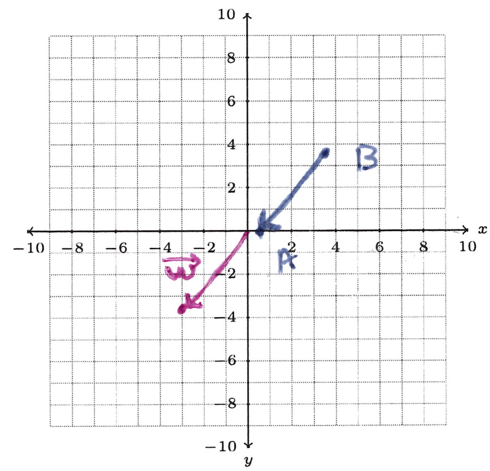
5. Compute a vector, $\vec{w}$, which is given by the directed line segment $\overrightarrow{BA}$ with points $A = \left(\frac{1}{2}, 0\right)$ and $B = \left(\frac{7}{2}, \frac{7}{2}\right)$. Then sketch both the directed line segment $\overrightarrow{BA}$ and vector, $\vec{w}$.

$\overrightarrow{BA}$ says B is the initial point
A is the terminal point

$\vec{w} = \langle x_2 - x_1, y_2 - y_1 \rangle$
$= \langle \frac{1}{2} - \frac{7}{2}, 0 - \frac{7}{2} \rangle$
$\vec{w} = \langle -3, -\frac{7}{2} \rangle$



6. Given vectors $\vec{u} = \langle 3, -4 \rangle$, $\vec{v} = \langle 6, 11 \rangle$, and $\vec{w} = \left\langle -\frac{2}{5}, -\frac{3}{2} \right\rangle$, compute each of the following.

(a) The magnitude of $\vec{u}$.

$|\vec{u}| = \sqrt{(x_1)^2 + (x_2)^2}$

$|\vec{u}| = \sqrt{(3)^2 + (-4)^2}$
$= \sqrt{9 + 16}$
$= \sqrt{25}$

$|\vec{u}| = 5$

(b) The length of $\vec{w}$.

"magnitude"

$|\vec{w}| = \sqrt{\left(-\frac{2}{5}\right)^2 + \left(\frac{-3}{2}\right)^2}$
$= \sqrt{\frac{4}{25} + \frac{9}{4}}$

$|\vec{w}| = \sqrt{\frac{16 + 225}{100}}$
$|\vec{w}| = \frac{\sqrt{241}}{10}$

(c) $\vec{v} - \vec{u}$ $= \langle 6, 11 \rangle - \langle 3, -4 \rangle$
$= \langle 6 - (3), 11 - (-4) \rangle$
$\vec{v} - \vec{u} = \langle 3, 15 \rangle$

(d) $8\vec{w}$ $= 8 \cdot \langle -\frac{2}{5}, -3/2 \rangle$
$= \langle 8(-2/5), 8(-3/2) \rangle$
$8\vec{w} = \langle -16/5, -12 \rangle$

(e) $6\vec{u} + 9\vec{v}$ $= 6\langle 3, -4 \rangle + 9\langle 6, 11 \rangle$
$= \langle 18, -24 \rangle + \langle 54, 99 \rangle$
$6\vec{u} + 9\vec{v} = \langle 72, 75 \rangle$

(f) $-\frac{1}{2}\vec{u} + 4\vec{w} - \frac{2}{5}\vec{v}$ $= -\frac{1}{2}\langle 3, -4 \rangle + 4\langle -\frac{2}{5}, -\frac{3}{2} \rangle - \frac{2}{5}\langle 6, 11 \rangle$
$= \langle -\frac{3}{2}, 2 \rangle + \langle -\frac{8}{5}, -6 \rangle + \langle -\frac{12}{5}, -\frac{22}{5} \rangle$
$-\frac{1}{2}\vec{u} + 4\vec{w} - \frac{2}{5}\vec{v} = \langle -\frac{55}{10}, -\frac{42}{5} \rangle$ or $\langle -\frac{11}{2}, -\frac{42}{5} \rangle$

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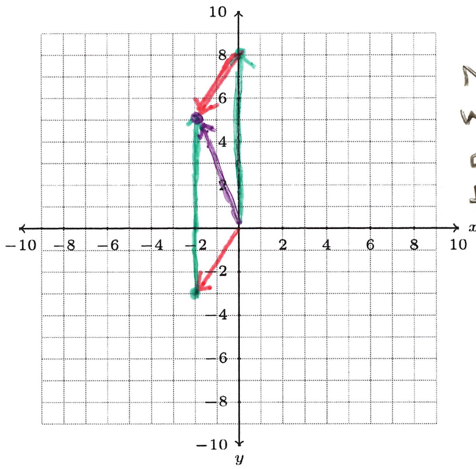
7. Using a graph, show $\langle -2, -3\rangle + \langle 0, 8\rangle$ has the same resultant vector as $\langle 0, 8\rangle + \langle -2, -3\rangle$.

$\langle -2,-3\rangle + \langle 0,8\rangle = \langle -2,5\rangle$

$\langle 0,8\rangle + \langle -2,-3\rangle = \langle -2,5\rangle$



Notice when we add, the terminal point of the first is the initial point of the second.

8. Given vectors $\vec{u} = \vec{i} + 3\vec{j}$, $\vec{v} = -2\vec{i}$, and $\vec{w} = \langle -7, 8\rangle$, compute each of the following and write your final answer using the standard basic vectors, when appropriate.

(a) Write $\vec{w}$ using standard basic vectors **ij**. $\vec{i} = \langle 1,0\rangle$ and $\vec{j} = \langle 0,1\rangle$

$\vec{w} = -7\vec{i} + 8\vec{j}$

(b) $|\vec{u}|$

$\vec{u} = \vec{i} + 3\vec{j} = \langle 1,3\rangle$

$|\vec{u}| = \sqrt{(x_1)^2 + (x_2)^2}$

$|\vec{u}| = \sqrt{(1)^2 + (3)^2}$

$= \sqrt{1+9}$

$|\vec{u}| = \sqrt{10}$

(c) $\vec{u} - 2\vec{w} = \vec{i} + 3\vec{j} - (-7\vec{i} + 8\vec{j})$

$= (1 - (-7))\vec{i} + (3 - 8)\vec{j}$

$= 8\vec{i} - 5\vec{j}$

(d) $|\vec{u} + 3\vec{v}|$ First compute $\vec{u} + 3\vec{v}$.

$\vec{u} + 3\vec{v} = \vec{i} + 3\vec{j} + 3(-2\vec{i})$

$= (1 + (-6))\vec{i} + (3 + 0)\vec{j}$

$= -5\vec{i} + 3\vec{j}$

$|\vec{u} + 3\vec{v}| = \sqrt{(-5)^2 + (3)^2}$

$= \sqrt{25 + 9}$

$= \sqrt{34}$

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9. Given $\vec{a} = -1\vec{i} + 5\vec{j}$,

(a) Compute a unit vector that has the same direction as $\vec{a}$.

$|\vec{u}| = \frac{1}{|\vec{v}|} \cdot \vec{v}$

First we need $|\vec{a}| = \sqrt{(-1)^2 + (5)^2}$
$= \sqrt{1+25}$
$= \sqrt{26}$

$\frac{\vec{a}}{|\vec{a}|} = \frac{-1}{\sqrt{26}}\vec{i} + \frac{5}{\sqrt{26}}\vec{j}$

(b) Compute a vector that has the same direction as $\vec{a}$ and has a length of $\frac{1}{4}$.

We multiply the unit vector in part a by $\frac{1}{4}$

$\frac{1}{4}\left(\frac{\vec{a}}{|\vec{a}|}\right) = \frac{1}{4}\left(\frac{-1}{\sqrt{26}}\vec{i} + \frac{5}{\sqrt{26}}\vec{j}\right)$
$= \frac{1}{4}\left(\frac{-1}{\sqrt{26}}\right)\vec{i} + \frac{1}{4}\left(\frac{5}{\sqrt{26}}\right)\vec{j}$
$= \frac{-1}{4\sqrt{26}}\vec{i} + \frac{5}{4\sqrt{26}}\vec{j}$

(c) Compute a vector that is parallel to $\vec{a}$ with a length 5.

Vectors $\vec{a}$ and $\vec{b}$ are parallel is $\vec{b} = c \cdot \vec{a}$ where c is a constant

$\vec{b} = 5 \cdot \left(\frac{\vec{a}}{|\vec{a}|}\right) = 5\left(\frac{-1}{\sqrt{26}}\vec{i} + \frac{5}{\sqrt{26}}\vec{j}\right)$
$= \frac{-5}{\sqrt{26}}\vec{i} + \frac{25}{\sqrt{26}}\vec{j}$

10. Write the component form of vector, $\vec{v}$, whose initial point is the origin of the two dimensional Cartesian Plane and makes an angle of $\frac{7\pi}{6}$ with the positive x-axis. The magnitude of $\vec{v}$ is 5.

$\vec{v} = |\vec{v}| \cdot \langle \cos(\theta), \sin(\theta) \rangle$

$\theta = \frac{7\pi}{6}$
$|\vec{v}| = 5$

$\vec{v} = 5\langle \cos(\frac{7\pi}{6}), \sin(\frac{7\pi}{6}) \rangle = \langle 5\cos(\frac{7\pi}{6}), 5\sin(\frac{7\pi}{6}) \rangle$

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11. Two chains have been attached to a chunk of concrete buried in the ground and then each attached to a different backhoe. If the backhoes drive in opposite directions from the concrete with one of the chains creating a 30° angle with the ground and having a magnitude of 120 lbs, while the other creates a 45° angle with the ground and has a magnitude of 100 lbs. What is the resultant force **F** acting on the chunk of concrete? Then compute the magnitude and direction of the force.

$$\vec{F}_1 = 120\cos(30°)\vec{i} + 120\sin(30°)\vec{j}$$
$$= 120 \cdot \frac{\sqrt{3}}{2}\vec{i} + 120 \cdot \frac{1}{2}\vec{j}$$
$$= 60\sqrt{3}\,\vec{i} + 60\vec{j}$$

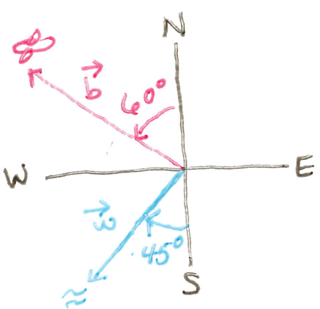
$$\vec{F}_2 = 100 \cdot \cos(135°)\vec{i} + 100\sin(135°)\vec{j}$$
$$= 100\left(-\frac{\sqrt{2}}{2}\right)\vec{i} + 100\left(\frac{\sqrt{2}}{2}\right)\vec{j}$$
$$= -50\sqrt{2}\,\vec{i} + 50\sqrt{2}\,\vec{j}$$

$$\vec{F} = \vec{F}_1 + \vec{F}_2 = (60\sqrt{3}\,\vec{i} + 60\vec{j}) + (-50\sqrt{2}\,\vec{i} + 50\sqrt{2}\,\vec{j})$$
$$= (60\sqrt{3} - 50\sqrt{2})\vec{i} + (60 + 50\sqrt{2})\vec{j}$$

$$|\vec{F}| = \sqrt{(60\sqrt{3} - 50\sqrt{2})^2 + (60 + 50\sqrt{2})^2}$$
$$\approx 134.864$$

$$\theta = \arctan\left(\frac{60 + 50\sqrt{2}}{60\sqrt{3} - 50\sqrt{2}}\right)$$
$$\approx 75.743°$$

12. The wind is blowing at a speed of 18 mph in the direction S45°W. A red tailed hawk is flying N60°W at an airspeed of 100 mph. Determine the true course and ground speed of the hawk.



$$\vec{b} = 100\cos(150°)\vec{i} + 100\sin(150°)\vec{j}$$
$$= 100\left(-\frac{\sqrt{3}}{2}\right)\vec{i} + 100 \cdot \left(\frac{1}{2}\right)\vec{j}$$
$$= -50\sqrt{3}\,\vec{i} + 50\vec{j}$$

$$\vec{w} = 18\cos(-135°)\vec{i} + 18\sin(-135°)\vec{j}$$
$$= 18 \cdot \left(-\frac{\sqrt{2}}{2}\right)\vec{i} + 18 \cdot \left(-\frac{\sqrt{2}}{2}\right)\vec{j}$$
$$= -9\sqrt{2}\,\vec{i} - 9\sqrt{2}\,\vec{j}$$

$$\vec{T} = \vec{b} + \vec{w}$$
$$= (-50\sqrt{3}\,\vec{i} + 50\vec{j}) + (-9\sqrt{2}\,\vec{i} - 9\sqrt{2}\,\vec{j})$$
$$\vec{T} = (-50\sqrt{3} - 9\sqrt{2})\vec{i} + (50 - 9\sqrt{2})\vec{j}$$

$$|\vec{T}| = \sqrt{(-50\sqrt{3} - 9\sqrt{2})^2 + (50 - 9\sqrt{2})^2}$$
$$\approx 106.093$$

$$\theta = \tan^{-1}\left(\frac{50 - 9\sqrt{2}}{-50\sqrt{3} - 9\sqrt{2}}\right)$$
$$\approx -20.568°$$

So we add 180° to get to QII = 159.43°

True Course: N 69.43° W

Ground Speed: 106.093 mph

Trigonometric Identity Reminders

13. State the three Pythagorean Identities Good to know, based on right triangle $a^2+b^2=c^2$

Relating Sine and Cosine Functions

$\cos^2(x) + \sin^2(x) = 1$ for any real number x

Relating Tangent and Secant Functions

$1 + \tan^2(x) = \sec^2(x)$ for $x \neq \frac{\pi}{2} + \pi k$, k is any integer

Relating Cotangent and Cosecant Functions

$\cot^2(x) + 1 = \csc^2(x)$ for $x \neq \pi k$, k is any integer

14. State the Reciprocal Identities based on fractions.

Relating Sine and Cosecant Functions

$\csc(x) = \frac{1}{\sin(x)}$ and $\sin(x) = \frac{1}{\csc(x)}$ for $x \neq \pi k$, k is any integer

Relating Cosine and Secant Functions

$\sec(x) = \frac{1}{\cos(x)}$ and $\cos(x) = \frac{1}{\sec(x)}$ for $x \neq \frac{\pi}{2} + \pi k$, k is any integer

Relating Tangent and Cotangent Functions

$\cot(x) = \frac{1}{\tan(x)}$ and $\tan(x) = \frac{1}{\cot(x)}$ for $x \neq \frac{\pi}{2} k$, k is any integer

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