FINAL EXAM REVIEW

Pr 1. You wish to buy a car for \$25,000. The dealership offers you three different loans. Loan A has a monthly APR of 5%. Loan B has an annual interest rate of 7%, compounded quarterly, and Loan C has an annual interest rate of 6%, compounded continuously. Which loan has the smallest effective interest rate?

Loan A: $r_{eff} = (1+\frac{r}{n})^n - 1$

$(1+\frac{.05}{12})^{12} - 1 \approx 5.116\%$

Loan B: $r_{eff} = \left((1+\frac{.07}{4}\right)^4 - 1 \approx 7.19\%$

Loan C: $r_{eff} = e^r - 1 = e^{.06} - 1 \approx 6.18\%$

Loan A has the lowest effective interest rate.

Pr 2. You would like to have \$750,000 in your retirement account when you retire in 30 years. Your retirement account earns 5.6% annual interest, compounded monthly. How much do you need to deposit at the end of each month to meet your retirement goal, if you make an initial deposit of \$5000? How much of the \$750,000 did you invest over the 30 years?

N = 30 × 12 = 360
I% = .056 = 5.6%
PV = −5000
FV = 750000
PMT = −776.89
P/Y = 12

We need to invest \$834.29 per month.

750000
− 360 × PMT
Total interest

360 × PMT = Amount invested
≈ \$279680.40

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Pr 3. You purchased a home five years ago for \$240,000. The bank required a 10% down payment, and gave you a 30-year loan with a 4.2% interest rate, compounded monthly.

(a) What is the monthly payment?

(b) What is the current balance on the loan?

(c) You have the opportunity to refinance with a 15-year loan with a 3.6% interest rate. What will be the new monthly payment?

(d) If you refinance, how much will you have saved by the time the house is paid off?

a) N = 12 × 30

PV ≠ 240000
240000

PV = +216000
I = 4.2
PMT =
FV = 0
P/Y = C/Y = 12

PMT = $1,056.27

216000
− .1 × 240000

240000
−24000
216000

b) N = 5 × 12 = 60
FV = → $195,990
I = 4.2
PV = 216000
PMT = −1056.27

c) want to refinance
N = 12 × 15
I = 3.6%
PV = 195990
FV = 0
PMT = −$1410.74

d) Total savings? Total spent under original loan = A
Total spent under new plan = B

B−A

A = 1056.27 × 360

$$B = \underbrace{1056.27 \times 60}_{\text{Time spent before refinace}} + 1410.74 \times 180$$

= $62,947.80

Pr 4. Determine the value of w, x, and y given $\begin{bmatrix} 2 & w-3 \\ 2 & 4x \end{bmatrix} - \begin{bmatrix} y & -6 \\ -8 & 12 \end{bmatrix}^T = 2\begin{bmatrix} -1 & 6 \\ 4 & -4 \end{bmatrix}$

$$\begin{bmatrix} 2 & w-3 \\ 2 & 4x \end{bmatrix} - \begin{bmatrix} y & -8 \\ -6 & 12 \end{bmatrix} = \begin{bmatrix} 2\cdot(-1) & 2\cdot 6 \\ 2\cdot 4 & 2\cdot(-4) \end{bmatrix}$$

$$\begin{bmatrix} 2-y & w-3-(-8) \\ 2-(-6) & 4x-12 \end{bmatrix} = \begin{bmatrix} -2 & 12 \\ 8 & -8 \end{bmatrix}$$

$-y = -$
$\boxed{y=4}$

$2-y=-2$

$8=8$ ✓

Pr 5. Compute $\begin{bmatrix} 2 & 3x & 5 \\ 6w & 0 & 2y \end{bmatrix} \begin{bmatrix} -6 & 3m \\ 3n & 4 \\ -p & 0 \end{bmatrix}$.

$$\begin{array}{cccc} 1 & 2 & 3 & \\ & & & \end{array} \quad \begin{array}{cc} 1 & 2 \end{array} \quad \begin{array}{c} 1 \\ 2 \\ 3 \end{array}$$

2 × 3 3 × 2

A·B has 2×2

$$= \begin{bmatrix} 2(-6) + 3x(3n) + 5(-p) & 2(3m) + 3x4 + 5\cdot 0 \\ 6w(-6) + 0(3n) + 2y(-p) & 6w(3m) + 0\,4 + 2y0 \end{bmatrix}$$

0

$$= \begin{bmatrix} -12 + 9xn - 5p & 6m + 12x \\ -36w - 2yp & 18wm \end{bmatrix}$$

Pr 6. An automobile purchased for use by the manager of a firm at a price of $29,490 is to be depreciated using a linear model over ten years. Suppose the value depreciates by 39% after 5 years. When will the car

v(t) = linear

reach its scrap value of $1000?

$V(5) = 29490 - .39 \times 29490$
$= 17988.90$

$V(0) = 29490$

Solve $V(t) = 1000$?

$V(t) =$ linear eq'n in t
$= mt + b$

$29490 = V(0) = m \cdot 0 + b = b$
$b = 29490$

$V(5) = m5 + 29490 = 17988.90$
$5m = 17988.90 - 2949$
$m = -2300.22$.

$V(t) = -2300.22 \times t + 2949$

$1000 = -2300.22t + 29490$

$t = \dfrac{1000 - 29490}{-230022}$

$t \approx 12.39$ years

Pr 7. Dave sells organic bath soap at his stand at the local farmers market. He makes the soap for \$1 per bar, and sells them at \$6 per bar. Suppose that it costs him \$ 30 in fixed costs. Determine the break-even point.

$(x, R(x))$ where $C(x) = R(x)$

$C(x) = 1 \cdot x + 30 = x + 30$
$R(x) = 6x$

Solve $6x = x + 30$
$-x \quad -x$
$\frac{5x}{5} = \frac{30}{5} \to x = 6$
$R(6) = 6 \times 6 = 36$

$(6, \$36)$

Pr 8. Determine the value of k so that the following system of linear equations has infinitely many solutions.

$$-x + ky = 8$$
$$3x - 6y = -24$$

$\to -x = -ky + 8$
$x = ky - 8$

substitute $3(ky - 8) - 6y = -24$
$3ky - \cancel{24} - 6y = -24$
$(3k - 6)y = 0$

if $3k - 6 \neq 0$, then there is a unique solution $(8, 0)$

if $3k - 6 = 0$, $0 = 0$, so we have infinitely many sol'ns

$3k = 6$
$\boxed{k = 2}$

Pr 9. Set up and solve the following problem as a system of linear equations.

Donald has \$15,000 to invest. He decides to invest in three different companies. The Huey company costs \$250 per share and pays dividends of \$3 per share each year. The Dewey company costs \$60 per share and pay dividends of \$1.00 per share each year. The Louie company costs \$80 per share and pays \$2.00 per share per year in dividends. ~~Link~~ Donald wants to have twice as much money in the Dewey company as in the Louie company. ~~Link~~ also wants to earn \$200 in dividends per year. How much should Link invest in each company to meet his goals?

x = # of shares of Huey
y = # of shares of Dewey
z = # of shares of Louie

$$15000 = 250x + 60y + 80z$$

$$200 = 3x + 1y + 2z$$

$$y = 2z \quad \text{or} \quad y - 2z = 0$$

$2D = L$ or $2L = D$

$$\begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \left[\begin{array}{ccc|c} 250 & 60 & 80 & 15000 \\ 3 & 1 & 2 & 200 \\ 0 & 1 & -3 & 0 \end{array}\right] \end{array}$$

} use calc rref to solve...

Pr 10. A local burger truck makes 4 types of burgers. The slim costs \$3, has one patty and one slice of cheese. The big cheesy costs \$7, has two patties, three slices of cheese, and one strip of bacon. The standard costs \$5, has one patty, one slice of cheese, and three pieces of bacon. The bacon-me-crazy costs \$7, has one patty, one slice of cheese, and 6 strips of bacon. Suppose that we have 1200 strips of bacon, 1000 burger patties, and 800 slices of cheese. How many of each type of burger should we make in order to maximize the profit? Set up the linear optimization problem, but do not solve it.

ℓ = # of slims made and sold.
c = # of big cheesies made and sold.
s = # of standards made and sold.
b = # of bacon-me-crazies " ".
P = total profit.

Maximize $P = 3\ell + 7c + 5s + 7b$

subject to:
$\ell + 2c + s + b \le 1000$ (patties)
$\ell + 3c + s + b \le 800$ (cheese)
$c + 3s + 6b \le 1200$ (bacon).

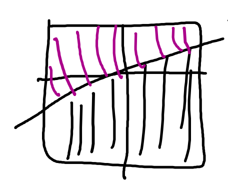
$\ell \ge 0,\ c \ge 0,\ s \ge 0,\ b \ge 0.$

Pr 11. Solve the following linear optimization problem using the method of corners.

Maximize $x - y$ subject to:

- $2x + y \le 8$
- $2x - 3y \le 4$
- $x \ge 0, y \ge 0$

True shading



$2x + y \le 8 \rightarrow 2x + y = 8$

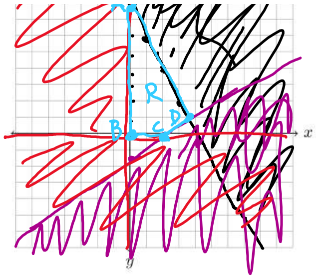
$y = -2x + 8$

Test point: $(0,0)$

$2 \cdot 0 + 0 = 0 \le 8$ ✓

reverse shading

$2x - 3y = 4 \rightarrow$ set $x = 0$ $\quad -3y = 4$



$y = \frac{-4}{3}$

$2\cdot 0 - 3\cdot 0 = 0 \le 4$ Set $y=0$ $2x=4 \to x=2$

4 vertices $(0,0)$,   y-intercept for $2x+y=8$

$(0,8)$, $(2,0)$, $(7/2,1)$

3rd point x-intercept for $2x-3y \le 4$

D :
$2x + y = 8$
$2x - 3y = 4$

Sub Eq 2 from Eq 1,

$y = 8$
$-(3y) = -4$

$4y = 4 \to y = 1$

$2x + 1 = 8$
$2x = 7$
$x = 7/2$

Pt:	$x-y$
$(0,0)$	$0-0 = 0$
$(0,8)$	$0-8 = \underline{-8}$
$(2,0)$	$2-0 = 2$
$(7/2,1)$	$7/2-1 = \underline{5/2}$

Maximum at $(7/2,1)$ with value $5/2$.

Pr 12. For the following simplex tableau, identify the basic and non-basic variables. State the solution corresponding to the tableau, and determine if it is an optimal solution. If it is not, identify the pivot row, pivot column, and pivot entry.

$8/2=4$
$5/\frac{1}{2}=10$

(a)

x	y	s_1	s_2	P	constant
0	$\boxed{2}$	1	0	0	8
1	$\frac{1}{2}$	0	$\frac{1}{2}$	0	5
0	-2	0	$\frac{3}{2}$	1	15

basic variables : x, s_1, P

non basic : y, s_2

Set non basics to 0

$0x + s_1 = 8$ $s_1 = 8$

$x = 5$

$(5,0)$
$x = 5$
$y = 0$
$s_1 = 8$
$s_2 = 0$
$P = 15$

not optimal: Pivot row = row 1
pivot column is 2
Pivot entry is 2.

(b)

x	y	s_1	s_2	P	constant
1	0	1	0	0	8
-1	1	0	1	0	0
-2	-3	0	0	1	0

pivot column

$\to 8/0$ DNE
$\to 0/1 = 0$

basic : s_1, s_2, P

Non-basic: $x, y \to$ solution $(0,0)$

P

(c)

x	y	z	s_1	s_2	P	constant
0	2	1	0	0	0	9
1	$\frac{1}{2}$	0	$\frac{1}{3}$	0	0	2
0	$\frac{1}{2}$	0	2	$\frac{3}{2}$	1	42

$\to$ cannot pivot, so it is the optimum solution.

basic variables : x, z, P

nonbasic's : y, s_1, s_2

$x = 2$ $(2,0,9)$ with

$y=0$
$z=9$
$P=42$

Value $P=42$.

Pr 13. In an experiment, a fair standard 2-sided coin is flipped, noting which side faces up, and then a card is drawn from a well-shuffled deck, noting the suit. Write the sample space for the experiment.

$S = \{ (H,H), (H,D), (H,C), (H,S), (T,H), (T,D), (T,C), (T,S) \}$

Pr 14. A survey of 100 Aggies was taken to gather information on how they commute to campus. A breakdown of those surveyed is shown in the table. Suppose a randomly selected Aggies. What is the probability the person chosen is

	Drive	Bus	Other	**Total**
Freshmen	15	10	14	**39**
Sophomore	11	8	12	**31**
Junior	9	5	4	**18**
Senior	6	4	2	**12**
Total	**41**	**27**	**32**	**100**

(a) P(rides the bus) $= \frac{27}{100} = .27$

(b) P(is a Junior or Senior) $= \frac{18+12}{100} = .30$

(c) P(is a Freshman or does not drive) $= \frac{39+27+32-10-14}{100} = .74$

(d) P(is a Senior and rides the bus) $= \frac{4}{100} = .04$

Pr 15. Given $P(A) = 0.4$, $P(B) = 0.7$, and $P(A \cup B) = 0.9$, compute $P\left[(A \cap B)^C\right]$.

$P(x^c) = 1 - P(x)$

$P((A \cap B)^c) = 1 - P(A \cap B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$.9 = .4 + .7 - P(A \cap B)$

$.9 = 1.1 - x$

$x + .9 = 1.1$

$x = .2$

$P((A \cap B)^c) = 1 - .2 = \boxed{0.8}$

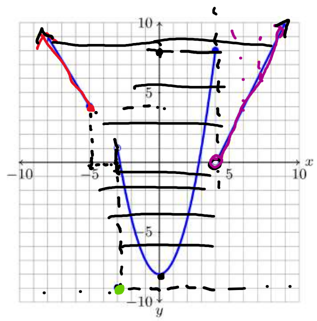
Pr 16. Your insurance company has a policy to insure personal property. Assume your personal property is worth \$2,000, and according to campus statistics there is a 1% chance that your property will be stolen during the next year and a 10% chance that your property is damaged beyond repair through natural causes during the next year. If your property is stolen the policy will give you \$2,000, while if it is damaged beyond repair you receive get \$1,000. What is the insurance company's expected profit on this policy, if the premium for the policy is \$300?

	stolen	damaged	okey-dokey
X	300 − 2000	300 − 1000	300
P(E)	.01	.1	.89

$1 - .1 - .01 =$

expected value $=$
$.01 \times (-1700) + .1(-700) + .89(300) =$

Pr 17. State the domain and range of the function given in the graph below, using interval notation.



→ domain: $(-\infty, -5] \cup [-3, \infty)$

range: $\{-9\} \cup [-8, \infty)$

Pr 18. The price-demand function (in dollars) for a particular item is given by $p(x) = -0.05x + 50$, where x is the number of items. The company who produces these items has a production cost of \$2 per item and fixed costs of \$120. What price should the company charge for the item in order to maximize profit?

$P(x) = R(x) - C(x)$

$R(x) = p(x) \cdot x = (-.05x + 50)\,x = -.05x^2 + 50x$

$C(x) = 2x + 120$

$P(x) = -.05x^2 + 50x - (2x + 120)$

$= -.05x^2 + 48x - 120$

vertex (h, k) max/min profit

maximum/min. quantity

$h = -\frac{b}{2a} = -\frac{48}{2(-.05)}$

$= \frac{48}{2 \times .05}$

$= \frac{48}{.10}$

Answer: $P(h) = p(480) = -.05(480) + 50$

$= \$26$ $\quad = 480$

Max'm profit $= 26 \times 480$

Pr 19. Compute and simplify the difference quotient of $g(x) = \frac{3x}{2x-3}$.

$$\frac{g(x+h)-g(x)}{h} = \frac{\frac{3(x+h)}{2(x+h)-3} - \left(\frac{3x}{2x-3}\right)}{h}$$

$$= \frac{1}{h}\left[\frac{3(x+h)}{2(x+h)-3} - \frac{3x}{2x-3}\right]$$

$$= \frac{1}{h}\left[\frac{3(x+h)}{(2(x+h)-3)}\frac{(2x-3)}{(2x-3)} - \frac{3x(2(x+h)-3)}{(2x-3)(2(x+h)-3)}\right]$$

$$= \frac{1}{h}\left[\frac{(3x+3h)(2x-3) - (3x(2x+2h-3))}{(2(x+h)-3)(2x-3)}\right]$$

$$= \frac{1}{h}\left[\frac{(6x^2-9x+6hx-9h) - 6x^2 - 6xh + 9x}{(2(x+h)-3)(2x-3)}\right] =$$

$$= \frac{1}{\cancel{h}}\left[\frac{-9\cancel{h}}{(2(x+h)-3)(2x-3)}\right] = \boxed{\frac{-9}{(2(x+h)-3)(2x-3)}} = \frac{-9}{(2x+2h-3)(2x-3)} =$$

Pr 20. State the domain of $f(x) = \frac{\ln(11-3x)}{e^x\sqrt{2x+13}}$ using interval notation.

$\frac{1}{e^x}$ need $e^x \neq 0$ always true

$(-\infty, \infty)$

$\frac{1}{\sqrt{2x+13}}$ in denom.

$2x+13 \geq 0$

$2x+13 \neq 0$

$2x+13 > 0$

$2x > -13$

$x > -\frac{13}{2}$

$(-\frac{13}{2}, \infty)$

Domain for $\ln(11-3x)$

$11-3x > 0$

$11 > 3x$

$\frac{11}{3} > x, \quad x < \frac{11}{3}$

$(-\infty, \frac{11}{3})$

$\leftarrow \left(-\frac{13}{2}, \frac{11}{3}\right)$

$-13/2 \quad 0 \quad 11/$

Pr 21. Algebraically solve: $8 \cdot 4^{3x+2} = 16$.

$a^N = a^M \Rightarrow N = M$

shortcut

$4^{3x+2} = 2^1$

base: 2,

$8 = 2^3$

$4 = 2^2$

$16 = 2^4$

$2^3 \cdot (2^2)^{(3x+2)} = 2^4$

$(a^b)^c = a^{b \cdot c}$

$a^b a^c = a^{b+c}$

$2^3 \cdot 2^{2 \cdot (3x+2)} = 2^4$

$2^{3+6x+4} = 2^4 \longrightarrow 3+6x+4 = 4$

$\rightarrow 6x = -3$

$\boxed{x = -1/2}$

Pr 22. You place \$1000 as an initial deposit in a savings account earning annual interest at a rate of 3.5% and leave it there for 4 years. How long will it take for the savings account to reach \$1200, assuming that the account is compounded continuously?

$A = Pe^{rt}$

$A(0) = P = 1000$

$A = 1000e^{.035t}$

$A(t) = 1200?$

$1200 = 1000e^{.035t}$

$$e^{.035t} = \frac{1200}{1000} = 1.2$$

$$.035t = \ln(1.2)$$

$$t = \frac{\ln(1.2)}{.035} \approx 5.20$$

over 5 years...